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BREAK-COLLAPSE METHOD FOR RESISTOR NETWORKS-RENORMALIZATION GROUP APPLICATIONS

by

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ABSTRACT

The break-collapse method recently introduced for the q-state Potts model is adapted for resistor networks. This method greatly simplifies the calculation of the conductance of an arbitrary two-terminal d-dimensional array of conductances, obviating the use of either Kirchhoff's laws or the star-triangle or similiar transformations. Related properties are discussed as well. An illustrative real-space renormalization-group treatment of the random resistor problem on the square lattice is presented; satisfactory results are obtained.

I - INTRODUCTION

A considerable amount of effort is being devoted to the study of random resistor networks (see Deutscher 1981 and references therein for connections with various physically interesting systems). A convenient way for quantitatively discussing such problems is through the real-space renormalization-group (RG) framework (Stinchcombe and Watson 1976, Straley 1977 a,b, Reynolds et al 1977, 1980, Kirpatrick 1977, Rosman and Shapiro 1977, Yeomans and Stinchcombe 1978, Bernasconi 1978, Kogut and Straley 1978, Lobb and Frank 1979, Lobb et al 1981, Kenkel and Straley 1982, Mujeeb and Stinchcombe 1982 and Derrida and Vannimenus 1982). Typically within this procedure the equivalent conductances of two-terminal arrays (or graphs) of conductors have to be calculated. The purpose of the present work is to show that the break-collapse method (BCM), recently introduced (Tsallis and Levy 1981) for calculating analogous percolation, Ising and Potts arrays, can be adapted to resistor systems, thus providing a simple way to calculate the above mentioned equivalent conductances. It avoids the use of Kirchhoff's laws (which lead to large systems of linear equations to be solved) and iterative star-triangle or similar transformations (whose implementation is greatly dependent on the topology of the particular array to be solved, a fact which might become a considerable disadvantage for complex arrays, e.g. $d=3$ ones).

In Sectin II we state (without proof) the resistor BCM and related properties; in Section III we illustrate its use for the central operational task within a RG calculation for the quenched random resistor network on the square lattice.

II - BREAK-COLLAPSE METHOD AND RELATED PROPERTIES

Let us denote the conductance of a single linear resistor as g ($g \equiv I/V$ where I and V are respectively the current and voltage across the resistor). If we have a parallel (series) array of two conductances g_1 and g_2, the equivalent conductance $g_p(g_s)$ is given by the well-known expression

$$g_p = g_1 + g_2 \qquad \text{(parallel)} \tag{1}$$

$$g_s = \frac{g_1 g_2}{g_1+g_2} \qquad \text{(series)} \tag{2}$$

This last expression can be rewritten in the parallel form, i.e.

$$g_s^D = g_1^D + g_2^D \tag{2'}$$

with

$$g_i^D \equiv g_0^2/g_i \qquad (i = 1,2,s) \tag{3}$$

where g_0 is an arbitrary reference conductance and D stands for dual (see Tsallis 1981 and Alcaraz and Tsallis 1982 for the analogous concepts in the context of the q-state Potts and Z(N) models).

Algorithms (1) and (2) enable the calculation of the equivalent conductance G of any two-terminal array as long as it is reducible through sequential series and parallel operations (examples of such graphs are presented in Figs. 1 (b,c)). However, this is not sufficient if the array is irreducible (e.g., Figs. 1(a,d,e)). Is is precisely this more general situation which can be solved by using the BCM. In particular, for an arbitrary two-terminal connected graph with bond conductances $\{g_i\}$, the conductance of the graph is given by $G(\{g_i\}) = N(\{g_i\})/D(\{g_i\})$ where both the numerator N and the denominator D are multilinear functions of the $\{g_i\}$ (Mason and Zimmermann 1960). If we now choose the j-th bond of the graph and "break" ("collapse") it, i.e. we impose $g_j = 0$ ($g_j \to \infty$), we have a new equivalent conductance G_j^b(G_j^c) given by

$$G_j^b(\{g_i\}') = N_j^b(\{g_i\}')/D_j^b(\{g_i\}') \tag{4}$$

$$G_j^c(\{g_i\}') = N_j^c(\{g_i\}')/D_j^c(\{g_i\}') \tag{5}$$

where the superscript b(c) refers to a quantity with the j-th bond broken (collapsed), and the set $\{g_i\}'$ excludes g_j. The multilinearity of both N and D leads to

$$N(\{g_i\}) = N_j^b(\{g_i\}') + g_j N_j^c(\{g_i\}') \tag{6}$$

and

$$D(\{g_i\}) = D_b^j(\{g_i\}') + g_j D_j^c(\{g_i\}') \tag{7}$$

The sequential use of these equations (together with Eqs. (1) and (2)) is what we call BCM; it greatly simplifies the calculation of conductances. Let us illustrate the procedure on the example of Fig. 1(a) (b = 2 Wheatstone bridge). After operating on the central bond of this figure, the broken and collapsed arrays respectively indicated in Fig. 1(b) and 1(c) are obtained. By exclusively using Eqs. (1) and (2), we obtain

$$G^b = \frac{N^b}{D^b} = \frac{g_1g_2g_3 + g_1g_2g_4 + g_1g_3g_4 + g_2g_3g_4}{g_1g_2 + g_1g_3 + g_2g_4 + g_3g_4} \tag{8}$$

and

$$G^c = \frac{N^c}{D^c} = \frac{g_1g_3 + g_2g_3 + g_1g_4 + g_2g_4}{g_1 + g_2 + g_3 + g_4} \tag{9}$$

therefore, by using Eqs. (6) and (7),

$$G = \frac{g_1g_2g_3 + g_1g_2g_4 + g_2g_3g_4 + g_1g_3g_4 + (g_1g_3 + g_2g_3 + g_1g_4 + g_2g_4)g_5}{g_1g_2 + g_1g_3 + g_2g_4 + g_3g_4 + (g_1 + g_2 + g_3 + g_4)g_5}$$

which is the well-known Wheatstone bridge result. The multilinear character of the numerator and denominator of (10) is written explicitely in terms of g_5.

As a corollary of Eqs. (6) and (7) we obtain a practical expression for the derivative, namely

$$\frac{\partial G}{\partial g_j} = \frac{N_j^c - GD_j^c}{D} \tag{11}$$

Another immediate consequence of Eqs. (6) and (7) is the following binominal-type form when two bonds in the conductance array are operated on,

$$G(\{g_i\} = \frac{N(\{g_i\})}{D(\{g_i\})}$$

$$= \frac{N^{bb}_{jk}(\{g_i\}'') + g_j N^{cb}_{jk}(\{g_i\}'') + g_k N^{bc}_{jk}(\{g_i\}'') + g_j g_k N^{cc}_{jk}(\{g_i\}'')}{D^{bb}_{jk}(\{g_i\}'') + g_j D^{cb}_{jk}(\{g_i\}'') + g_k D^{bc}_{jk}(\{g_i\}'') + g_j g_k N^{cc}_{jk}(\{g_i\}'')} \qquad (12)$$

where we have simultaneously operated on the j-th and k-th bonds (the set $\{g_i\}''$ excludes now both g_j and g_k); the extension of Eq. (12) to any number of operated bonds is straightforward.

Let us also quote an interesting property concerning planar arrays and duality. If we consider any pair of dual arrays (i.e. superimposable in such a way that each bond of one array crosses one and only bond of the other; see details in Essam and Fisher 1970 and references therein) and denote by G and G^D, respectively, their equivalent conductances, we verify that

$$G^D(\{g_i^D\}) = [G(\{g_i\})]^D \qquad (13)$$

i.e.

$$G^D(\{g_0^2/g_i\}) = g_0^2/G(\{g_i\}) \qquad (13')$$

This property can be illustrated on the pair of arrays indicated in Figs. 1 (d,e).

Finally, we note that the break-collapse properties of the resistor network follow quite closely those presented in

Tsallis and Levy 1981 for the q-state Potts model $\mathcal{H} = -qJ\sum_{i,j}\delta_{\sigma_i,\sigma_j}$, $\sigma_i = 1,2,\ldots,q$). This is of course not surprising since the resistor problem can be obtained as the $T \to 0$ limit of the $q \to 0$ Potts model (Stephen 1976, Lubensky 1978 and Wu 1982). To be more precise, if we associate with each bond the transmissivity (Tsallis 1981 and references therein)

$$\bar{t} \equiv (1-e^{-qJ/k_BT})/[1+(q-1)e^{-qJ/k_BT}] \quad , \tag{14}$$

the parallel and series algorithms are given by

$$\bar{t}_p = \frac{\bar{t}_1 + \bar{t}_2 + (q-2)\bar{t}_1\bar{t}_2}{1+(q-1)\bar{t}_1\bar{t}_2} \xrightarrow{(q\to 0)} \frac{\bar{t}_1 + \bar{t}_2 - 2\bar{t}_1\bar{t}_2}{1 - \bar{t}_1\bar{t}_2} \tag{15}$$

and

$$\bar{t}_s = \bar{t}_1\bar{t}_2 \tag{16}$$

By introducing $\bar{t}_i = 1 - g_0/g_i$ it is straightforward to verify that, in the limit $g_0/g_i \to 0$ (i.e. $\bar{t}_i \to 1$, hence $T \to 0$), Eq. (15) (Eq. (16)) leads to Eq. (1) (Eq. (2)).

III - RENORMALIZATION GROUP APPLICATION: RANDOM RESISTOR NETWORK

Consider a square lattice with the following independent (quenched model) conductance probability law for each bond

$$P(g) = (1-p)\delta(g-g_1) + p\delta(g-g_2) \qquad (0 \le g_1 \le g_2) \tag{17}$$

The average conductivity $\sigma(g_1,g_2;p)$ has the following properties: (i) $\sigma(0,g_2;p) \propto (p-p_c)^t$ and $\sigma(g_1,\infty;p) \propto (p_c-p)^{-s}$ as $p \to p_c = 1/2$, where the critical exponents satisfy, at $d=2$, $t=s$; (Straley 1977 (b)); (ii) $[\sigma(0,g_2;1)]^{-1}[d\sigma(0,g_2;p)/dp]_{p=1} = [\sigma(g_1,\infty,0]^{-1}[d\sigma(g_1,\infty;p)/dp]_{p=0} = 2$ (Bernasconi 1978); (iii) because of the square-lattice self-duality, $g(g_1,g_2;p)g(g_1,g_2;1-p) = g_1g_2$ (Straley 1977 (b)).

We intend to approximately calculate $\sigma(0,g_2;p)$ (resistor-insulator mixture) and $\sigma(g_1,\infty;p)$ (resistor-superconductor mixture) within the RG framework by renormalizing the $b=2$ Wheatstone bridge (see Fig. 1 (a)) into a single bond (it is now well known that both graphs being self-dual, this choice is an appropriate one for the square lattice). The renormalized distribution law $P_H(g)$ associated with the $b=2$ graph is, through use of Eq. (10), given by

$$\begin{aligned} P_H(g) &= [(1-p)^5 + (1-p)^4 p]\,\delta(g-g_1) \\ &+ 4\ (1-p)^4 p\ \delta\left(g - \frac{3g_1^2 + 5g_1g_2}{5g_1 + 3g_2}\right) \\ &+ 2\ (1-p)^3 p^2 \delta\left(g - \frac{g_1^2 + 4g_1g_2 + 3g_2^2}{2g_1 + 6g_2}\right) \\ &+ 2\ (1-p)^3 p^2 \delta\left(g - \frac{2g_1g_2}{g_1+g_2}\right) \\ &+ 4\ (1-p)^3 p^2 \delta\left(g - \frac{g_1^3 + 5g_1^2g_2 + 2g_1g_2^2}{2g_1^2 + 5g_1g_2 + g_2^2}\right) \end{aligned}$$

$$+ 2\ (1-p)^3 p^2 \delta\left(g - \frac{g_1^3 + 4g_1^2 g_2 + 3g_1 g_2^2}{3g_1^2 + 4g_1 g_2 + g_2^2}\right)$$

$$+ 2\ (1-p)^2 p^3 \delta\left(g - \frac{g_2^3 + 4g_2^2 g_1 + 3g_2 g_1^2}{3g_2^2 + 4g_2 g_1 + g_1^2}\right)$$

$$+ 4\ (1-p)^2 p^3 \delta\left(g - \frac{g_2^3 + 5g_2^2 g_1 + 2g_2 g_1^2}{2g_2^2 + 5g_2 g_1 + g_1^2}\right)$$

$$+ 2\ (1-p)^2 p^3 \delta\left(g - \frac{2g_2 g_1}{g_2 + g_1}\right)$$

$$+ 2\ (1-p)^2 p^3 \delta\left(g - \frac{g_2^2 + 4g_2 g_1 + 3g_1^2}{2g_2 + 6g_1}\right)$$

$$+ 4\ (1-p) p^4 \delta\left(g - \frac{3g_2^2 + 5g_2 g_1}{5g_2 + 3g_1}\right)$$

$$+ \left[(1-p) p^4 + p^5\right] \delta(g - g_2) \qquad (18)$$

Following along the lines of previous works (Bernasconi 1978 and Mujeeb and Stinchcombe 1928 and references therein), we shall approximate this distribution law by a <u>binary</u> one, namely

$$P'(g) = (1-p')\delta(g-g_1') + p'\delta(g-g_2') \qquad (19)$$

where p_1', g_1' and g_2' are the renormalized parameters. In the general (p,g_1,g_2)-problem we should need <u>three</u> RG recursion relations to calculate $\sigma(g_1,g_2;p)$, but for the particular cases of interest, namely $g_1 = 0$, $\forall g_2$ and $g_2^{-1} = 0$, $\forall g_1$, <u>two</u> RG recursion relations suffice. In both cases there is a δ-function (at $g = 0$ for the conductor-insulator case, and at $g^{-1} = 0$ for the conductor-superconductor case) whose position remains invariant under renormalization. Following previous works (Stinchcombe and Watson 1976, Yeomans and Stinchcombe 1978, Mujeeb and Stinchcombe 1982) we shall use the weight of this particular δ-function in order to obtain the recursion relation for the occupancy probability. We have, for the both $g_1 = 0$ and $g_2^{-1} = 0$ cases,

$$p' = 2p^2 + 2p^3 - 5p^4 + 2p^5 \qquad (20)$$

first obtained by Reynolds et al 1977 for bond-percolation on square lattice. The second recursion relation is provided by

$$\langle f(g) \rangle_{p'} = \langle f(g) \rangle_{p_H} \qquad (21)$$

where the choice of the function f(g), appears to by arbitrary. Stinchcombe and Watson 1976 and Mujeeb and Stinchcombe 1982 have used $f(g) = g$ for the $g_1 = 0$ problem; Yeomans and Stinchcombe 1978 used $f(g) = g, \ln g$ for the same problem; Bernasconi 1978 has used $f(g) = \ln g$ for the full (p,g_1,g_2)-problem; Lobb and Frank 1979 have used $f(g) = g, 1/g, \ln g$ for the $g_1 = 0$ case. Herein we use $f(g) = g$ for the $g_1 = 0$ problem, $f(g) = 1/g$ for the $g_2^{-1} = 0$ one, and finally

$$f(g) = S(g) \equiv \frac{g}{g + g_0} \qquad (22)$$

for both problems ($g_0 \equiv$ reference conductance). The function S(g) is the simplest one which monotonically varies from 0 to 1 when g increases from 0 to ∞, and which satisfies the property

$$S^D(g) \equiv S(g^D) = S(g_0^2/g) = 1 - S(g) \tag{23}$$

The use of a variable satisfying this type of property (probability-like transformation under duality) has proved to be extremely useful in the treatment of Ising and Potts problems (Levy et al 1980, Tsallis 1981, Tsallis and de Magalhães 1981, de Magalhães and Tsallis 1981, Alcaraz and Tsallis 1982, de Oliveira and Tsallis 1982, de Magalhães et al 1982). Note also that, in the limit $g \to 0 (g\to\infty)$, $S(g) \sim g/g_0 (S(g) \sim 1-g_0/g)$.

<u>Conductor-Insulator Problem</u>: (g-RG framework)

Equation (21) with $f(g) = g$ provides

$$p' g_2' = (p^2 + \frac{2}{3} p^3 - \frac{14}{15} p^4 + \frac{4}{15} p^5) g_2 \tag{24}$$

which together with Eq. (20) provides closed recursion relations. These equations provide the following fixed points: $(p, g_2^{-1}) = (0,0)$ (semi-stable), $(0,\infty)$ (fully stable), $(1/2,0)$ (fully unstable), $(1/2,\infty)$ (semi-stable), and $(1, g_2^{-1})$ (fully stable $\forall\ g_2^{-1}$). The Jacobian matrix $\partial(p', 1/g_2')/\partial(p, 1/g_2)$ equals for $(p, g_2^{-1}) = (1/2, 0)$,

$$\begin{pmatrix} 13/8 & 0 \\ 0 & 30/17 \end{pmatrix} \tag{25}$$

and, for $(p, g_2^{-1}) = (1,1)$,

$$\begin{pmatrix} 0 & 0 \\ -8/5 & 1 \end{pmatrix} \tag{25'}$$

Consequently $\nu = \ln 2/\ln (13/8) \simeq 1.428$ (Reynolds et al 1977 compared to the exact den Nijs 1979 value of 4/3). Furthermore, by taking into account that $\sigma(g_1,g_2;p)$ scales as $1/g_2$ (Stinchcombe and Watson 1976), we obtain $t = \ln(30/17)/\ln(13/8) \simeq 1.170$ (compared to estimates which range from 1.0 to 1.43; see Gefen et al 1981 for a discussion on the present situation) and $[\sigma(0,g_2;1)]^{-1}[d\sigma(0,g_2;p)/dp]_{p=1} = 8/5$ (to be compared with the exact value 2). The full p-dependence of $\sigma(0,g_2;p)$ is presented in Fig. 2.

<u>Conductor-Superconductor Problem:</u> (g^{-1}-RG framework)

Equation (21) and $f(g) = 1/g$ provides

$$\frac{1-p'}{g_1'} = \frac{(1-p)^2 + \frac{2}{3}(1-p)^3 - \frac{14}{15}(1-p)^4 + \frac{4}{15}(1-p)^5}{g_1} \tag{26}$$

Notice that through the transformation $p \to 1-p$ and $g_2 \to 1/g_1$, Eqs. (24) and (26) are the same; this property should hold for <u>all</u> b-sized "Wheatstone bridges". The following fixed points are obtained: $(p,g_1^{-1}) = (0,g_1^{-1})$ (fully stable, $\forall\, g_1^{-1}$), $(1/2,0)$ (semi-stable), $(1/2,\infty)$ (fully unstable), $(1,0)$ (fully stable), and $(1,\infty)$ (semi-stable). The Jacobian matrix $\partial(p',1/g_1')/\partial(p,1/g_1)$ equals, for $(p,g_1^{-1}) = (1/2,\infty)$

$$\begin{pmatrix} 13/8 & 0 \\ 0 & 17/30 \end{pmatrix} \tag{27}$$

and, for $(p,g_1^{-1}) = (0,1)$, is the same as in Eq. (25'). Consequently ν is the same as before, $s = t \simeq 1.170$ and $[\sigma(g_1,\infty;0)]^{-1}[d\sigma(g_1,\infty;p)/dp]_{p=0} = 8/5$ (the exact value being 2). The p-dependence of $\sigma(g_1,\infty;p)$ is presented in Fig. 2.

Conductor-Insulator and -Superconductor Problems: (S-RG framework)

Equation (21) with $f(g) = S(g) \equiv g/(g+g_0)$ provides for the $g_1 = 0$ case (where we choose $g_0 = g_2$)

$$\frac{1}{g_2'} = \left[\frac{p'}{\frac{2}{3}p^2 + \frac{p^3}{2} - p^4 + \frac{p^5}{3}}\right] \frac{1}{g_2} \qquad (28)$$

and, for the $g_2^{-1} = 0$ case (where we choose $g_0 = g_1$)

$$g_1' = \left[\frac{1-p'}{\frac{2}{3}(1-p)^2 + \frac{(1-p)^3}{2} - (1-p)^4 + \frac{(1-p)^5}{3}} - 1\right] g_1 \qquad (29)$$

which together with Eq. (20) provide closed recursion relations. As before, through the transformation $(p,g_2) \to (1-p,1/g_1)$, Eqs.(28) and (29) are the same; this property should hold for all b-sized "Wheatstone bridges". The fixed points are, for the $g_1 = 0$ case ($g_2^{-1} = 0$ case) the same as obtained in the g - RG(g^{-1}-RG). Analysis of the relevant Jacobians provides: (i) the same value of ν as before; (ii) $t = s = \ln(31/17)/\ln(13/8) \simeq 1.237$, to be compared to the recent accurate estimate of $t = 1.28 \pm 0.03$ (Derrida and Vannimenus 1982); (iii) $[\sigma(0,g_2;1]^{-1}[d\sigma(0,g_2;p)/dp]_{p=1} = [\sigma(g_1,\infty;0]^{-1}[d\sigma(g_1,\infty;p)/dp]_{p=0} = 2$, which is the exact value. The full p-dependences are presented in Fig. 2.

IV - CONCLUSION

In conclusion, the calculation of any two-terminal array of conductances can be greatly simplified by performing trivial topological operations (bond "breaking" and "collapsing") and applying the algorithm described herein (denoted by break-col lapse method). It avoids the use of Kirchhoff's laws and of non-linear transformations such as the star-triangle mapping. Although the use of the break-collapse method has been exhibited on the standard Wheatstone bridge, different and larger arrays can be solved as well. The study of a few d-dimensional anisotropic random resistor problems is presently in progress and will be published elsewhere.

As an illustration of this method, we have constructed, along the lines of previous works, three different and quite simple real-space renormalization-groups (noted g-RG, g^{-1} - RG and S - RG) to study the full concentration-dependence of the mean conductivity of the quenched random resistor problem on the square lattice in both limiting situations where the normal resistors are mixed either with insulating ($g_1 = 0$) of with superconducting ($g_2^{-1} = 0$) bonds. All three RG's lead to satisfactory results. The S-RG (where mean values are taken on the variable $S(g) \equiv g/(g+g_0)$) excellent results are obtained in spite of the small RG cluster that has been used. It enables the "single-shot" treatment of <u>both</u> $g_1 = 0$ and $g_2^{-1} = 0$ cases, and provides, besides the exact critical probability $p_c = 1/2$, the <u>exact limiting slopes</u> (at $p = 0$ and $p = 1$) as well as a critical exponent $t = s \simeq 1.24$ which compares well with a quite accurate recent result by Derrida and Vannimenus 1982, namely 1.28 ± 0.03.

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CAPTION FOR FIGURES

FIG. 1 - Two-terminal planar arrays of conductances $\{g_i\}$ (o and ● respectively denote terminal and internal nodes) (a) self-dual $b = 2$ Wheatstone bridge; (b) and (c) are respectively "broken" and "collapsed" graphs of (a); (d) and (e) constitute a dual pair of graphs.

FIG. 2 - Concentration-dependence of the square-lattice mean conductivity in both limiting cases $g_1 = 0$ (resistor-insulator mixture) and $g_2^{-1} = 0$ (resistor-superconductor mixture).

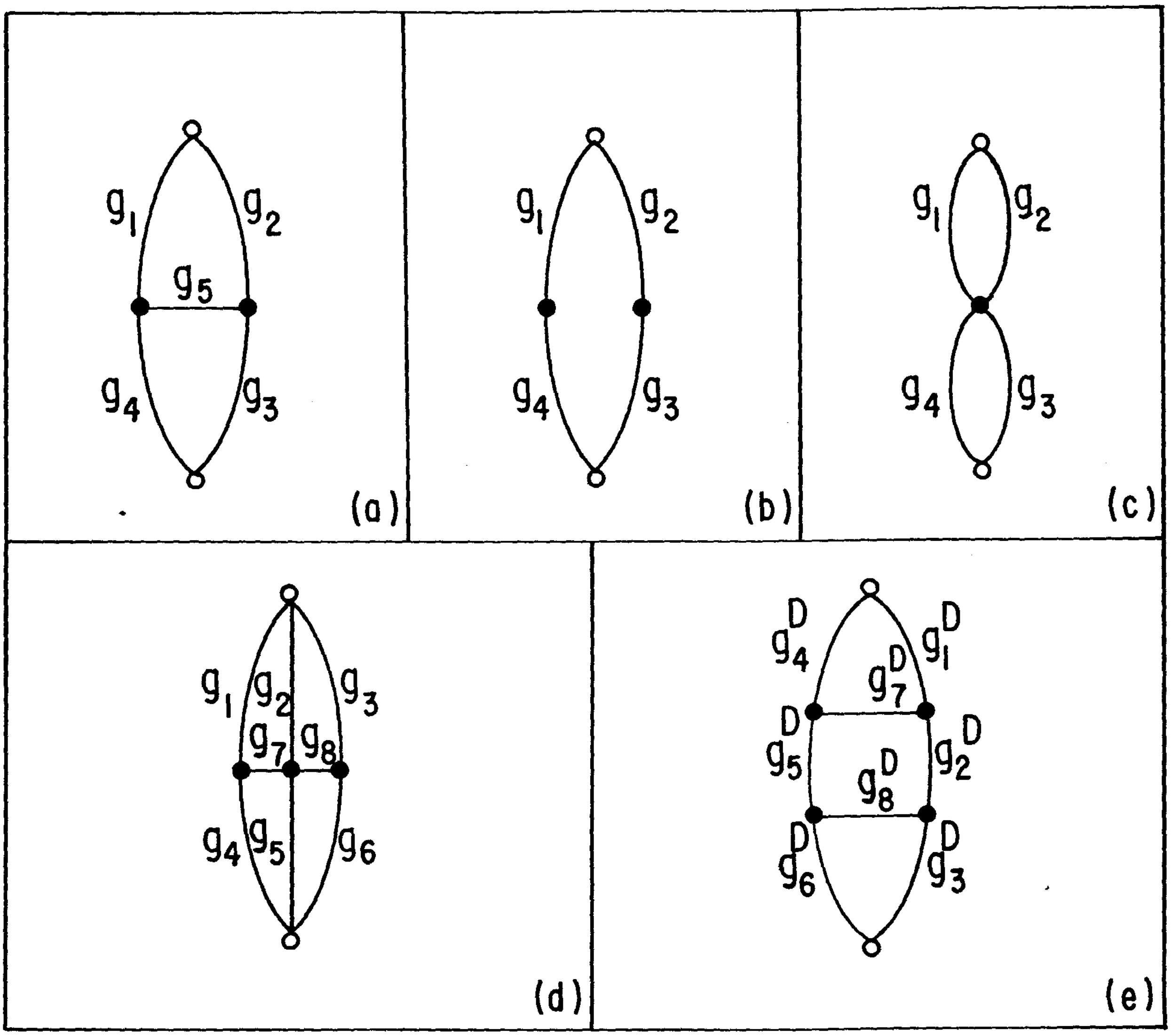
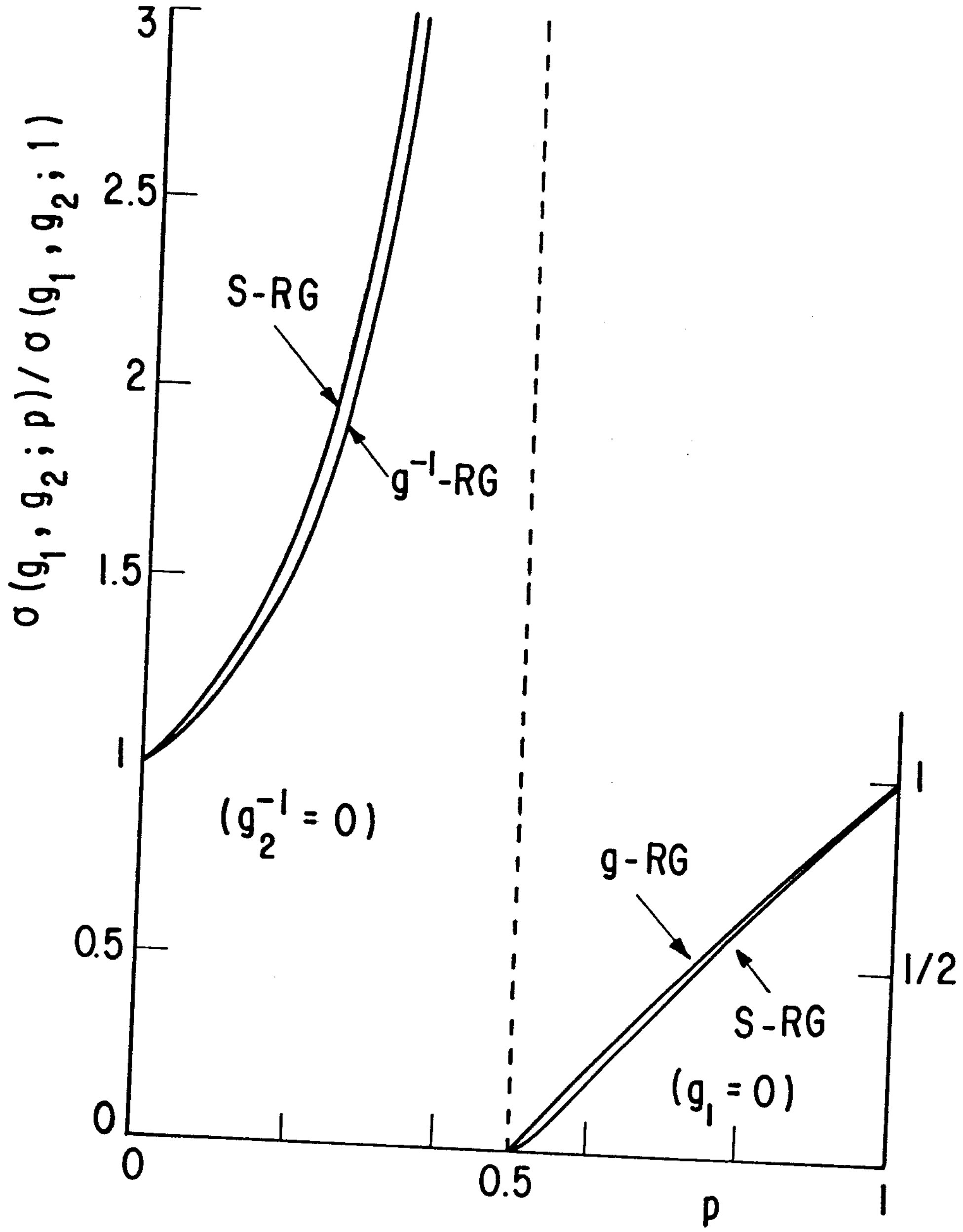

g1 g2 g5 g4 g3 (a)
g1 g2 g4 g3 (b)
g1 g2 g4 g3 (c)
g1 g2 g3 g7 g8 g4 g5 g6 (d)
g4D g1D g7D g5D g2D g8D g6D g3D (e)

FIG. 1



3
2.5
2
1.5
1
0.5
0
σ (g₁ , g₂ ; p) / σ (g₁ , g₂ ; 1)
S-RG
g⁻¹-RG
(g₂⁻¹ = 0)
g-RG
S-RG
(g₁ = 0)
1
1/2
0
0.5
1
p

FIG. 2