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ABSTRACT

We give a naïve perturbative proof of the existence of infrared renormalons for large distance asymptotically free theories. We argue that the extension of the result for small distance asymptotically free theories is not obvious. Indeed we do not find infrared renormalons for QCD.

Key-words: Infrared; Renormalons; QCD.

Infrared divergences for a theory containing massless particles appear in the Borel transformed Green functions as singularities on the real axis of the Borel variable b (infrared renormalons). This follows from some assumptions on the small momentum behaviour of the effective coupling constant $\bar{g}(\lambda,g)$ of the theory, or in the context of an "infrared renormalization" from the introduction of multilocal counterterms[1,2,3,4].

In this note we give a naïve perturbative derivation of that property, valid for large distance (infrared) asymptotically free theories. The extension of the result to ultraviolet asymptotically free theories, as non-abelian gauge field theories, presumes that $\bar{g}(\lambda,g)$ has the same functional dependence on λ and g as $\lambda \to 0$ no matter the particular regime under investigation (theory asymptotically free at large or at small distances). This assumption seems to be in contradiction with some results from lattice QCD[5] and from the formal theory of the QCD effective coupling constant as developped in ref. 6; if we take $\bar{g}(\lambda,g)$ for $\lambda \sim 0$ from the results of ref. 6 we do not find infrared renormalons.

In the following we use Euclidean metric throughout; the Minkowskian case may be recovered by a Wick rotation. Let us consider a N-point insertion in a perturbatively defined Green function, as depicted in Fig. 1. We may write, ignoring for simplicity spin and tensor indices,

$$G(g) = \int \frac{d^4p_1 \ldots d^4p_{N-1}}{p_1^2 \ldots p_N^2} E(g)\Gamma_N(g), \tag{1}$$

where E(g) is diagramatically disjoint to the N-point insertion, and Γ_N is the truncated N-point Green function. Using the convolution formula,

$$B[f_1\dots f_n] = \int_0^b db_1\dots\int_0^{b} db_n B_{f_1}(b_1)\dots B_{f_n}(b_n)\,\delta(b-b_1-\dots-b_n) \quad (2)$$

where $B_{f_i}(b_i)$ stands for "Borel transform of f_i", the Borel transform of eq. (1) is given by,

$$B[G] = \int\dots\int \frac{d^4p_1\dots d^4p_{N-1}}{p_1^2\dots p_N^2}\int_0^b db_1 B_E(b_1)B_\Gamma(b-b_1) \quad (3)$$

For large distance asymptotically free theories, the effective coupling constant is small for small values of the renormalization group (RG) scaling parameter λ, and is given in the leading logarithm approximation by,

$$\bar{g}(\lambda,g) = \frac{g}{1-g\beta_2\ell n\lambda} \quad (4)$$

where $\beta_2 > 0$ is the first non-zero coefficient in the perturbative expansion of the β-function, $\beta(g) = \beta_2 g^2 + \theta(g^3)$. We perform for Γ_N in eq. (1) the usual RG scale transformation. Then making a perturbative expansion in $\bar{g}$ we have, corresponding to a particular Feynman graph with L loops and V vertices, a contribution to the Borel transform (3), as follows:

$$\int \ldots \int \frac{d^4 p_1 \ldots d^4 p_{N-1}}{p_1^2 \ldots p_N^2} \int_0^b db_1 B_E(b_1) \left(\frac{p_i}{q_i}\right)^{4-N} e^{d_A (b-b_1)^{V-1}} \left(\frac{p_i}{q_i}\right)^{(b-b_1)\beta_2} \times$$

$$\times \int \ldots \int d^4 k_1 \ldots d^4 k_L \, F(q,k) , \qquad (5)$$

where we have used the convolution theorem (2), the q_i's, $i = 1,\ldots N-1$, are fixed momenta, and d_A is the anomalous dimension, $d_A = \int_0^\lambda d\lambda' \gamma[\bar{g}(\lambda', g)]$.

Naïve power counting shows that "superficial" infrared divergences (all the p_i's going simultaneously to zero) of $B[G]$ manifest themselves as singular varieties on the real (b, b_1) plane, or as singularities on the real b axis (infrared renormalons)-see (Fig. 2). We have superficial infrared convergence for

$$b - b_1 < \frac{-N}{\beta_2} . \qquad (6)$$

Now, as it was already remarked[2], the essential point is that the existence of infrared renormalons is due to the peculiar functional dependence of $\bar{g}$ on λ and g, which allows the self consistent reproduction of the factor $\lambda^{(b-b_1)\beta_2}$ in the Borel transform of $\bar{g}^V$, through the convolution theorem.

In order to extend the result to the case of ultraviolet asymptotically free theories ($\beta_2 < 0$), it is currently assumed that eq. (4) remains valid even if $\bar{g}$ itself is not small, which leads to infrared renormalons on the real positive b-axis. For QCD this assumption seems not to be completely justifiable. Lattice calculations[5], or more recently, a formal work[6] on the infrared behaviour of $\bar{g}$ for QCD, gives a power dependence of $\bar{g}(\lambda,g)$ on λ as $\lambda \to 0$. Actually, in the case of small g, one easily deduces, from the results of ref. 6, that,

$$\bar{g}(\lambda,g) \underset{\lambda\to 0}{\sim} \left[\exp\left(-\frac{k}{2\beta_3 g^2}\right)\right]\left(\frac{1}{\lambda}\right)^k \qquad (7)$$

where k is a positive number, and $\beta_3 > 0$ comes from the perturbative expansion for $\beta(g)$, $\beta(g) = -\beta_3 g^2 + O(g^5)$.

To proceed, we take, for example, a 3-point insertion (3-gluon vertex) in a perturbatively defined (in g) Green function (Fig. 3). Since eq. (7) has no expansion around $g = 0$, the standard Borel transform is not defined. Instead we use the modified Borel transform[7]

$$B'_G(b) = \int_0^a \frac{dg}{g^2} \exp(b/g) G(g) \quad , \qquad (8)$$

where a is some (small) constant, and we get for the Green function with a 3-gluon vertex insertion,

$$B'[G] = \int \frac{d^4p_1 d^4p_2}{p_1^2 p_2^2 p_3^2} \left(\frac{q_i}{p_i}\right)^k \Gamma_3(p) \int_0^b db_1 B_E(b_1) \psi(b-b_1) \quad , \qquad (9)$$

with $\psi(b-b_1) = \int_0^a \frac{dg}{g^2} \exp\left(\frac{b-b_1}{g} - \frac{k}{2\beta_3 g^2}\right)$.

We see from eq. (9) that the Borel variables b, b_1, are not linked to the infrared ($\{p_i \to 0\}$) behaviour of $B'[G]$. The use of the modified Borel transform and the power dependence of $\bar{g}$ on λ (eq. (7)) make the p's and the b, b_1 dependence in eq.

(9) to be completely factorized, leading to the (at least apparently) non-existence of infrared renormalons in QCD.

This conclusion relies on the validity of formula (7) which is obtained from an exact formal equation of ref. [6],

$$\frac{1}{\bar{g}^2(\lambda)} = \frac{1}{g^2} + 2\beta_3 \ln\lambda - a\ln\frac{\bar{g}^2(\lambda)}{g^2} + f(\bar{g}^2) - f(g^2), \tag{10}$$

taking for respectively large $\bar{g}$ and small g the appropriate behaviour of the (otherwise unknown) function f,

$$f(\bar{g}) \sim c\ln\bar{g}^2$$
$$f(g) \sim \sum_{k=1}^{\infty} \frac{c_j}{j}(g^2)^j . \tag{11}$$

The numbers a, β_3 and c are such that $k = \frac{c-a}{\beta_3} > 0$. Taking in (10) only the leading terms in $\bar{g}$ and g leads to eq. (7).

In other words, the existence of infrared renormalons relies on the currently assumed behaviour of the Borel transformed effective coupling constant, $\bar{b}(b,\lambda)$, for small λ,

$$\bar{b}(b,\lambda) \sim \lambda^{b\beta_3} \quad .$$

For $\bar{g}$ as given by (7) or more generally with a power dependence[5] on λ, this is not the case.

FIGURE CAPTIONS

FIG. 1. N-point insertion in a Green function. Wavy lines are used to represent massless propagators.

FIG. 2. Infrared divergences of a Borel transformed Green function. The hatched region corresponds to superficial infrared convergence in the case of a N-point insertion.

FIG. 3. 3-gluon insertion in a Green-function.

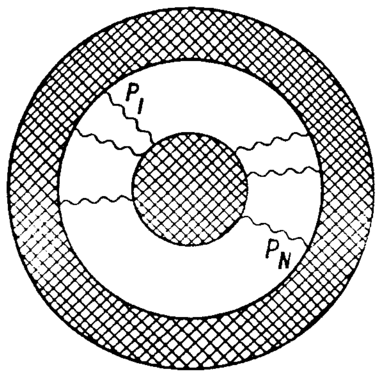

FIG. 1

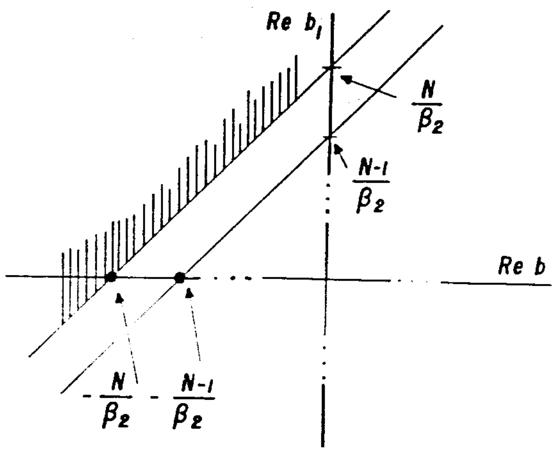

FIG. 2

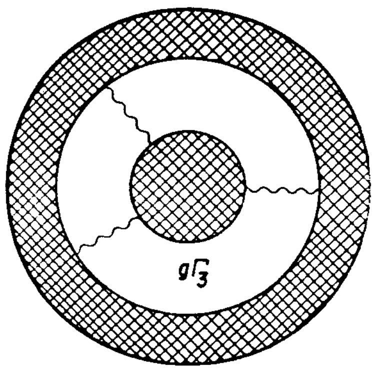

FIG. 3

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