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A $\lambda\phi^4$ Geometrodynamical field theory of Quantum-Gravity as a Dynamics of self-avoiding universes

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Abstract

We introduce in terms of Astekar Variables, a self-avoiding Quantum Universes representation for a $\lambda\phi^4$ Third Quantized Geometrodynamical Field Theory of Gravitation

Introduction

One of the most important not solved problems in Geometrodynamics of Third Quantization for Einstein Gravitation Theory is related to a description of process involving joining and splitting of Universes. I address in this paper a formal Path-Integral solution for this problem by proposing a $\lambda\phi^4$ - Field Theory of Universes amenable to a description in terms of a Dynamics of Self-Avoiding Quantum Universes. My framework is a suitable generalization for quantum gravity processes of a similar geometrical procedure used years ago by Symanzik in his non-perturbative self-avoiding loop representation for the usual $\lambda\phi^4$ - Point Like Field Theory ([2]) and by myself in the proposed Self-Avoiding String Field Theory of ref. ([3]). In section 2 I present my proposed Geometrodynamical Propagator in terms of Astekar-Sen variables. In section 3 I present mine many universe path integral. Finally, in appendix A, B, I clarify some remarks presented in the bulk of this paper.

The Wheeler - De Witt Geometrodynamical Propagator

The starting point in Wheeler - De Witt Geometrodynamics is the Probability Amplitude for metrics propagation in a cylindrical Space Time $R^3 \times [0, T]$ ([1]), the so called Wheeler Universe

$$G[{}^3g^{IN};{}^3g^{OUT}] = \int_{{}^3g^{IN}}^{{}^3g^{OUT}} d\mu[h_{\mu\nu}]\exp[-S(h_{\mu\nu})] \tag{1}$$

where the integration over the four metrics Functional Space on the cylinder $R^3 \times [0,T]$ is implemented with the Boundary conditions that the metric field $h_{\mu\nu}(x,t)$ induces on the Cylinder Boundaries the Classically Observed metrics ${}^3g^{IN}(x)$ and ${}^3g^{OUT}(x)$ respectivelly. The Covariant Functional measure averaged with the Einstein action $S(h_{\mu\nu}) = \int_{R^3\times[0,T]} d^3x\,dt\left(\sqrt{g}\,R(g)\right)$ is given explicitly in ref. ([4]).

Unfortunatelly the use of eq.(1) in terms of metrics variables is difficulted by the "Conformal Factor Problem" in the Euclidean Framework (see appendix A for this result). In order to overcome such difficulty I follow ref. [5] by using from the begining, the Astekar Variables to describe the Geometrodynamical Propagation.

Let me, thus, consider Einstein Gravitation Theory Parametrized by the $SU(2)$ Three-Dimensional Astekar - Sen connection $A^a_\mu(x,t)$ associated to the Projected Spin Connection on the Space - Time Three - Dimensional Boundaries ([5]).

$$A^{a,IN}_\mu(x) = -i\omega^{0a}_\mu(x,0) + \frac{1}{2}\epsilon^a_{bi}\omega^{bc}_\mu(x,0) \tag{2}$$

$$A^{a,OUT}_\mu(x) = -i\omega^{0a}_\mu(x,T) + \frac{1}{2}\epsilon^a_{bi}\omega^{bc}_\mu(x,T) \tag{3}$$

An appropriate action on the Functional Space of Astekar-Sen connections is proposed by myself to be given explicitly by a slight modification of that proposed in ref.([5]). My proposed action is given by a covariant σ-model like Path Integral with a scalar intrinsic field $E(x,t)$ on $R^3 \times [0,T]$. Here μ^2 denotes a scalar "mass" parameter which my be vanishing (massless Wheeler-Universes).

$$\begin{aligned} S_{\mu^2}[A^a_\mu(x,t),E(x,t)] &= \\ &= \frac{1}{16\pi G}\int_0^T dt\int_{R^3} d^3x (E(x,t))^{-1}\left[\left(\frac{\partial}{\partial t}A_{a,\mu}\right)G^{\mu a,\nu b}[A]\left(\frac{\partial}{\partial t}\right)A_{b,\nu}\right] + \\ &+ \mu^2\int_0^T dt\int_{R^3} d^3x\, E(x,t) \end{aligned}$$

where the invariant metric on the Wheeler - De Witt Super Space of Astekar Connections is given by

$$G^{\mu a,\nu b}[A] = (b(A))^{-1}(J^{\mu a}J^{\nu b} - J^{\mu b}J^{\nu a})(A) \tag{5}$$

with

$$J^{\mu a}(A) = \frac{1}{2}\epsilon^{\mu\alpha\rho}F^{a}_{\alpha\rho}(A) \tag{6}$$

and

$$b(A) = \det(J(A))_{\mu;a} \tag{7}$$

My proposed Quantum Geometrodynamical Propagator will be given now by the following Formal Path Integral

$$G[A^{IN}, A^{OUT}] = \int_{A^a_\mu(x,0)=A^{a,IN}_\mu(x);\, A^a_\mu(x,T)} d^{INV}(A^a_\mu(x,t)) \times \int \left(\Pi_{(x,T)\in R^3\times[0,T]}(dE(x,t))\right) \times \\ \times \exp\left(-S_{\mu^2}[A^a_\mu; E]\right) \tag{8}$$

where the invariant functional measure over the Astekar -Sen Connections is given by the invariant functional metric ([5])

$$dS^2_{INV} = \int_{R^3\times[0,T]} d^3x\,dt[\delta A_{\mu,a}\,\tau^{\mu a,\nu b}[A](\delta A_{\nu,b})](x,t) \tag{9}$$

In order to show that the Geometrodynamical Propagator eq.(8) satisfies the Wheeler - De Witt Equation, I follow my procedure to deduce Functional Wave Equations from Geometrical Path Integrals by exploiting the effective Functional Translation Invariance on the Functional Space of the Scalar Intrinsic metrics $(E(x,t))$ at the Boundary $t \to 0^+$ (see ref. [6]). As a consequence, have that the propagator eq.(8) satisfies the Wheeler - De Witt Equation with the "mass" parameter μ^2.

$$\epsilon_{abc}F^c_{\mu\nu}(A^{IN})(x)\frac{\delta^2}{\delta A^{a,IN}_\mu(x)\delta A^{b,IN}_\nu(x)}G(A^{IN};A^{OUT}) = \\ = -\mu^2 G(A^{IN};A^{OUT}) + \delta^{(F)}(A^{IN,a}_\mu - A^{OUT,a}_\nu) \tag{10}$$

where we have used the Euclidean commutation relation (which does not have $\left(= \sqrt{-1}\right)$ to obtain eq.(10).

$$\left[\left(\frac{G^{\mu a,\nu b}[A]}{E} \times \left(\frac{\partial}{\partial t}A_{\nu,b}\right)\right)(x,t); A_{\mu,a}(x',t)\right] = \delta^{(3)}(x-x') \tag{11}$$

It is instructive to remark that classical Canonical Momentum written in eq.(11) is given by the

Schröringer Functional Representation in the quantum mechanical equation (10).

$$\Pi^{\mu a}(x) = \frac{\delta}{\delta A_{\mu}^{a,IN}(x)} \tag{12}$$

It is worth point out that the usual Covariant Polyakov Path Integral for Klein - Gordon Particles my be considered as the 0-dimensional reduction of the Geometrodynamical Propagator eq. (8) (see eq. 2.1: second reference of [4]).
At this point I remark that by fixing the Gauge $E(x,t) = \frac{E}{\mu^2}$, with μ^2 the "mass" parameter, we arrive at the analogous Proper-Time Schwinger Representation for this Geometrodynamical Quantum Gravity propagator

$$G_{\tilde{E}}[A^{IN}, A^{OUT}] = \int_0^{\infty} dt\, e^{-(\tilde{E}t)} \times \int d^{INV}(A_{\mu}^{a}) \times \exp(-S[A_{\mu}^{a}(x,t)]) \tag{13}$$

where $E = (E, \mu^2) \times vol(R^3)$ is the renormalized mass parameter in the Schwinger Proper-Time representation.
In the next section I will use the Proper-Time dependent Propagator given below, as usually is done in the Symanzik's Loop Space approach for Quantum Field Theories ([2]) to write a Third-Quantized Theory for Gravitation Einstein theory in terms of Astekar-Sen variables.

$$G[A^{IN}, A^{OUT}; T] = \int_{A_a^{\mu}(x,0)=A_{\mu}^{a,IN}(x);\, A_{\mu}^{a}(a,T) A_{\mu}^{a,OUT}(x)} d^{INV}(A_{\mu}^{a}) \times$$

$$\times \exp\left\{-\frac{1}{16\pi G}\int_0^T dt \int_{R^3} d^3x \left[\left(\frac{\partial}{\partial t}A_{a,\mu}\right) \times G^{\mu a, \nu b}[A]\left(\frac{\partial}{\partial t}A_{\nu,b}\right)\right](x,t)\right\} \tag{14}$$

Unfortunatelly exactly solutions for eq.(10) with $\mu^2 \neq 0$ or eq.(14) were not found yet. However its σ –like structure and $SU(2)$ Gauge Invariance may afford to truncated aproximate solutions as usually done for the Wheeler-De Witt equations by means of the Mini-Super Space Ansatz. Finally let me comment on the introduction of a Quantized Matter Field represented by a massless field $\varphi(x,t)$ on the Space Time.
By considering the effect of the introduction of this quantized field as a fluctuation on the Geometrodynamical Propagator eq.(8) one should consider the following functional representing the interaction of this massless quantized matter and the Astekar-Sen connection as one can easily see by making $E(x,t)$ variations

$$S_{INT}[A_\mu^a;E,\varphi] = \int_0^T dt \int_{R^3} d^3x \Big\{ \Big[\varphi\Big(-\frac{\partial}{\partial t}\Big(E\frac{\partial}{\partial t}\Big)\Big)\varphi \Big](x,t) + \\ + \Big(\varphi\Big[\partial_\mu\Big(\frac{1}{E}G^{a\mu,b\rho}[A]\frac{\partial}{\partial t}A_{\rho b}\times G_{av}^{\mu\sigma}[A]\frac{\partial}{\partial t}A_{\sigma\mu}\Big)\partial_v\Big]\varphi\Big)(x,t)\Big\}$$

Now the effect on integrating at the scalar matter field in eq. (14) is the appearence of the further effective action to be added on the σ –like action of our Proposed Geometrodynamical Propagator.

$$S^{EFF}[A_\mu^a,E,T] = -\frac{1}{2}\lg\det{}_F\Big\{-\frac{\partial}{\partial t}\Big(E\frac{\partial}{\partial t}\Big)+\partial_\mu\Big(\frac{1}{E}G^{a\mu,bF}[A]\frac{\partial}{\partial t}A_{\rho b}\times G_{av}^{\mu\sigma}[A]\frac{\partial}{\partial t}A_{\sigma\mu}\Big)\partial_v\Big\} \tag{16}$$

The Coupling with (Weyl) Fermionic Matter is straight forward and leading to the Left-Handed Fermionic Functional Determinant in the presence of the Astekar-Sen connection $A_\mu^a(x,t)$ ([7],[9]).

The joint probability for the massless field propagator in the presence of a fluctuating geometry parametrized by the Astekar-Sen connection is given by

$$G[A_\mu^{IN};A_\mu^{OUT};<\varphi(x_1,t_1)\varphi(x_2,t_2)>] = \int d^{INV}[A_\mu^a]d[E]\times A_\mu^a(x,-\infty) =$$

$$= A_\mu^{a,IN}(x);A_\mu^a(a,+\infty) = A_\mu^{a,OUT}(x)\exp\Big\{-\tfrac{1}{16\pi G}\int_{-\infty}^{+\infty}dt\int_{R^3}d^3x\times$$

$$\times\Big(\tfrac{1}{E(x,t)}\times\big(\tfrac{\partial}{\partial t}A_{a,\mu}\big)G^{\mu a,vb}[A]\big(\tfrac{\partial}{\partial t}A_{v,b}\big)(x,t)\Big)+\mu^2\int_{-\infty}^{+\infty}dt\int_{R^3}d^3xE(x,t)\Big\}\times$$

$$\times\det{}^{-\frac{1}{2}}\Big[-\tfrac{\partial}{\partial t}\big(E\tfrac{\partial}{\partial t}\big)+\partial_\mu\big(\tfrac{1}{E}G^{a\mu,b\rho}[A]\tfrac{\partial}{\partial t}A_{\mu,b}\times G_{av}^{\mu\sigma}[A]\times\tfrac{\partial}{\partial t}A_{\sigma,m}\big)\partial_v\Big]\times$$

$$\times\lim_{(x,t)\to 0}\frac{\delta}{\delta J(x_1,t_1)}\frac{\delta}{\delta J(x_2,t_2)}\times\exp\Big\{-\tfrac{1}{2}\int_{-\infty}^{+\infty}dt\,dt'\int_{R^3}d^3x\,d^3y\times$$

$$\Big\{EJ\eta_{abc}\epsilon^{\mu v\rho}\Big(G^{a\mu,b'\rho'}\tfrac{\partial}{\partial t}A_{\rho',b'}\times G^{bv,b''\rho''}\tfrac{\partial}{\partial t}A_{\rho'',b''}\times G^{c\rho;b'''\rho'''}\tfrac{\partial}{\partial t}A_{\rho''',b'''}\Big)\Big\}(x,t)\times$$

$$\Big[-\tfrac{\partial}{\partial t}\big(E\tfrac{\partial}{\partial t}\big)+\partial_\mu\big(\tfrac{1}{E}G^{a\mu,b\rho}[A]\tfrac{\partial}{\partial t}A_{\rho b}G_{av}^{\mu\sigma}\tfrac{\partial}{\partial t}A_{\sigma\mu}\big)\partial_v\Big]\Big\}((x,t),(y,t))\times$$

$$\times\Big\{EJ\eta_{abc}\epsilon^{\mu v\rho}\Big(G^{a\mu,b'\rho'}\tfrac{\partial}{\partial t}A_{\rho',b'}\times G^{bv,b''\rho''}\tfrac{\partial}{\partial t}A_{\rho''b''}\times G^{c\rho;b'''\rho'''}\tfrac{\partial}{\partial t}A_{\rho''',b'''}\Big)(y,t')\Big\} \tag{17}$$

A $\phi\lambda^4$ Geometrodynamical Field Theory for Quantum Gravity

Let me start the analysis by considering the generating functional of the following Geometrodynamical Field Path Integral as the simplest generalization for Quantum Gravity of Similar well-defined Quantum Field Theory Path Integrals os Strings and Particles ([2],[3]).

$$Z[J(A)] = \int D^F(\phi[A]) \times \exp\left\{-\int dv(A)\phi[A] \times \left(\int d^3x\left(\epsilon_{abc} F^c_{\mu\nu}(A) \frac{\delta^2}{\delta A^a_\mu \delta A^b_\nu}\right)(x)\right)\phi[A]\right\} \times$$

$$\times \exp\left\{-\lambda \int d^3x d^3y \int dv(A) dv(\overline{A})\ \left(\phi^2[A(x)]\phi^2[\overline{A}(y)]\right) \times \delta^{(3)}\left(A_\mu(x) - \overline{A}_\mu(y)\right)\right\}$$

$$\exp\left\{-\int dv(A) J(A)\phi[A]\right\} \tag{18}$$

The notation is a follows: (i) The Universe Third Quantized field is given by a functional $\phi[A]$ defined over the space of all Astekar-Sen connections configurations $M = \{A^a_\mu(x); x \in R^3\}$. The sum over the functional space M is defined by the Gauge and Diffeomorphism invariant and Topological - Non - Trivial Path Integral of a Chern - Simons Field Theory on the Astekar - Sen connections

$$dv(A) = \int (\Pi_{x\in R^3} dA^a_\mu(x)) \times \exp\left\{-\int d^3x\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)(x)\right\} \tag{19}$$

(iii) The Third Quantized Functional Measure in eq.(16) is given formally by the usual Feynman product measure

$$D^F(\phi[A]) = \prod_{A\in M} d\phi[A] \tag{20}$$

and (iv) The $x\phi^4$ - like interaction vertex is given by a self-avoiding Geometrical Interaction among the Astekar - Sen Field Configurations in the Extrinsic Space R^3.

$$\lambda \sum_{a=1}^{3} \delta^{(3)}\left(A^a_\mu(x) - \overline{A}^a_\mu(y)\right) \tag{21}$$

The proposed interaction vertex was defined in such way that allows the replacement of the Four Universe interaction in eq.(18) by an independent interaction of each Astekar-Sen correction with an extrinsic triplet of Gaussian stochastic field $W^a(x)$ followed by an average over W^a. Similar procedure is well know in many-body and many Random Surface paht integral quantum field theory ([2],[3]). So, I can write equation (18) in the following form

$$Z[J(A)] = \left\langle \int D^F(\phi[A]) \times \exp\left\{ -\int dv(A) \left[\phi[A] \left(\mathrm{L}(A) - i\lambda \int d^3x \left(\sum_{a=1}^{3} W^a(A_\mu^a(x)) \right) \right) \times \right\rangle \right.\right.$$

$$\times \phi[A_\mu^a(x)] + J(A)\phi[A]]\}\rangle_W \tag{22}$$

Here, $W^a(A^a)$ means the external a-component of the triplet of the external stochastic field $\{W_\mu^a\}$ projected on the Astekar-Sen Connection $\{A_\mu^a\}$, namelly

$$W^a(A^a) = \int d^3 \times W^a(A_1^a(x), A_2^a(x), A_3^a(x)) \tag{23}$$

and has the white noise stochastic correlation function

$$\langle W^a(x^\mu) W^b(y^\nu) \rangle = \lambda \delta^{(3)}(x^\mu - y^\mu)\delta^{ab} \tag{24}$$

The $\mathrm{L}(A)$ operator on the functional space of the universe field is the Wheeler-De Witt operator defining the quadratic action in eq.(18).

In the free case $\lambda \equiv 0$. The Third Quantized Universe Path Integral eq.(18) is exactly soluble with the following Generating Functional

$$\frac{Z[J(A)]}{Z[0]} = \exp\left\{ +\frac{1}{2} \int dv(A)\, dv(\overline{A})\, J(A) \left(\int d^3x \, \epsilon_{abc} F_{\mu\nu}^c(A) \frac{\delta^2}{\delta A_\mu^a \delta A_\nu^b} \right)^{-1} (A,\overline{A}) \times J(\overline{A}) \right\} \tag{25}$$

Here the functional inverse of the Wheeler-De Witt is given explicity by the Geometrodynamical Propagator eq.(13) with $\widetilde{E} = 0$ (see eqs.(13)-(14)).

$$\left(\int d^3x \, \epsilon_{abc} F_{\mu\nu}^c(A) \frac{\delta^2}{\delta A_\mu^a \delta A_\nu^b} \right)^{-1} (A,\overline{A}) = \int_0^\infty dT \, G\left[A,\overline{A},T\right] \tag{26}$$

In order to reformulate the Third Quantized Universe Field Theory as a dynamics of Self-Avoiding Geometrodynamical Propagators, I evaluate formally the Gaussian $\phi[A]$ Functional Path Integral in eq.(18) with the following result

$$Z[J(A)] = \left\langle \det^{-\frac{1}{2}} \left[\int_{R^3} d^3x \, \epsilon_{abc} F^c_{\mu\nu}(A) \frac{\delta^2}{\delta A^a_\mu \delta A^b_\nu} + i\lambda \left(\sum_{a=1}^{3} W^a(A^a(x)) \right) \right] \times \right.$$

$$\left. \times \exp\left\{ +\tfrac{1}{2} \int dv(A) \, dv(\bar{A}) \times J(A) \left[\int_{R^3} d^3x \, \epsilon_{abc} F^c_{\mu\nu}(A) \frac{\delta^2}{\delta A^a_\mu \delta A^b_\nu} + i\lambda \left(\sum_{a=1}^{3} W^a(A^a_\mu(x)) \right)^{-1} \right] (A, \bar{A}) \times J(\bar{A}) \right\} \right.$$

(

Let me follow the previous studies implemented for particles and strings ([2],[3]) by defining the Functional Determinant of the Wheeler - De Witt operator by the Proper-Time technique

$$-\tfrac{1}{2} \log \det \left[\mathrm{L}(A) + i\lambda \int_{R^3} d^3x \left(\sum_{a=1}^{3} W^a(A^a_\mu(x)) \right) \right] = -\int_0^\infty \tfrac{dT}{T} \left\{ \int dv(A) dv(\bar{A}) \, \delta^{(F)}(A - \bar{A}) \times \right.$$
$$\left. \times \left\langle A \right| \exp\left[-T \left(\mathrm{L}(A) + i\lambda d^3x \left(\sum_{a=1}^{3} W^a(A^a_\mu(x)) \right) \right) \right] \left| \bar{A} \right\rangle \right\} \tag{28}$$

with the Geometrodynamical Propagator (see eq.(14)) in the presence of the Extrinsic Potential $\{W^a_\mu(x)\}$ being given explicity by the Path Integral below

$$\left\langle A \middle| \exp\left[-T(\mathrm{L}(A)) + i\lambda \int_{R^3} d^3x \left(\sum_{a=1}^{3} W^a(A^a_\mu(x)) \right) \right] \middle| \bar{A} \right\rangle =$$

$$= \int d^{INV}[B^a_\mu(x,t)] \exp\left\{ -\tfrac{1}{16\pi G} \int_0^T dt \int_{R^3} d^3x \left[\left(\tfrac{\partial}{\partial t} B_{a,\mu} \right) \times G^{\mu a, \nu b}[B] \left(\tfrac{\partial}{\partial t} B_{\nu b} \right) \right](x,t) \right\} \times$$

$$\times \exp\left\{ -i\lambda \int_0^T dt \int_{R^3} d^3x \left(\sum_{a=1}^{3} W^a(B^a_\mu(x,t)) \right) \right\} \tag{29}$$

By substituing eq.(29) and eq.(28) into eq.(27) and making a Loop Expansion of the functional determinant, I obtain eq.(18) as a Theory of an Ensemble of Geometrodynamical Propagators interacting with the Extrinsic Gaussian Stochastic Field $\{W^a(x)\}$. The Gaussian average $\langle \quad \rangle_W$ may be straightforwardly evaluated at each loop expansion order and producting the self-avoiding interaction among the Geometrodynamical Propagators (the Wheeler Quantum Universes) and leading to the picture of Joining and Sppliting of These Wheeler Universes as necessary for the description of the Universe in its Space-Time Third Quantized foam picture of Wheeler.

For instance, by neglecting the functional determinant in eq.(27), I have the following expression for the Geometrodynamical Third Quantized Propagator.

$$\langle\Phi[A_\mu^{a,IN}]\Phi[A_\mu^{a,OUT}]\rangle^{(0)} = \int_0^\infty dT \times \int_{B_\mu^a(x,0)=A\,;\,B_\mu^a(x,T)=\bar{A}} d^{INV}[B_\mu^a(x,t)] \times$$

$$\times \exp\left\{-\frac{1}{16\pi G}\int_0^T dt \int d^3x \left[\left(\frac{\partial}{\partial t}B_{a,\mu}\right)G^{\mu a,\nu b}[B]\left(\frac{\partial}{\partial t}B_{\nu b}\right)\right](x,t)\right\} \times \tag{30}$$

$$\times \exp\left\{-\frac{\lambda^2}{2}\int_0^T dt \int_0^T dt' \int_{R^3} d^3x d^3y \left(\sum_{a=1}^{3}\delta^{(3)}(B_\mu^a(x,t) - B_\mu^a(y,t'))\right)\right\}$$

Next corrections will involve self - avoiding interactions among differents Wheeler Universes associated to different Astekar-Sen connections associated to different Geometrodynamical Propagators appearing from the Functional Determinant Loop Expansion eq.(28).

Finally, I comment that calculations will be done sucessfully only if one is able to handle correctly the Geometrodynamical Propagator eq.(14) on eq.(17) and, thus, proceed to generalized for this Quantum Gravity case the analogous framework used in the Theory of Random Lines and Surfaces ([2],[3]). Work is on the progress to solve eq.(10) and eq.(14) by means of strings theories ([7]).

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Appendix A

In this appendix we briefly describe the Functional Integral of the Geometrodynamical Propagator in terms of the metrics field $h_{\mu\nu}(x,t)$ on $R^D x(0,T)$. In this framework, the metric boundary condition and the conformal factor (regarded as an independent dynamical degree of freedom as in 2D Quantum Gravity - ([6]) are taken into account by using the metric decomposition

$$h_{\mu\nu}(x,t) = \rho(x,t)\left[-N^2(x,t)(dt)^2 + g_{ij}(x,t)(dx^i + N^i(x,t)dt)(dx^j + N^j(x,t)dt)\right] \tag{$A-1$}$$

where $\rho(x,t)$ is the conformal factor; $\{N(x,t), N^i(x,t)\}$ are the lapse-shift pieces of the metric field satisfying the boundary metric piece of eq.$(A-1)$ with the boundary condition

$$h_{ij}(x,0) = {}^3g^{IN}(x)\,;h_{ij}(x,T) = {}^3g^{OUT}(x)$$

The Differomorphism invariant functional measure in eq.(1) may be expressed in terms of decomposition eq.$(A-1)$ by evaluating the associated invariant functional volume from the De Witt functional metric expressed in terms of this new parametrization

$$dS_F^2 = \int d^D x\, dt \left\{ N\sqrt{h}\, (\delta g_{ab}(N;N;h,\rho)) \left(g^{aa'} g^{bb'} \right) \left(N;N^{\,i};h,\rho \right) \times \delta g_{a'b'}(N;N^{\,i};h,\rho) \right\} \qquad (A-2)$$

Here we have used a notation which emphasizes the dependence on the new parametrization eq.$(A-2)$. For the case where one supposes that the conformal factor does not play any dynamical-role we have the relationship

$$g^{00} = -\frac{1}{N^2}\,;\, g^{0i} = -\frac{N^i}{N^2}; \text{ TCItag}$$

$$g^{ij} = h^{ij} - \frac{N^i N^j}{N^2}\,;\delta g_{00} = 2(\delta N^\kappa) g_{\kappa i} N^i - 2N\delta N - N^\kappa (\delta h_{\kappa i}) N^{\,i};$$

$$\delta g_{i0} = h_{ij}(\delta N^j) - (\delta h_{ij} N^j)$$

$$\delta g_{ij} = \delta h_{ij}$$

After substituing eq.$(A-3)$ into eq.$(A-2)$ we, thus, obtain the associated volume element (The Functional Covariant Metric) in terms of the A.D.M. variables. Unfortunatly the present use of such parametrization in Path Integrals for Quantum Gravity is an open question.
In the case of a dynamical conformal factor

$$g_{\mu\nu}(x,t) = e^{-2\omega(x,t)}\, \bar{g}_{\mu\nu}(x,t) \qquad (A-4)$$

One is lead to consider the following Higher Derivative Invariant Path Integral

$$\int d_{\bar{g}}^{COV} \mu[g] \int d_{\bar{g}}^{COV} \left[e^{-\frac{\omega D}{2}} \right] \times \left\{ \exp\left\{ -\int d_x^D\, dt \left(\sqrt{g} R \right) \right\} \Big|_g = e^{-2\omega \bar{g}} \right\} \times \exp\{ -I(\omega, \bar{g}) \} \qquad (A-5)$$

with the Conformal Factor Four Dimensional Anomaly Action ([4],[9])

$$\begin{aligned} I(\omega,\bar{g}) = \int d^D x\, dt \sqrt{\bar{g}}\, \{ [c_1 R_{\mu\nu} R^{\mu\nu} + \Delta_{\bar{g}} R](\bar{g}) \times \omega + c_3 [R_{\mu\nu} (\nabla_a^\mu \nabla_\omega^\nu)](\bar{g}) + \\ -4\omega^\mu \omega_\mu \Delta_{\bar{g}} \omega + \partial(\omega^\mu \omega_\mu)^2 + 3(\Delta_{\bar{g}} \omega)^2 \} (x,t) \end{aligned} \qquad (A-6)$$

and the covariant functional measure for the conformal factor

$$d_{\bar{g}}^{COV}\left[e^{-\frac{\omega D}{2}}\right] = \prod_{x\in R^4}\left(\sqrt[4]{\bar{g}}\, e^{-\frac{\omega D}{2}}\right) d\omega(x,t) \qquad (A-7)$$

It is worth point out that neither of the above displayed Quantum Gravity Propagator Parametrizations allows to express the interaction with our external disorder field eq.(23) in a local form.

Finally we remark that our proposed action in eq.(14) is similar (but not equal) with that action proposed in eq.(8) - ref.[8] with zero cosmological constant and by integrating their conjugate field variable with a Feynman Measure in the Gauge of a vanishing Gauss law and zero shift function. Further study will be necessary to clarify the above cited similarities ([5],[8]).

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