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Z_k string fluxes and monopole confinement in non-Abelian theories

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Abstract

Recently we considered $N = 2$ Super Yang-Mills with a mass breaking term and showed the existence of BPS Z_k-string solutions for arbitrary simple gauge groups which are spontaneously broken to non-Abelian residual gauge groups. We also calculated their string tensions exactly. In doing so, we have considered in particular the hypermultiplet in the representation of a diquark condensate. In the present work we shall analyze some of the different phases of the theory and find that the magnetic fluxes of the monopoles and Z_k-strings of the theory are proportional to one another, allowing for monopole confinement in one of the phase transitions of the theory. Then we will calculate the threshold length for a string to break in a new pair of monopole-antimonopole. We will further show that some of the resulting confining theories can obtained by adding a deformation term to $N = 2$ or $N = 4$ superconformal theories and, as such, may satisfy a gauge/string correspondence.

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1 Introduction

It is long believed that the quark confinement would be dual to a non-Abelian generalization of Meissner effect, as proposed by 't Hooft and Mandelstam many years ago[1]. Following their ideas an important progress has been made by Seiberg and Witten [2] which starting from an $N = 2$ $SU(2)$ supersymmetric theory obtained an effective $U(1)$ $N = 2$ super QED with an $N = 2$ mass breaking term. In this theory, the $U(1)$ is broken to a discrete group and as it happens the theory develops string solutions and electric charge confinement occurs. After that, many interesting works appeared [3] analyzing various aspects of different $N = 2$ theories with a mass breaking term and (solitonic) string solutions. However, since the dynamics of a non-Abelian theory is hard to control, in these theories the gauge group are usually completely broken to its discrete center by Higgs mechanism. Therefore they do not possess $SU(3)\times U(1)_{\rm em}$ as subgroup of the unbroken gauge group and the monopoles belong to $U(1)$ singlets and not to representations of non-Abelian groups.

In order to avoid these problems, recently we considered [4] (for a review see [5]) $N = 2$ super Yang-Mills with a breaking mass term, with arbitrary simple gauge group and non-Abelian unbroken gauge symmetry. One of the spontaneous symmetry breaking is produced by a complex scalar ϕ that could be for example in the symmetric part of the tensor product of k fundamental representations. In particular if $k = 2$, this scalar is in the representation of a diquark condensate and therefore it can be thought as being itself the diquark condensate or the monopole condensate in the dual theory, like in the Abelian-Higgs theory. When this scalar acquires an expectation value it gives rise to the monopole confinement and to the quark mass, when $k = 2$. We have showed the existence of BPS Z_k-string solutions for these theories and calculated exactly their string tensions. In the present work, we analyze many properties of these theories. In section 2 we show that by varying continuously a mass parameter m we can pass from an unbroken phase to a phase with free monopoles and then to a phase with Z_k-strings and confined monopoles. In section 3 we analyze the monopoles in the free-monopole phase. These monopole solutions are expected to fill irreducible representations of the dual unbroken gauge group[6]. In this phase we recover $N = 2$ supersymmetry and we show that some of these theories are conformal invariant in this phase. In section 4 we analyze the magnetic fluxes of the BPS strings which appear in the superconducting phase. We show that the fluxes of the magnetic monopoles and strings are proportional to one another and therefore the monopoles can get confined. Wealso obtain the threshold length of a string to break in a new pair of monopole-antimonopole. The general topological aspects for monopole confinement during a phase transition have been given in [7] (see also [8]). Our aim here is to analyze the monopole confinement in our specific theory. From the values of the magnetic fluxes we calculate the threshold length for a string to break in a new pair of monopole-antimonopole. We also show that some of the confining theories, are obtained by adding deformations terms to superconformal theories. We conclude with a summary of the results.

2 Phases of the theory

As is quite well known, in the broken phase of the Abelian-Higgs theory in 3+1 dimensions, there exist string solutions with string tension satisfying the inequality

$$T \geq \frac{q_\phi}{2} a^2 |\Phi_{\rm st}| \,, \tag{1}$$

where a is a breaking parameter in the potential, $\Phi_{\rm st}$ is the string's magnetic flux which satisfies the quantization condition

$$\Phi_{\rm st} = \frac{2\pi n}{q_\phi} \,, \; n \in Z \tag{2}$$

and q_ϕ is the electric charge of the scalar field. Considering that $\phi^\dagger$ is a condensate of electrons, then[1] $q_\phi = 2e$. Following t' Hooft and Mandelstam's [1] idea, if one considers a (Dirac) monopole-antimonopole system in the Abelian-Higgs theory, the magnetic lines can not spread over space but must rather form a string which gives rise to a confining potential between the monopoles. This idea only makes sense since the (Dirac) monopole magnetic flux is $\Phi_{\rm mon} = g = 2\pi/e$, which is consistent with the string's magnetic flux quantization condition, allowing one to attach to the monopole two strings with $n = 1$.

Let us now generalize these ideas to a non-Abelian theory. For simplicity let us consider a gauge group G which is simple, connected and simply-connected, and adopt the same conventions as in [4]. Following our previous work, we shall consider a Yang-Mills theory with a complex scalar S in the adjoint representation and another complex scalar ϕ. We consider a scalar in the adjoint representation because in a spontaneous symmetry breaking it produces a exact symmetry group with a $U(1)$ factor, which allows the existence of monopole solutions. Additionally, another motivation for having a scalar in the adjoint representation is because with it, we can form an $N = 2$ vector supermultiplet and, like in the Abelian-Higgs theory, the BPS string solutions appear naturally in a theory with $N = 2$ supersymmetry and a $N = 2$ mass breaking term. Moreover, in a theory with the field content of $N = 2$, the monopole spin is consistent with the quark-monopole duality[10] which is another important ingredient in 't Hooft and Mandelstam's ideas. A necessary condition for the existence of a string is to have a non-trivial first homotopy group. One way to produce a spontaneous symmetry breaking satisfying this condition is to introduce a complex scalar ϕ in a representation which contains the weight state $|k\lambda_\phi\rangle$ [9], where k is an integer greater or equal to two, and λ_ϕ a fundamental weight. We can have at least three possibilities: one is to consider ϕ in the representation with $k\lambda_\phi$ as highest weight, which we shall denote $R_{k\lambda_\phi}$. We can also consider ϕ to be in the direct product of k fundamental representations with fundamental weight λ_ϕ, which we shall denote $R^{\otimes}_{k\lambda_\phi}$. Finally a third possibility would be to consider ϕ in the symmetric part of $R^{\otimes}_{k\lambda_\phi}$, called $R^{\rm sym}_{k\lambda_\phi}$, which always contains $R_{k\lambda_\phi}$. This last possibility has an extra physical motivation that if $k = 2$, it corresponds to the representation of a condensate of two fermions (quarks) in the fundamental representation with fundamental weight λ_ϕ, and we can interpret ϕ as being this diquark condensate. In this case, when ϕ takes a non-trivial expectation value, it also gives rise to a mass term for these quarks. In order to have $N = 2$ supersymmetry we should need another complex scalar to be in the same hypermultiplet as ϕ. For simplicity's sake, however, we shall ignore it setting it to zero.

[1]Considering that $\hbar = 1 = c$

In our work [4], we considered the potential

$$V = \frac{1}{2}\left(Y_a^2 + F^\dagger F\right) \geq 0 \tag{3}$$

where

$$Y \equiv Y_a T_a = \frac{e}{2}\left\{\left(\phi^\dagger T_a \phi\right) T_a + \left[S^\dagger, S\right] - m\left(\frac{S+S^\dagger}{2}\right)\right\}, \tag{4}$$

$$F \equiv e\left(S^\dagger - \frac{\mu}{e}\right)\phi, \tag{5}$$

T_a being orthogonal Lie algebra generators which satisfy

$$\text{Tr}\ (T_a T_b) = x_\phi \psi^2 \delta_{ab} \tag{6}$$

where x_ϕ is the Dynkin index of ϕ's representation and ψ^2 is the length square of the highest root which we shall take to be 2. That potential is the bosonic part of $N = 2$ super Yang-Mills with one flavor (with one of the aforementioned scalars of the hypermultiplet put equal to zero). The parameter μ gives a bare mass to ϕ and m gives a bare mass to S which softly breaks $N = 2$ SUSY. The parameter m may also be responsible for spontaneous gauge symmetry breaking and, as in the Abelian-Higgs case, we can consider it as a function of temperature, $m = m(T)$.

The vacua are solutions to the conditions

$$Y = 0 = F\,. \tag{7}$$

In order to the topological string solutions to exist, we look for vacuum solutions of the form

$$\phi^{\text{vac}} = a|k\lambda_\phi >, \tag{8}$$

$$S^{\text{vac}} = b\lambda_\phi \cdot H\,, \tag{9}$$

where a and b are complex constants, k is a integer greater or equal to two and λ_ϕ is an arbitrary fundamental weight. If $a \neq 0$, this configuration breaks $G \to G_\phi$ in such a way that[9] $\Pi_1(G/G_\phi) = Z_k$, which is a necessary condition for the existence of Z_k-strings. Let us consider that $\mu > 0$. Following [4], from the vacuum conditions (7) one can conclude that

$$|a|^2 = \frac{mb}{k},$$

$$\left(kb\lambda_\phi^2 - \frac{\mu}{e}\right)a = 0\,.$$

There are three possibilities:

(i) If $m < 0 \ \Rightarrow\ a = 0 = b$ and the gauge group G remains unbroken.

(ii) If $m = 0 \ \Rightarrow\ a = 0$ and b can be any constant. When $m = 0$, $N = 2$ supersymmetry is restored. In this case, S^{vac}breaks[9]

$$G \to G_S \equiv (K \times U(1))\,/Z_l\,, \tag{10}$$

where K is the subgroup of G associated to the algebra whose Dynkin diagram is given by removing the dot corresponding to λ_ϕ from that of G. The $U(1)$ factor is generated by $\lambda_\phi \cdot H$ and Z_l is a discrete subgroup of $U(1)$ and K.

(iii) If $m > 0 \Rightarrow$

$$|a|^2 = \frac{m\mu}{k^2 e \lambda_\phi^2}, \quad (11)$$

$$b = \frac{\mu}{k e \lambda_\phi^2}, \quad (12)$$

and G is further broken to[9]

$$G \to G_\phi \equiv (K \times Z_{kl})/Z_l \supset G_S . \quad (13)$$

In particular, for $k = 2$, we have for example,

$$\begin{aligned} \text{Spin}(10) &\to (SU(5) \times Z_{10})/Z_5 , \\ SU(3) &\to (SU(2) \times Z_4)/Z_2 . \end{aligned}$$

Therefore by continuously changing the value of the parameter m we can produce a symmetry breaking pattern $G \to G_S \to G_\phi$. It is interesting to note that, unlike the Abelian-Higgs theory, in our theory the bare mass μ of ϕ is not required to satisfy $\mu^2 < 0$ in order to have spontaneous symmetry breaking. Therefore in the dual formulation, where one could interpret ϕ as the monopole condensate, we don't need to have a monopole mass satisfying the problematic condition $M^2_{\text{mon}} < 0$ mentioned by 't Hooft[11].

Let us analyze in more detail the last two phases.

3 The $m = 0$ or free-monopole phase

In this phase $a = 0$ and b is an arbitrary non-vanishing constant, which we shall consider it to be given by (12), in order to have the same value as the case when $m < 0$. The non-vanishing expectation value of S^{vac} defines the $U(1)$ direction in G_S, (10), and one can define the corresponding $U(1)$ charge as [12]

$$Q \equiv e \frac{S^{\text{vac}}}{|S^{\text{vac}}|} = e \frac{\lambda_\phi \cdot H}{|\lambda_\phi|} . \quad (14)$$

Since in this phase $\Pi_2(G/G_S) = Z$, it can exist Z-magnetic monopoles. These solutions can be written in the following form[13]: for each root α, such that $2\alpha^{\text{V}} \cdot \lambda_\phi \neq 0$ (where $\alpha^{\text{V}} \equiv \alpha/\alpha^2$), we can define the generators

$$T_1^\alpha = \frac{E_\alpha + E_{-\alpha}}{2} , \quad T_2^\alpha = \frac{E_\alpha - E_{-\alpha}}{2i} , \quad T_3^\alpha = \frac{\alpha \cdot H}{\alpha^2} \quad (15)$$

which satisfy the SU(2) algebra

$$\left[T_i^\alpha, T_j^\alpha \right] = i \epsilon_{ijk} T_k^\alpha .$$

Using spherical coordinates we define the group elements

$$g_p^\alpha(\theta, \phi) \equiv \exp(ip\varphi T_3^\alpha) \exp(i\theta T_2^\alpha) \exp(-ip\varphi T_3^\alpha) , \quad p \in Z . \quad (16)$$

Let $S = M + iN$, where M and N are real scalar fields. The asymptotic form for the scalars of the $Z-$monopole are obtained by performing a gauge transformation on the vacuum solution (8), (9) by the above group elements. This results, at $r \to \infty$

$$M(\theta,\phi) = g_p^\alpha v \cdot H \left(g_p^\alpha\right)^{-1} , \tag{17}$$

$$N(\theta,\phi) = 0 \ , \quad \phi(\theta,\phi) = 0 \, . \tag{18}$$

where $v \equiv b\lambda_\phi$. The U(1) magnetic charge of these monopoles are [13]

$$g \equiv \frac{1}{|v|}\int dS_i M^a B_i^a = \frac{4\pi}{e}\frac{pv\cdot\alpha^{\rm V}}{|v|} \tag{19}$$

where $B_i^a \equiv -\epsilon_{ijk}G_{jk}^a/2$ are the non-Abelian magnetic fields. These monopoles fill supermultiplets of $N = 2$ supersymmetry [14] and satisfy the mass formula

$$m_{\rm mon} = |v||g| \, . \tag{20}$$

Not all of these monopoles are stable. The stable or fundamental BPS monopoles are those which $p = 1$ and $2\alpha^{\rm V} \cdot \lambda_\phi = \pm 1$[15]. From now on we shall only consider these fundamental monopoles, which are believed to fill representations of the gauge subgroup K[6].

It is interesting to note that for the particular case where the gauge group is $G = SU(2)$ and ϕ is in the symmetric part of the tensor product of two fundamental representations, which correspond to the adjoint representation, the supersymmetry of the theory is enhanced to $N = 4$, and the theory has vanishing β function. There are other examples of vanishing β functions when $m = 0$. The β function of $N = 2$ super Yang-Mills with a hypermultiplet is given by

$$\beta(e) = \frac{-e^3}{(4\pi)^2}\left[h^{\rm V} - x_\phi\right]$$

where $h^{\rm V}$ is the dual Coxeter number of G and x_ϕ is the Dynkin index of ϕ's representation(6). If ϕ belongs to $R_{2\lambda_\phi}^{\rm sym}$,

$$x_\phi = x_{\lambda_\phi}\left(d_{\lambda_\phi} + 2\right) \, .$$

where x_{λ_ϕ} and d_{λ_ϕ} are, respectively, the Dynkin index and the dimension of the representation associated to the fundamental weight λ_ϕ. On the other hand if ϕ belongs to the direct product of k fundamental representations, $R_{k\lambda_\phi}^{\otimes}$,

$$x_\phi = k d_{\lambda_\phi} x_{\lambda_\phi} \, ,$$

For $SU(n)$ (which has $h^{\rm V} = n$), if ϕ is in the tensor product of the fundamental representation of dimension $d_{\lambda_{n-1}} = n$ with itself (which has Dynkin index $x_{\lambda_{n-1}} = 1/2$), $x_\phi = n$ and the β function vanishes. Therefore the theory is superconformal (if we take $\mu = 0$). Therefore, for these theories, when $m = 0$, they are $SU(n-1) \otimes U(1) \sim U(n-1)$ $N = 2$ superconformal theories.

4 The $m > 0$ or superconducting phase

In the "$m > 0$" phase, the U(1) factor is broken and the corresponding force lines cannot spread over space. Since G is broken in such a way that $\Pi_1(G/G_\phi) = Z_k$, these force lines may form topological Z_k-strings. We indeed showed in [4] the existence of BPS Z_k-strings in the limit $m \to 0_+$ and $\mu \to \infty$, with $m\mu =$ const. We shall show now that, as in the Abelian Higgs theory, the U(1) magnetic flux $\Phi_{\rm mon}$ of the above monopoles is proportional to the Z_k-string magnetic flux $\Phi_{\rm st}$, and therefore these $U(1)$ flux lines coming out of the monopole can be squeezed into Z_k-strings, which can give rise to a confining potential.

4.1 Z_k-string magnetic flux

Since

$$Q\phi^{\rm vac} = ek|\lambda_\phi|\,\phi^{\rm vac}\,,$$

the electric charge of $\phi^{\rm vac}$ is

$$q_\phi = ek|\lambda_\phi|\,. \tag{21}$$

On the other hand, the string tension satisfies the bound [4]

$$\begin{aligned} T &\geq \frac{me}{2}\left|\int d^2x\, M^a B_3^a\right| \\ &= \frac{q_\phi}{2}|a|^2|\Phi_{\rm st}| \end{aligned} \tag{22}$$

where

$$\Phi_{\rm st} \equiv \frac{1}{|v|}\int d^2x\, M^a B_3^a \tag{23}$$

is the $U(1)$ string magnetic flux and the integral is taken over the plane perpendicular to the string. This flux definition is gauge invariant and consistent with the flux definition for the monopole (19). One notes that (22) is very similar to the Abelian result (1). Let us use BPS string ansatz in [4]:

$$\begin{aligned} \phi(\varphi,\rho) &= f(\rho)e^{i\varphi M_n}a|k\lambda_\phi>\,, \\ mS(\varphi,\rho) &= h(\rho)ka^2e^{i\varphi M_n}\lambda_\phi\cdot He^{-i\varphi M_n}\,, \\ W_i(\varphi,\rho) &= g(\rho)M_n\frac{\epsilon_{ij}x^j}{e\rho^2} \quad \to \quad B_3(\varphi,\rho) = \frac{M_n}{e\rho}g'(\rho)\,, \\ W_0(\varphi,\rho) &= W_3(\varphi,\rho) = 0\,, \end{aligned} \tag{24}$$

with the boundary conditions

$$f(\infty) = g(\infty) = h(\infty) = 1\,,$$

$$f(0) = g(0) = 0\,,$$

and considering

$$M_n = \frac{n}{k}\frac{\lambda_\phi\cdot H}{\lambda_\phi}\,.$$

Then, from the BPS condition $D_\pm S = 0$ together with the boundary conditions results that $h(\rho) = 1$.Therefore we obtain that for the BPS strings

$$\Phi_{\rm st} = \oint dl_I A_I = \frac{2\pi n}{q_\phi} \quad , \qquad n \in Z_k \, , \tag{25}$$

where $A_I \equiv W_I^a M^a/|v|$, $I = 1, 2$. This flux quantization condition is also very similar to the Abelian result (2) and generalizes, for example, the string magnetic flux for $SU(2)$[16] and for $SO(10)$[17]. In [18], it is also calculated fluxes for the $SU(n)$ theory, but with the gauge group completely broken to its center and a different definition of string flux which is not gauge invariant. We can rewrite the above result as

$$\Phi_{\rm st} q_\phi = 2\pi n \ , \quad n \in Z_k \, .$$

Let us now check that $\Phi_{\rm st}$ is consistent with the $U(1)$ magnetic flux $\Phi_{\rm mon}$ of the monopoles. From (19), using (21) and the fact that

$$\alpha^{\rm V} = \sum_{i=1}^{r} m_i \alpha_{\rm i}^{\rm V} \ , \quad \alpha_i^{\rm V} = \frac{\alpha_i}{\alpha_i^2} \ , \quad m_i \in Z,$$

it follows that

$$\Phi_{\rm mon} = g = \frac{2\pi k m_\phi p}{q_\phi} \, .$$

Therefore, for the fundamental (anti)monopoles, which have $p = 1$ and $m_\phi = \pm 1$, $\Phi_{\rm st}$ is consistent with $\Phi_{\rm mon}$ if $n = k$. This can be interpreted that for one fundamental monopole we can attach k Z_k-strings with $n = 1$. That result is consistent with the fact that k Z_k-strings with $n = 1$ have trivial first homotopy, as do the monopoles.

4.2 Monopole confinement

In the $m < 0$ phase, one would expect [7] that the monopoles produced in the $m = 0$ phase develop a flux line or string and get confined. We can see this more concretely in the following way: as usual, in order to obtain the asymptotic scalar configuration of a (spherically symmetric) monopole, starting form the vacuum configuration (8), (9) one performs the spherically symmetric gauge transformation (16) and obtains that at[2] $r \to \infty$,

$$S(\theta, \varphi) = g_p^\alpha b \lambda_\phi \cdot H \left(g_p^\alpha \right)^{-1} \tag{26}$$

$$\phi(\theta, \varphi) = g_p^\alpha a |k \lambda_\phi > \tag{27}$$

However, $\phi(\theta, \varphi)$ is singular. In order to see this let's consider for simplicity $p = 1$, $k = 2$, and α to be those positive roots such that $2\lambda_\phi \cdot \alpha^{\rm V} = 1$. In this case, the orthonormal weight states

$$|2\lambda_\phi > \ , \ |2\lambda_\phi - \alpha > \ , \ |2\lambda_\phi - 2\alpha >$$

[2] Note that when we take $m = 0 \Rightarrow a = 0$ we recover the asymptotic scalar field configuration for the Z-monopole in the "m=0 phase" (17), (18)

form a spin 1 irrep of the $su(2)$ algebra (15) and the orthonormal states

$$|\pm> \equiv \frac{1}{2}\left(|2\lambda_\phi> \pm i\sqrt{2}|2\lambda_\phi - \alpha> - |2\lambda_\phi - 2\alpha>\right) , \tag{28}$$

$$|0> \equiv \frac{1}{\sqrt{2}}\left(|2\lambda_\phi> + |2\lambda_\phi - 2\alpha>\right) , \tag{29}$$

satisfy

$$\begin{aligned} T_2^\alpha|\pm> &= \pm|\pm> , \\ T_2^\alpha|0> &= 0 . \end{aligned}$$

We can then write

$$|2\lambda_\phi> = \frac{1}{2}\left(|+> + |-> + \sqrt{2}|0>\right) .$$

Then, (27) can be written as

$$\phi(\theta,\varphi) = a\left\{\cos^2\frac{\theta}{2}|2\lambda_\phi> - \frac{\sqrt{2}}{2}\sin\theta e^{-i\varphi}|2\lambda_\phi - \alpha> + \sin^2\frac{\theta}{2}e^{-2i\varphi}|2\lambda_\phi - 2\alpha> .\right\}$$

Therefore at $\theta = \pi$,

$$\phi(\pi,\varphi) = ae^{-2i\varphi}|2\lambda_\phi - 2\alpha>$$

which is singular. This generalizes Nambu's result [19] for the $SU(2)\times U(1)$ case. In order to cancel the singularity we should attach a string in the $z < 0$ axis with a zero in the core, as in our string ansatz (24). One could construct an ansatz for $\phi(r,\theta,\phi)$ by multiplying the above asymptotic configuration by a function $F(r,\theta)$ such that $F(r,\pi) = 0$.

The string tension for k strings with $n = 1$ must satisfy

$$T \geq k\pi|a|^2 .$$

Then, the threshold length $d^{\rm th}$ for a string to break producing a new monopole-antimonopole pair, with masses (20), is derived form the relation

$$\frac{4\pi}{e}\frac{k|a|^2|\lambda_\phi|}{m} = E^{\rm th} = Td^{\rm th} \geq k\pi|a|^2 d^{\rm th} ,$$

which results in

$$d^{\rm th} \leq \frac{4|\lambda_\phi|}{me} .$$

The monopole-antimonopole pair tends to deconfine when $m \to 0_+$, as one would expect, when $T \to 0$ and $d^{\rm th} \to \infty$.

It is expected that a confining theory obtained by a deformation of superconformal gauge theory in 4 dimensions should satisfy a gauge/string correspondence [20], which would be a kind of deformation of the CFT/AdS correspondence [21]. In the gauge/string correspondences it is usually considered confining gauge theories with $SU(N)$ broken to its center Z_N. We have seen that some of our confining theories are obtaining by adding a deformation to $U(N-1)$ superconformal theories which breaks the gauge group further to $SU(N-1)\otimes Z_2$ (up to a global factor). It would be interesting to know if those theories also satisfy some gauge/string correspondence.

5 Summary and conclusions

In this work we have extended some of the ideas of 't Hooft and Mandelstam of quark confinement for non-Abelian theories. We have considered $N = 2$ super Yang-Mills with arbitrary simple gauge group, with one flavor and with an $N = 2$ mass breaking term. We have shown that, by continuously varying the mass breaking parameter m, we can pass from an unbroken phase to a phase with free monopoles and then to a phase with Z_k-strings. This last phase occurs due to the fact that the scalar ϕ, which can be interpreted as a diquark condensate when $k = 2$, acquires a non-vanishing expectation value. We showed that the magnetic fluxes of the monopoles and of the strings are proportional to one another and therefore the monopoles can undergo confinement. We showed that threshold length for a string to break in a new pair of monopole-antimonopole is proportional to m^{-1}. We also have shown that some of these confining theories are obtained by deforming some $N = 2$ or $N = 4$ super conformal theories.

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