CBPF-NF-037/82

A SIMPLE INTERPOLATION FOR HIGH AND LOW MOMENTUM TRANSFERS IN QCD

by

J.A. Mignaco and I.Roditi

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - BRASIL

A SIMPLE INTERPOLATION FOR HIGH AND LOW MOMENTUM TRANSFERS IN QCD

J.A.Mignaco and I.Roditi

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - BRASIL

ABSTRACT

It is shown that a simple rational approximation for the function $\beta(g)$ of the renormalization group provides a useful expansion parameter for the perturbative regime in QCD and, simultaneously, allows for confinement in the functional integral approximation to this problem.

In recent years 't Hooft [1] proposed a transformation for g, the QCD running coupling constant, which enabled him to obtain very easily the analytic behaviour of Green's functions in the complex g^2-plane. This comes out because the Callan-Symanzik [2] $\beta(g^2)$ function so obtained retains only the two first terms giving a simpler renormalization group trend. Going along the same way but in another context Adler [3] studied the up to one-loop exact $\beta(g^2)$ function. In other words he made a transformation on g that gave him a Callan-Symanzik function with only the first term retained, and using a heuristic approach was able to obtain a linear mean field potential producing confinement. Later, with T. Piran, improved this approximation keeping the nice properties of the lowest order [4]. However it can be shown [5] that within the framework of optimised [6] perturbation theory Adler's one loop transformation gives poor results while 't Hooft's is a well suited one. On the other hand if we use the two-loop exact 't Hooft's $\beta(g^2)$ function we apparently miss the nice feature of confinement [7] achieved with the semiclassical methods.

A moderate proposal containing only the information conveyed by the two prescription independent terms is a transformation which is somewhat in the "middle way" as it retains the nice porperties of both Adler's and 't Hooft's transformations. At this we arrive by choosing a transformed g that gives for the $\beta(g^2)$ function its first Padé approximant

$$\beta_{RA}(g^2) = - \frac{b_0 g^4}{1 - \frac{b_1}{b_0} g^2}$$

where b_0 and b_1 are the usual renormalization scheme independent coefficients of the QCD $\beta(g^2)$ function [8]

$$b_0 = \frac{2}{(4\pi)^2}(11 - \frac{2}{3}N_f) \tag{2}$$

$$b_1 = \frac{2}{(4\pi)^4}(102 - \frac{38}{3}N_f)$$

where N_f is the number of flavors.

In order to settle more plainly the ideas let us first study the above transformations from the point of view of their usefulness in the renormalization group improved perturbation theory. We shall relate the couplings obtained with them to one obtained by a $\overline{\text{MOM}}$ scheme [9], when they differ by a small amount we find another good expansion parameter [5].

Following 't Hooft (see also [10]) we can relate two schemes by a finite renormalization

$$g^2 = \hat{g}^2 + a\hat{g}^4 + c\hat{g}^6 + \dots \tag{3}$$

Substituting this in $\beta(g^2)$ defined as usual [1]

$$\mu \frac{d}{d\mu} g^2 = \beta(g^2) = - b_0 g^4 - b_1 g^6 - b_2 g^8 + \dots \tag{4}$$

and equating coefficients it is found that for the transformed $\hat{\beta}(\hat{g}^2)$

$$\mu \frac{d}{d_\mu} \hat{g}^2 = \hat{\beta}(\hat{g}^2) = - \hat{b}_0 \hat{g}^4 - \hat{b}_1 \hat{g}^6 - \hat{b}_2 g^8 + \dots, \tag{5}$$

we have

$$\hat{b}_0 = b_0$$

$$\hat{b}_1 = b_1 \tag{6}$$

$$\hat{b}_2 = b_0(a^2 - c) + b_1 a + b_2$$

and so on for the other $\hat{b}$'s. As is known only the two first $\hat{b}$'s are renormalization scheme independent. 't Hooft's $\beta_{tH}(g^2_{tH})$ function is obtained by choosing $a = 0$[‡1] and the other coefficients in such a way that all the $\hat{b}_{i's}$ $(i \geq 2)$ become zero. So for $\hat{b}_2 = 0$ we get

$$c = \frac{b_2}{b_0} \tag{7}$$

To obtain the $\beta_{RA}(g^2_{RA})$ proposed by us, we can choose $a = 0$ and equate consistently the other constants with those of the expansion

$$\beta_{RA}(g^2_{RA}) = - b_0 g^4_{RA} - b_1 g^6_{RA} - \frac{b_1^2}{b_0} g^8_{RA} + \ldots \tag{8}$$

---

‡1 a is and entirely arbitrary parameter in this game, its arbitrariness may be shown to follow from the inability of the theory to produce a value for the initial condition for $\beta(g^2)$. In lattice gauge theory, a discussion of its relevance for the choice of an action in the lattice has been given by Gliozzi et al [11]

so that for $\hat{b}_2 = \frac{b_1^2}{b_0}$ we get

$$c = \frac{b_2}{b_0} - \frac{b_1^2}{b_0^2} \tag{9}$$

and so on. Taking for reference the $\overline{MS}$ scheme [9.12] we have to third order

$$g^2_{\overline{MS}} = g^2_{tH}\left(1 + \frac{b_2}{b_0} g^4_{tH}\right) \tag{10}$$

$$g^2_{\overline{MS}} = g^2_{RA}\left(1 + \left(\frac{b_2}{b_0} - \frac{b_1^2}{b_0^2}\right) g^4_{RA}\right)$$

From the calculation of Tarasov et al [13]

$$b_2 = \frac{2}{(4\pi)^6}\left(\frac{2857}{2} - \frac{5033}{18} N_f + \frac{325}{54} N_f^2\right) \tag{11}$$

and it can be seen that $\frac{b_2}{b_0}$ is indeed a small number as Hagiwara [5] noticed, but we now see that $\frac{b_2}{b_0} - \frac{b_1^2}{b_0^2}$ for $N_f = 3$ is a smaller number. So both can be related to the $\overline{MOM}$ scheme [9,5] to third order by

$$g^2_{\overline{MOM}}(\mu) = g^2_{\overline{MS}}(\mu e^{-t_{\overline{MOM}}}) \tag{12}$$

$$\times\left\{1 + \left[8,319 - 1,257\, N_f + \frac{213}{16(33-2N_f)}\right] - \frac{g_{\overline{MS}}{}^4}{(4\pi^2)^2}\right\}$$

$$t_{\overline{MOM}} = 1 - \frac{12 - N_f}{3(33-2N_f)} \tag{13}$$

giving good results so far as $g^2_{\overline{MOM}}$ gives good results.

As for Adler's transformation it gives such large coefficients in sucessive orders that is no useful as an expansion parameter [ ].

Now we come to the second part of this work. Adler and Adler and Piran use a [3] [4] heuristic approximation for the effective action $S_{eff}[A]$ that incorporates the renormalization group aspects of the theory, as proposed by Pagels and Tomboulis [14]

$$S_{eff}[A] = \int d^4x \mathcal{L}_{eff}(x) \tag{14}$$

$$\mathcal{L}_{eff} = \frac{1}{4} \frac{F^2}{g^2(t)} \qquad F^2 = tr(F_{\mu\nu}F_{\mu\nu}) \tag{15}$$

$$t \equiv \frac{1}{4} \log (\frac{F^2}{\mu^2}) \tag{16}$$

Here $\mathcal{L}_{eff}$ is the effective lagrangian for the fields $F_{\mu\nu}$ including quark-source effects, and $g^2(t)$ is the running coupling constant defined by

$$t = \int_{g_0^2}^{g^2(t)} \frac{dg^2}{\beta(g^2)} \tag{17}$$

Using $\beta_A(g^2) = - b_0 g_A^4$ it is found

$$\mathcal{L}^A_{eff}(F^2) = \frac{1}{8} b_0 F^2 \log (\frac{F^2}{ex^2}) \tag{18}$$

so that at $F^2 = x^2$ there is a negative minimum

$$\mathcal{L}^A_{eff}(x^2) = - \frac{1}{8} b_0 x^2 \tag{19}$$

going to infinity with x^2. From it obtains a linear mean field potential

$$V_{mf} \sim R + cte \tag{20}$$

where $R > 2r$ and r is a regularization length[‡2].

A possible improvement of this result would suggest to go to a higher order in $\beta(g^2)$. At this point the results become controversial.

Using 't Hooft's prescription, Elizalde [7] got, for $\mathcal{L}_{eff}$:

$$\mathcal{L}^{tH}_{eff}(F^2) = \frac{1}{8} b_0 F^2 \{ \log\left(\frac{F^2}{ex^2}\right) + 4 \frac{b_1}{b_0^2} \log\left(\frac{b_0^2}{4b_1} \log \frac{F^2}{ex^2}\right) \} \tag{21}$$

He did not find a minimum, neglecting terms of $(\log(\frac{F^2}{ex^2}))^{-2}$. Keeping these terms, however, a mininum appears for.

$$y = \log\left(\frac{F^2}{ex^2}\right) \simeq .636 \tag{22}$$

or

$$F^2 \simeq x^2 e^{1.636} \tag{22a}$$

This number again is not well in the domain where we should expect the approximation to be valid.

Adler and Piran [4] proposed a leading $\log$ coinciding with (18). Its improvement formula does not lead to an effective

---

‡2 Notice however that one should expect the approximations used to be valid in the realms $|F^2/ex^2| << 1$, or $|F^2/ex^2| >> 1$.

lagrangian as (21). However, the first derivative of the effective lagrangian with respect to $\frac{1}{2}F^2$ they write in the form:

$$\varepsilon(E) = \frac{1}{2} b_o \left[\ell og(\frac{E}{\kappa}) + 2 \frac{b_1}{b_o^2} \ell og\, \ell og(\frac{E}{\kappa}) \right] \qquad (23)$$

which has a zero for

$$E_{min} = 1.680\,\kappa \qquad (23a)$$

Again, for this value the approximation is not entirely justified. Any expression like (21) leads to the neglect, in this region, of terms which have values comparable to the ones cancelling in (23).

The expression for $\beta_{RA}(g^2)$ proposed in (1) leads to an effective lagrangian:

$$\mathcal{L}^{RA}_{eff}(F^2) = \frac{1}{8} b_o F^2 \{\ell og\,(\frac{F^2}{ex^2}) + 4\frac{b_1}{b_o^2}\ \ell og\,(\frac{b_o}{b_o^2}\ell og\,(\frac{F^2}{ex^2}))\}$$

$$= \frac{1}{8} b_o x^2 e^{\,y+1}\{y + \frac{4b_1}{b_o^2}\ \ell og\frac{b_o}{4}y\} \qquad (24)$$

This expression, keeping all terms, has a negative minimum for $y_o \simeq 2.458$

$$\mathcal{L}^{RA}_{eff} \simeq -\frac{1}{8} b_o x^2 \ .\ (4\pi)^2\ .\ (0.339) \qquad (25)$$

It is amusing to notice that

$$F^2 = x^2 e^{3.458} \qquad (26)$$

$$\simeq 30x^2$$

which is in better agreement with the expected validity of the arguments that led to the original approximation (15).

We think in the light of these results that it is needed to be cautious before drawing a definite conclusion about the existence of a minimum for the effective action, but is to be mentioned that Adler and Piran [4] obtain remarkable results for the quark-quark potential from (23).

We have considered a prescription that allows to express the Callan-Symanzik function $\beta(g)$ in terms of the two coefficients which are prescription independent. It is, moreover, itself a prescription independent quantity, since it is uniquely determined as a Padé approximant.

It displays as desirable feature to provide a good approximation for the $\overline{\text{MOM}}$ prescription of Braaten and Leveille [9,5], so it may be as well used for calculation in the perturbative regime of QCD.

It also works in the other extreme regime of the theory, producing a minimum of the effective action as is wanted.

Whether this properties arise as a happy accident or have a deeper meaning, we cannot tell. But, undoubtedly, the RA prescription offers a competitive alternative to the others schemes that have been proposed, not being worse than any of them, and often rather better.

References

[1] G.'t Hooft, in Deeper Pathways in High Energy Physics, Orbis Scientiae 1977, edited by B.Kursunoglu et al (Plenun Press, New York, 1977), The whys of subnuclear Physics, Erice 1977), edited by A.Zichichi (Plenum Press, New York, 1979).

[2] C.G.Callan, Phys. Rev. D2 (1970) 1541
K.Symanzik, Commun. Math. Phys. 18 (1970) 227

[3] S.L.Adler, Phys. Rev. D23 (1981) 2905

[4] S.L.Adler and T.Piran, Phys. Lett. 117 B(1982)91

[5] K.Hagiwara, Phys. Lett. 118B (1982) 141

[6] W.Celmaster and R.J.Gonsalves, Phys. Lett. 42 (1979) 1435, R.Barbieri, L.Caneschi, G.Curci and E.D'Emilio, Phys. Lett. 81B (1979) 207
G.Grumberg, Phys. Lett. 95B (1980) 70
W.Celmaster and D.Sivers, Phys. Rev. D23 (1981) 227
P.M.Stevenson, Phys. Rev. D23 (1981) 2916

[7] E.Elizalde, Phys. Lett. 115B (1982) 307

[8] D.J.Gross and F.Wilczek, Phys. Rev. Lett. 30 (1973) 1343
H.D.Politzer, Phys. Rev. Lett, 30 (1973) 1346

[9] E. Braaten and J.Leveille, Phys. Rev. D24 (1981) 1369

[10] N.N.Khuri and O.A.Mc Bryan, Phys. Rev. D20 (1979) 881
H.Epstein and J.Iliopoulos, Annals of the New York Acad. of Sci. 373 (1981) 1.

[11] F.Gliozzi, F.Ravanini and S.Sciuto, Phys. Lett.118B (1982) 402

[12] A.J.Buras, E.G.Floratos, DG.Ross and C.T.Sachrajda, Nucl. Phys. B131 (1977) 308
W.A.Bardeen, A.J.Buras, D.W.Duke and T.Muta, Phys. Rev. D18 (1978) 3998

[13] O.V.Tarasov, A.A. Vladimirov and A.Yu.Zharkov, Phys. Lett. 93B (1980) 429

[14] H.Pagels and E.Tomboulis, Nucl. Phys. B143 (1978) 485