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PHASE TRANSITIONS IN AN EXTENDED HUBBARD MODEL

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INTRODUCTION.

Recently[1] phase transitions of the isotropic non-degenerate Hubbard Hamiltonian have been studied within the renormalization group formalism using the $\varepsilon = 4-d$ expansion to first order in ε. The purpose of these papers was to consider the effects of the spin-charge coupling on the critical properties of the electron gas. In the usual non-degenerate Hubbard model the charge field cannot become soft when the intra-atomic electron-electron interaction is positive. The question which arises is to know what happens to a second order magnetic transition when the order parameter is coupled to this non-soft degree of freedom. One of the conclusions of papers[1] is that spin-charge coupling may inhibit a first order transition.
The effects of simple constrains imposed on the charge field was considered as well as the possibility of Fischer renormalization of the critical exponents.

The present paper deals with the "extended" Hubbard model, an extension of the conventional Hubbard model which consists in taking account both of the intra-atomic Coulomb repulsion and of the Coulomb interaction between electrons at different sites. This model was considered among others by Röpke et al.[2] using functional integral methods. It is much richer than the original one. Instead of one interaction parameter, one can consider three energy parameters. In molecular field approximations a variety of solutions paramagnetic, spin ferromagnetic and antiferromagnetic, charge-ordered states (see also Ref.3) are obtained depending on the value and sign of these parameters. Moreover the extended Hubbard model was used by Bari[4] in the atomic limit to investigate the influence of electron-phonon interaction on the metal-insulator transition. This model was also considered by Ihle and Lorenz[5] to discuss the phase diagram of the ordered charge transition which can be first or second order depending on the value of inter-and intra-atomic electron-electron interaction.

In the present work, we derive the free-energy functional of the extended Hubbard Hamiltonian using the Hubbard-Stratonovich transformation[6] and taking account of the vector nature of the spin-field. If one considers that all neighbours are equivalent the number of fields is four two vectorial spin fields and two scalar charge fields. Two are soft (one spin and one charge field) and the two others are non-soft. One has to solve the problem of four coupled fields.

We show that as far as the ferromagnetic transition is concerned universality holds and that introducing a short-range interaction between neigbours yields the same functional as for the simple Hubbard model considered in 1.

However here we shall mainly focus on the charge-ordered transition. The spin-charge coupling is irrelevant (see section 3) in the renormalization group theory sense and we investigate the influence of the coupling of the two charge fields on the transition, by considering the relevance of the various coupling terms in the free energy functional expressions. For the three dimension model one of the coupling is marginal and requires a seperate analysis. We show however that the effect of the interaction between the two charge fields can result in a shift of the quadratic and quartic coupling parameters in the Landau-Ginsburg-Wilson functional, r and u, and consequently as in the ferromagnetic transition of the simple Hubbard model, one has the possibility of a tricritical point, first and second order transition depending on the details of the metallic band structure and the value of the interaction energy parameters.

2. THE FREE ENERGY FUNCTIONAL FOR THE EXTENDED HUBBARD MODEL.

We start with the Hamiltonian

$$H = \sum_{i,j\sigma} T_{ij}\, a^{+}_{i\sigma} a_{j\sigma} + \frac{1}{2} \sum_{\substack{\sigma\sigma' \\ i,j,i',j'}} V_{iji'j'}\, a^{+}_{i\sigma} a^{+}_{j\sigma'} a_{j'\sigma'} a_{i'\sigma} \qquad (1)$$

where as usual T_{ij} is the hopping integral between i and j, $a^{+}_{i\sigma}$ and $a_{i\sigma}$ are the creation and destruction operators for electrons on site i and with spin σ, $V_{iji'j'}$ represents the Coulomb and exchange interactions between electrons on sites i, i', j and j'.

We shall consider only one site (i) and two sites terms ($i \neq j$) and we define $V_{iiii} = U$, $V_{ijij} = K_{ij}$ and $V_{ijji} = I_{ij}$.

The one site terms read

$$V_1 = \frac{U}{2} \sum_{i\sigma} n_{i\sigma} n_{i-\sigma} = \frac{U}{4} N + \frac{U}{2} \sum_i \left(\frac{1}{4} n_i n_i - \vec{S}_i \vec{S}_i \right) \qquad (2)$$

using the relation

$$n_{i\uparrow} n_{i\downarrow} = \frac{1}{4} (n_{i\uparrow} + n_{i\downarrow}) + \frac{1}{2} (n_{i\uparrow} + n_{i\downarrow})^2 - \frac{1}{2} \vec{S}_i \vec{S}_i \qquad (3)$$

already discussed in Ref.1.

The two site terms can be written

$$V_2 = \frac{1}{2} \sum_{i,j} K_{ij} n_i n_j - \frac{1}{2} \sum_{i,j\sigma} I_{ij} \left(n_{i\sigma} n_{j\sigma} + a^{+}_{i\sigma} a_{i-\sigma} a^{+}_{j-\sigma} a_{j\sigma} \right) \qquad (4)$$

Defining $S^{+}_i = a^{+}_{i\uparrow} a_{i\downarrow}$ $S^{-}_i = a^{+}_{i\downarrow} a_{i\uparrow}$ and $S^{z}_i = \frac{1}{2} (n_{i\uparrow} - n_{i\downarrow})$ (5)

one can show that

$$a^{+}_{i\sigma} a_{i-\sigma} a^{+}_{j-\sigma} a_{j\sigma} = S^{+}_i S^{-}_j + S^{-}_i S^{+}_j \qquad (6)$$

and

$$\sum_{\sigma} n_{i\sigma} n_{j\sigma} = 2 S^{z}_i S^{z}_j + \frac{1}{2} n_i n_j \qquad (7)$$

This allows us to write the rotational invariant extended Hubbard Hamiltonian we consider in this paper in the form

$$H = H_0 + V_1 + V_2 = \sum_{i,j\sigma} T_{ij}\, a^{+}_{i\sigma} a_{j\sigma} + \sum_{i,j} \gamma_{ij}\, n_i n_j - \sum_{i,j} J_{ij}\, \vec{S}_i \vec{S}_j \qquad (8)$$

with $$\tilde{T}_{ij} = T_{ij} + \frac{U}{4}\delta_{ij} \qquad (9)$$

and $$\gamma_{ij} = \frac{1}{8} U \delta_{ij} + \left(\frac{1}{2} K_{ij} - \frac{1}{4} I_{ij}\right)(1 - \delta_{ij}) \qquad (10)$$

$$\gamma_{ij} = \frac{1}{2} U \delta_{ij} + I_{ij}(1 - \delta_{ij}) \qquad (11)$$

In that way we have taken into account the"vector" nature of the spin. In Ref.2, the spin flip terms are neglected and

$$U_1 = K_{ij} \quad , \quad U_2 = K_{ij} - I_{ij} \qquad (12)$$

Now we use the Hubbard - Stratonovich identity for a product of two operators in its vector form for the last term of (8)

$$e^{\vec{A}\cdot\vec{B}} = T_0 \int_{-\infty}^{+\infty} \mathcal{D}\vec{\phi}(\tau)\, \mathcal{D}\vec{\psi}(\tau)\, e^{\int_0^\beta \left[-\pi\vec{\phi}^2(\tau) - \pi\vec{\psi}^2(\tau) + i\sqrt{\pi}(\vec{A}-\vec{B})\vec{\phi}(\tau) + \sqrt{\pi}(\vec{A}+\vec{B})\vec{\psi}(\tau)\right] d\tau} \qquad (13)$$

and its scalar version for the second term of (8).
In (13) T_0 is the time-ordering operator needed to preserve the non-commutativity of the operators, which are now (imaginary) time dependent. In that way we obtain

$$e^{-\sum_{ij}\gamma_{ij} n_i n_j} = T_0 \int \prod_{ij} \mathcal{D}\eta_{ij}\, \mathcal{D}\lambda_{ij}\, e^{\sum_{ij}\int_0^\beta \left[-\pi\eta_{ij}^2 - \pi\lambda_{ij}^2 + 2\sqrt{\pi}\lambda_{ij}(i n_{ij} - \lambda_{ij}) n_i\right] d\tau} \qquad (14)$$

One has the symmetry relation

$$\lambda_{ij} = -\lambda_{ji} \qquad (\lambda_{ii} = 0)$$
$$\eta_{ij} = \eta_{ji} \qquad (15)$$

and using the same symmetry arguments, one has for the vectorial part

$$e^{\sum_{ij}\lambda_{ij}\vec{S}_i\cdot\vec{S}_j} = T_0 \int \prod_{ij} \mathcal{D}\vec{\phi}_{ij}\, \mathcal{D}\vec{\psi}_{ij}\, e^{\sum_{ij}\int_0^\beta \left[-\pi\vec{\psi}_{ij}^2 - \pi\vec{\phi}_{ij}^2 + 2\sqrt{\pi}\nu_{ij}(\vec{\psi}_{ij} + i\vec{\phi}_{ij})\vec{S}_i\right] d\tau} \qquad (16)$$

The partition function

$$\mathcal{Z} = \text{Trace}\left\{ T_\tau e^{-\beta H_0 - \int_0^\beta V(\tau)\,d\tau} \right\} \qquad (17)$$

can be written making use of (14) and (16)

$$\mathcal{Z} = \mathcal{Z}_0 \int \mathcal{D}\eta\, \mathcal{D}\lambda\, \mathcal{D}\vec{\phi}\, \mathcal{D}\vec{\varphi}\; e^{-\mathcal{H}(\eta,\lambda,\vec{\phi},\vec{\varphi})} \qquad (18)$$

with

$$\mathcal{H}(\eta,\lambda,\vec{\phi},\vec{\varphi}) = \sum_{i,j} \int_0^\beta d\tau\, \pi\left(\vec{\varphi}_{ij}^{\,2} + \vec{\phi}_{ij}^{\,2} + \eta_{ij}^2 + \lambda_{ij}^2\right) - \sum_{\sigma,\tau} \text{Trace}\, \log\left(1 - VG^0\right) \qquad (19)$$

$\mathcal{Z}_0$ and G_0 are the partition function and the Green's function respectively for the non interacting system when the potential V has matrix elements given by

$$i\sigma V_{ij\sigma'} = -\delta_{ij} \sum_j \Big\{ 2\left(i\sqrt{\pi\gamma_{ij}}\,(\eta_{ij} + i\lambda_{ij}) + \sqrt{\pi\nu_{ij}}\,\sigma\,(\varphi_{ij}^z + i\phi_{ij}^z)\right)\delta_{\sigma\sigma'} + \sqrt{\pi\nu_{ij}}\,(\varphi_{ij}^{-\sigma} + i\phi_{ij}^{-\sigma})\,\delta_{\sigma,-\sigma'} \qquad (20)$$

If one assumes that all neighbours are equivalent and using the symmetry relations (15) and similar relations for $\vec{\varphi}$ and $\vec{\phi}$, one can reduce the number of auxilliary fields by the following unitary transformations

$$\sum_j \sqrt{\pi\gamma_{ij}}\,\eta_{ij} = \sqrt{\pi(\gamma_0 + z\gamma_1)}\,x_1$$

$$\sum_j \sqrt{\pi\gamma_{ij}}\,\lambda_{ij} = -\sqrt{\pi z\gamma_1}\,x_2$$

$$\sum_j \sqrt{\pi\nu_{ij}}\,\vec{\phi}_{ij} = -\sqrt{\pi z\nu_1}\,\vec{y}_2 \qquad (21)$$

$$\sum_j \sqrt{\pi\nu_{ij}}\,\vec{\varphi}_{ij} = \sqrt{\pi(\nu_0 + z\nu_1)}\,\vec{y}_1$$

γ_0 and ν_0 are the diagonal and γ_1 and ν_1 the first neighbour off-diagonal interactions of Hamiltonian (8). One can see using the symmetry properties of η, λ, $\vec{\varphi}$ and $\vec{\varphi}'$ fields that the fictitious fields x_1, x_2, $\vec{y}_1$ and $\vec{y}_2$ correspond to the following real fields, chemical potential, staggered electrical field, uniform and staggered magnetic fields so that the potential (20) can be written :

$$
\begin{aligned}
i\sigma V_{i\sigma'} = & \Big\{ -2i\sqrt{\pi(\gamma_0+2\gamma_1)}\, x_1 - 2\sqrt{\pi}\,\gamma_1\, x_2 - \sigma\sqrt{\pi(\nu_0+2\nu_1)}\, y_1^{(3)} \\
& + i\sqrt{\pi 2\nu_1}\, y_2^{(3)} \Big\} \delta_{\sigma\sigma'} \\
& + \Big\{ -\sqrt{\pi(\nu_0+2\nu_1)}\, y_1^{-\sigma} + i\sqrt{\pi\nu_1}\, y_2^{-\sigma} \Big\} \delta_{\sigma,-\sigma'}
\end{aligned}
\qquad (22)
$$

The correspondence with the notations of Ref.2 being A $\leftrightarrow \vec{y}_2$, B $\leftrightarrow \vec{y}_1$ C $\leftrightarrow x_2$ and D $\leftrightarrow x_1$.

We develop Tr Log $(1 - VG^0)$ up to fourth order in V (second order in the interaction energies) (cf. Ref. 7) and using the Fourier transform representation with $q_i = (\vec{q}_i, \omega_i)$ to include both the momentum $\vec{q}_i$ and the Matsubara fermion frequencies ω_i.

We first consider the second order terms. The free energy reads :

$$
\begin{aligned}
\mathcal{F} = & \frac{1}{2}\int_q (1 + a_1 \chi^0(q))\, x_1(q)\, x_1(-q) + \frac{1}{2}\int_q (1 - a_2 \chi^0(q))\, x_2(q)\, x_2(-q) \\
& + \frac{1}{2}\int_q \mu_{12}^c\, x_1(q)\, x_2(-q) + \frac{1}{2}\sum_\alpha \int_q (1 - b_1 \chi^0(q))\, y_1^\alpha(q)\, y_1^\alpha(-q) \\
& + \frac{1}{2}\sum_\alpha \int_q (1 + b_2 \chi^0(q))\, y_2^\alpha(q)\, y_2^\alpha(-q) + \frac{1}{2}\sum_\alpha \int_q \mu_{12}^s\, y_1^\alpha(q)\, y_2^\alpha(-q)
\end{aligned}
\qquad (23)
$$

where α corresponds to the vector components. The function $\chi^0(q)$ is the non-enhanced susceptibility and

$$a_1 = U + 2z(U_1 + U_2)$$

$$a_2 = 2z(U_1 + U_2)$$

$$b_1 = U + 2z(U_1 - U_2) \qquad (24)$$

$$b_2 = 2z(U_1 - U_2)$$

the other coupling constants are given later in appropriate units.

In the limit U_1, $U_2 \to 0$, one recovers the results of Ref 1 i.e. two field .From (24), one can see that two fields one charge (x_2) and one spin (y_1) can become soft.

3. CHARGE ORDERED STATE TRANSITION.

Now we are interested in studying the system close to a charge instability eventually at low temperature but not very close to T= 0. This means that it is enough to consider the dependence of constants and fields at $\omega = 0$ with respect to small fluctuations of $\vec{q}$ around the wave-vector $\vec{K}_0$ corresponding to the maximum value of $\chi^0(\vec{q})$. Here we shall consider the case when the charge-wave density corresponds to an antiferromagnetic solution. One should have $\vec{k}_0 = \vec{g}/2 = \frac{\pi}{a}(1,0,0)$ the half lattice wave-vector. We therefore expand the susceptibility around its maximum

$$\chi^0(q) = \chi^0(k_0) - C|q - k_0|^2 \qquad (25)$$

and

$$(1 - a\chi^0(q)) = 1 - a\chi^0(k_0) + a'|\vec{q} - \vec{k}_0|^2 + \dots \simeq r_1 + |\vec{q} - \vec{k}_0|^2 \qquad (26)$$

with

$$r_1 \sim T - T_c^x \sim 1 - a\chi^0(k_0) \qquad (27)$$

In the same way one has

$$(1 - b\chi^0(q)) \simeq r_2 + |\vec{q} - \vec{k}_0|^2 \qquad (28)$$

with

$$r_2 = T - T_c^4 \sim 1 - c\chi^0(\vec{k}_0) \qquad (29)$$

If we put $\vec{q} - \vec{k}_0 = \vec{Q}$ and if we consider the expansion near $\vec{Q} = 0$, we must consider the fluctuations near $\vec{Q} = 0$ for all fields. We get

$$\mathcal{H}_0 = \frac{1}{2}\int_Q (r + Q^2)\, x_2(Q)\, x_2(-Q) + \frac{1}{2}\beta_1 \int_Q x_1(Q)\, x_1(-Q) + \mu_{12}^c \int_Q x_1(Q)\, x_2(-Q)$$

$$+ \frac{1}{2}\beta_2 \sum_\alpha \int_Q y_1^\alpha(Q)\, y_1^\alpha(-Q) + \frac{1}{2}\beta_3 \sum_\alpha \int_Q y_2^\alpha(Q)\, y_2^\alpha(-Q) + \mu_{12}^s \sum_\alpha \int_Q y_1(Q)\, y_2(-Q) \tag{10}$$

We have already neglected the Q^2- dependence of the Gaussian term of the non-soft field which will turn out to be irrelevant.
All third order terms will have the form

$$\int \xi'_{q_1} \xi_{q_2} \xi_{q_3}\, \delta(q_1 + q_2 + q_3)$$

where ξ stands for any of the fields $x_1, x_2, \vec{y}_1, \vec{y}_2$. Transforming to the new variables $q_i = Q_i + k_0$ one can see that one cannot have the moment conservation for $Q_i = 0$ and all third order terms vanish.

We now consider the fourth order terms and investigate the relevance of the various coupling terms.[6] We first integrate out all the variables $x_1(q), x_2(q), \vec{y}_1(q), \vec{y}_2(q)$ with momenta greater than $1/b$ where $b > 1$; we then rescale the momenta variables by b and the fields $x_1, x_2, \vec{y}_1, \vec{y}_2$ by C_1, C_2, C_3 and C_4 respectively. The choice of the C_i is made such that the coefficient of the q^2-term of the soft mode x_2 in the transformed Hamiltonian equals to unity and that the quadratic coupling constants of the non-soft fields are left unchanged therefore $C_2 = b^{3-\epsilon/2}$, $C_1 = C_3 = C_4 = b^{2-\epsilon/2}$ under renormalization group the terms $x_2^2 \cdot x_1^2$, $x_2^2 \cdot y_i^2$ transform like $b^{-2+\epsilon}$, the terms $x_2 \cdot x_1^3$, $x_2 \cdot x_1 \cdot y_i^2$ like $b^{-3+\epsilon}$ and the terms x_2^4, y_i^4, $x_1^2 \cdot y_i^2$ like $b^{-4+\epsilon}$ and are for that reason irrelevant after many interations. The same argument applies to the Q^2-term in the non-soft field as stated before.

We have considered the charge ordered transition. If we use the same discussion for the ferromagnetic transition the free-energy functional has the structure

$$(r+q^2)\vec{y}_1\vec{y}_1 + \vec{y}_2\vec{y}_2 + x_1x_1 + x_2x_2 + x_1x_2 + x_1\vec{y}_1\vec{y}_1 + x_2\vec{y}_1\vec{y}_1 + (\vec{y}_1\vec{y}_2)(\vec{y}_1\vec{y}_2) \quad (31)$$

with appropriate coupling constants and integrations. If one introduces the non-soft field $\bar{x} = x_1 + x_2$ and if one neglects $\vec{y}_2$ which is not coupled to any other field the functional for the soft field $\vec{y}_1$ reduces to the one obtained in Ref. 1 and this is consistant with the universality principle.

For the charge ordered transition if the dimensionality $d = 3$ $(\epsilon = 1)$ we keep only the terms x_2^4 and $x_2^3 \cdot x_1$ which transform like b^{ϵ} and b^{1} respectively. So putting $Q = q$, the Landau-Ginsburg-Wilson free energy functional reads

$$\mathcal{H} = \frac{1}{2}\int_{\vec{q}}(r+q^2)x_2(\vec{q})x_2(-\vec{q}) + \frac{1}{2}\beta_2\int_{\vec{q}}x_2(\vec{q})x_2(-\vec{q}) + \mu_{12}^c\int_{\vec{q}}x_1(\vec{q})x_2(-\vec{q})$$

$$+ \frac{1}{2}\beta_2\sum_\alpha\int_{\vec{q}}y_1^\alpha(\vec{q})y_1^\alpha(-\vec{q}) + \frac{1}{2}\beta_3\sum_\alpha\int_{\vec{q}}y_2^\alpha(\vec{q})y_2^\alpha(-\vec{q})$$

$$+ \mu_{12}^s\sum_\alpha\int_{\vec{q}}y_1^\alpha(\vec{q})y_2^\alpha(-q) + u\int_{\vec{q}_1\vec{q}_2\vec{q}_3}x_2(\vec{q}_1)x_2(\vec{q}_2)x_2(\vec{q}_3)x_2(-\vec{q}_1-\vec{q}_2-\vec{q}_3)$$

$$+ \gamma_{12}\int_{\vec{q}_1\vec{q}_2\vec{q}_3}x_2(q_1)x_2(q_2)x_2(q_3)x_1(-q_1-q_2-q_3) \quad (32)$$

In contrast to what happens for the magnetic transition in the simple Hubbard model, spin and charge fields are not coupled; all the coupling terms are now irrelevant and the terms containing spin fields can be dropped out from $\mathcal{H}$ if we consider only the charge instability.

Therefore one is left with the free energy functional

$$\mathcal{H} = \frac{1}{2}\int_{\vec{q}} (r + q^2)\, x_2(\vec{q})\, x_2(-\vec{q}) + \frac{1}{2}\beta \int_{\vec{q}} x_1(\vec{q})\, x_1(-\vec{q}) + \mu \int_{q} x_1(\vec{q})\, x_2(-\vec{q})$$

$$+ u \int_{\vec{q}_1,\vec{q}_2,\vec{q}_3} x_2(\vec{q}_1)\, x_2(\vec{q}_2)\, x_2(\vec{q}_3)\, x_2(-\vec{q}_1-\vec{q}_2-\vec{q}_3) + \gamma \int_{q_1,q_2,q_3} x_2(\vec{q}_1)\, x_2(\vec{q}_2)\, x_2(\vec{q}_3)\, x_1(-\vec{q}_1-\vec{q}_2-\vec{q}_3) \tag{33}$$

where $\mu = \mu^c_{12}$ and $\gamma = \gamma_{12}$. The coupling constants are given by

$$\beta = 1 + \frac{\pi^0}{2}\left[v + 2z(v_1+v_2)\right]$$

$$\mu = -\frac{i}{2}\pi^0\sqrt{2z(v_1+v_2)\left[v+2z(v_1+v_2)\right]} \tag{34}$$

$$u = -N''z^2\frac{(v_1+v_2)^2}{8}$$

$$\gamma = -i\frac{N''}{2\sqrt{2}}\sqrt{z^3(v_1+v_2)^3\left[v+2z(v_1+v_2)\right]}$$

where N'' is the fourth order electron loop (cf. Ref 7).

The coupling term $x_2^3.x_1$ is marginal in the renormalization group theory sense. Let us note however that if we define

$$x'_1(\vec{q}) = \sqrt{\frac{\beta}{2}}\, x_1(\vec{q}) + \frac{\sqrt{2}\,\mu}{2\sqrt{\beta}}\, x_2(\vec{q}) + \frac{\sqrt{2}\,\gamma}{2\sqrt{\beta}}\, x_2(-\vec{q}_1)\, x_2(-\vec{q}_2)\, x_2(\vec{q}+\vec{q}_1+\vec{q}_2) \tag{35}$$

and integrate over this new variable we are left with

$$\mathcal{H} = \frac{1}{2}\int_{\vec{q}} (r' + q^2)\, x_2(q)\, x_2(-q) + u' \int_{\vec{q}_1,\vec{q}_2,\vec{q}_3} x_2(\vec{q}_1)\, x_2(\vec{q}_2)\, x_2(\vec{q}_3)\, x_2(-\vec{q}_1-\vec{q}_2-\vec{q}_3)$$

(36)

where $u' = u - \frac{\mu\gamma}{\beta}$ $\qquad r' = r - \frac{\gamma^2}{2\beta}$

which is a simple Wilson type functional with coupling constants u' and r' modified by the two charge fields couplings. With (36) one obtains the usual Ising and Gaussian fixed points. The coupling between the two charge fields can affect the nature of the transition since u' can become negative inducing a first order transition. The quantities u, μ, γ, β depend on the band structure of the metallic band and in particular on the shape of the Fermi surface nesting. For instance using the results of the present paper one finds a condition for a first order transition

$$1 \gtrless \frac{3}{2}\pi^2 \left[U + 2z\,(v_1 + v_2)\right]$$

depending on the sign of N".

Therefore one can conclude that an investigation of the condition for the occurence of a first or second order transition using the functional (36) requires a detailed analysis of the susceptibility function and of the fourth-order bare fermion loop in the neighbourhood of the wave-vector $\vec{k}_o$ corresponding to the charge density transition.

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