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A NOTE CONCERNING THE GRAVITATIONAL UNDISTINGUISHIBILITY OF NEUTRINO FIELD AND ŞTOKESIAN FLUIDS

M. Novello

A Note Concerning the Gravitational Undistinguishibility of Neutrino Field and Stokesian Fluids -

M. NOVELLO
(Centro Brasileiro de Pesquisas Físicas)

The program of the Already Unified Field Theory proposed by Rainich and substained later by Wheeler and Misner[1] was based on the idea that it is possible to express the laws of electrodynamics and of gravitation in terms of pure geometry. The electromagnetic field, through its stress-energy tensor, modifies the curvature of the space-time in such specific way that it is possible, knowing only the geometry to go back and enumerate all properties of the electromagnetic field responsible for the bending of the space-time.

The apparent success of this program has led some authors to try to incorporate the massless neutrino field into such scheme. Some progress has been made on this way[2], but the final task of geometrization of the neutrino field has not yet been achieved.

Recently a fresh result of Tupper[3] has modified sensibly such picture. He analyses some specific examples of geometry which have pure electromagnetic field as it source, like the Bertotti-Robinson Universe, Reisner-Nordstron and Kerr-Newman solutions. In these cases it is possible to interpret the stress-energy tensor in terms of pure viscous fluid, i.e, a fluid which posses bulk viscosity and anisotropic pressure of the Cauchy linear type. Such interpretation makes the Already Unified Program to lose most of its appealing.

The purpose of this note is to show that the geometrization of the neutrino field has the same kind of dificulty as the electromagnetic case. In order to demonstrate this we will analyse a specific model of an Universe filled solely with

neutrino. We then show that a non-linear Stokesian fluid with heat conduction can give origin to the same geometry as the neutrino field.

The unique model of an Universe with neutrino is the Novello-Soarēs class of geometry[4]. We will consider here the steady-state anisotropic world of the type

$$(1)\quad ds^2 = dt^2 - 2\,e^{Mt}\,d\,\times dt - e^{Jt}\,(dy^2 + dz^2)$$

in which M and J are arbitrary parameters which are constrained to satisfy the inequality $J(J - M) < 0$.

We choose a local Lorentzian frame in which the tetrads are given by

$$e^{(0)}_{0} = 1;\ e^{(0)}_{1} = e^{(1)}_{1} = -A;\ e^{(2)}_{2} = e^{(3)}_{3} = C$$

In this tetrad frame the unique non null components of the contracted Riemann tensor are

$$(2)\quad R_{00} = R_{-11} = -R_{01} = 2\,J\,(J - M)$$

Novello and Soares have show that the source of this geometry is a neutrino field with current $j^{\mu} = \overline{\Psi}\,\gamma^{\mu}\Psi$ in the X-direction.

We claim here that it is possible to interpret the source of such geometry as a fluid with heat conduction and of the quadratic Stokesian type[5].

We write the stress-energy tensor in the tetrad frame:

$$(3)\quad T_{AB} = \rho\,V_A\,V_B - \frac{\rho}{3}\,(\eta_{AB} - V_A\,V_B) + q_{(A}\,V_{B)} + \Pi_{AB}$$

Where $\eta_{AB} = \mathrm{diag}\ (+ - - -)$,

and set $V_A = \delta^0_A$. The heat flux q_A and the anisotropic pressure Π_{AB} are determined by the phenomenological equations of state

(4a) $q_A = \lambda \dot{V}_A$

(4b) $\Pi_A^B = -2 \left[\Theta_A^C \quad \Theta_C^B - \frac{1}{3} \left(\Theta_{\mu\nu} \Theta^{\mu\nu}\right) \delta_A^B \right] \left[\frac{J}{J+M} \right]$

A straightforward calculation shows the expansion Θ , the acceleration $\dot{V}_A$ and the shear $\sigma^A_B \equiv \Theta^A_B - \frac{\Theta}{3} \delta^A_B$ are given by

$$\Theta = M + 2J$$

(5) $$\frac{1}{2} \sigma^1_1 = -\sigma^2_2 = -\sigma^3_3 = \frac{1}{3}(M - J)$$

$$\dot{V}_A = (0, M, 0, 0)$$

A direct computation, using these values in the expression of T_{AB} give

(6) $$T_{00} = T_{11} = -T_{01} = \rho$$

in which we choose the heat conduction coefficient λ to take the value $\lambda = \frac{2J}{M}(J-M)$. Einstein's equations are satisfied if we set $\rho = -2J(J - M)$.

This completes our proof. No reference is made to the neutrino field which is completelly substituted by the stokesian fluid.

We conclude that an observer which mesures the gravitational properties of Novello-Soares Universe is not able to answer the following question: is the source of such geometry a pure neutrino field or a Stokesian viscous fluid with a stress-energy tensor given by (3) and phenomenological equations characterized by (4) ?

The difficulty of the geometrical characterization of electromagnetic field demonstrated by Tupper, and of the neutrino field which we have shown in this paper rises the important question: the Rainich-Wheeler-Misner program of geometrization of the long-range fields must be abandoned ?

References

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