SPACE DIMENSIONALITY: WHAT CAN WE LEARN FROM STELLAR RADIATION AND FROM THE MÖSSBAUER EFFECT?[1]

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Abstract

Why is space 3-dimensional? After a brief review of the modern approaches to this query, enphasizing those papers which touched upon epistemological problems, it is stressed that there are two questions which deserve special attention. They are: the possibility that the large-scale dimensionality of space has been changing in the course of time; and the actual accuracy of the experimental determinations of the dimensionality of space covering a very large scale from the micro to the macro-cosmos. It is argued in this paper that both Stellar Radiation and the Mössbauer Effect can be used to sketch the answers to these questions.

Key-words: Space; Dimensionality; Astrophysics; Mössbauer effect.

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[1] Invited paper for a book in honour of Prof. *Jacques Danon*, edited by R.B. Scorzelli, I.S. Azevedo and E. Baggio Saitovitch, to be published by Éditions Frontières.
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Kant was the first to point out that the threefold dimension of space can be related to the structure of a particular physical law (in his case, Newton's gravitational law)[1]. Ehrenfest[2], in 1917, faced this problem from a different point of view, by formulating the incisive question*: "How does it become manifest in the fundamental laws of Physics that space has 3 dimensions*?". This approach inaugurated the modern discussion about *dimensionality*. However, it is only in the framework of Modern Field Theory that this query actually acquires its full physical meaning. Indeed, there is nowadays a widespread hope that theories in higher dimensions (*when supplemented with dimensional reduction*) may lead to a deeper and unified understanding of extremely High-Energy Physics. Obviously, the very fact of *imposing* the process of dimensional reduction is equivalent to assuming *a priori* the threefold dimension of space, which is just what should be questioned. To the best of our knowledge, there is as yet no satisfactory and unambiguous answer to the problem of dimensional reduction, even when the so called spontaneous compactification process is taken into account.

In the last four decades some progress has been achieved in the natural philosophy of space, including the dimensionality issue[3]. It is now clear that, besides Ehrenfest's problem, there is another complementary question which must be answered:

— *How do the fundamental laws of Physics* <u>*entail*</u> *space dimensionality*?

Contrary to Ehrenfest's question, in which the tridimensionality plays a special rôle *a priori*, here the dimensionality may be taken as an unknown quantity to be experimentally determined. Thus, it is implicit in this alternative program that one should look for physical laws which do not exhibit a singular behaviour in three dimensions and could be argued to be valid for any dimensionality. That such kind of law does exist was shown by the authors elsewhere[4], and are, for instance, Stefan-Boltzmann and Wien's laws of blackbody radiation, Planck and de Broglie relations.

In spite of the aforementioned progress, there still are questions which deserve some special attention, namely:

— *Is the dimensionality of space the same in the macro and micro-cosmos*? and

— *Has the large-scale dimensionality of space changed in the course of time?*

More precisely: what are the *temporal* and *spatial* "scales" to which the experimental constraints on space dimensionality known up to now actually apply?

These questions were briefly treated by the authors[4] and its relevance was recently stressed by Max Jammer in the third edition of his classical book on Space[3].

In this essay we intend to revisit this problem by focusing two phenomena involving very different spatial scale: Stellar Spectra and the Mössbauer Effect. Trying to render this paper more readable to those not familiarized with the dimensionality problem, we begin by pointing out the main features of Ehrenfest's fundamental papers[2]. There, several physical phenomena, for which qualitative differences between three-dimensional ($\Re^3$) and n-dimensional ($\Re^n$) spaces are evident, have been discussed. These aspects, which distinguish the $\Re^3$ Physics from the $\Re^n$ one, are called by him "singular aspects" and his works were aimed at stressing them. A crucial assumption is implicit in his approach, namely, that it is possible to make the formal extension $\Re^3 \rightarrow \Re^n$ for certain laws of Physics and, then, finding one or more principles which, in conjunction with these laws, can be used to single out the proper dimensionality of space. For this program to be carried out, in general, the *form* of a differential equation — which usually describes a physical phenomenon in three-dimensional space — is maintained and its validity for an arbitrary number of dimensions is *postulated*. This is supported by what Barrow & Tipler called *"Principle of Similarity"*[5], *i.e.*, alternative physical laws in hypothetical higher dimensional spaces should mirror their actual form in three dimensions as closely as possible; in practice, the form of the law is maintained, perhaps for anthropomorphic reasons.

The epistemological limits of Ehrenfest's approach and of their underlying postulates were discussed by the authors[4], and other philosophical questions were raised by us in other papers[6-10]. Considering our purpose here, it is sufficient to say in a nutshell that, besides the philosophical criticism, we have stressed[4] that even when a particular physical law does not show "singular aspects" as the number of dimensions changes, it is still possible to use it shed light on the dimensionality of space problem. One example, as mentioned before, is Planck's law.

Starting from the validity in $\Re^n$ of Thermodynamics — which depends on topological properties of space — it is straightforward to generalize Stefan's and Wien's law, leaving indeterminate the explicit form of $F(\nu/T)$. If we now make the same statistical assumption as Planck did in order to determine the explicit form of this function F, we still find that the energy of a quantum is $\varepsilon_0 = h\nu$, for any $\Re^n$, *i.e.*, for n=3, Planck's law does not show a "singular behaviour" in Ehrenfest's sense. In spite of this fact, it was shown by the authors for the first time[10] that blackbody phenomenology, extended to $\Re^n$, can be used to demonstrate the validity of the de Broglie relation, from which experimental limits on space dimensionality can be determined, for instance, by neutron diffraction.

In a subsequent paper, blackbody radiation was used by Grassi *et al.* to set upper limits to deviations from integer value (n=3) for the spatial number of dimensions, by analizing the 2.7 K background radiation[11]. They obtained $|n-3| < 0.02$. This was interpreted by Grassi *et al.* as "the limit between the fractal dimensions of local space and fractal dimensions of space on scales of the horizon". A more stringent limit, $|n-3| < 1.5 \times 10^{-9}$, was obtained from other astronomical observations[12]: data on the precession of the periastron of the planet Mercury and of the binary pulsar PSR 1913 + 16. The perihelion shift for Mercury gives a bound of the same order of magnitude in another paper[13]. The possibility of probing space dimensionality from blackbody radiation was later reconsidered[14], leading to $|n-3| \leq 10^{-3}$ and to the conclusion that thermal radiation methods could improve on existing low energy bounds (obtained from microscopic scale) for $|n-3|$ just if the cosmic background radiation could be measured with a precision greater than 10^{-9}, unattainable within present experimental capabilities. At the quantum level, more precise bounds were obtained from the anomalous (g–2) electron factor[13,15], $|n-3| < (5.3\pm2.5)\times10^{-7}$, and from Lamb shift in hydrogen[16] — which improved this limit by four order of magnitudes — yielding $|n-3| < 4\times10^{-11}$. All this results depends on the assumption that time is one dimensional.

We can now put together the results for the relation between any possible deviation $|\varepsilon| \equiv |n-3|$ of the integer space dimensionality n=3, and the length scale of the experimental observation: for scales of the order of the Bohr radius ($\approx 5\times10^{-11}$ m) one obtains $|\varepsilon| < 4\times10^{-11}$; for those of the order of the distance between the Sun and Mercury ($\approx 6\times10^{10}$ m) — and also for distances like that of the binary PSR ($\approx 10^{20}$ m) — one has $|\varepsilon| < 10^{-9}$; and for the horizon scale (taking the radius of the Universe $\approx 10^{26}$ m) one gets the weaker bound $|\varepsilon| < 2\times10^{-2}$.

From what was briefly reviewed up to now two main conclusions can be established: *i)* that the topological dimensionality of space discussed within the framework of Newtonian Mechanics[2,17], General Relativity[18,19], and Quantum Mechanics[4,19], seems to confirm the threefold nature of space for a very large range of spatial scales; *ii)* so far it is admitted that space can have a fractal dimensionality, the experimental upper limits for this assumption, ε, changes only one order of magnitude as a function of a typical length scale between $10^{-10} \div 10^{20}$ m, without considering the cosmic background radiation constraint, $|\varepsilon| < 10^{-3}$.

It is rather important to have data for much smaller length scales (compared to the atomic scale) as well as for an unexplored region outside the solar system (something between 10^{10} m and 10^{20} m).

Going to the micro-cosmos, it was estimated that $|\varepsilon|$ could be less than 5×10^{-7} on a length scale characterized by the Compton wavelength of the electron ($\approx 4\times10^{-13}$ m), and $|\varepsilon| < 10^{-5}$ from the anomalous magnetic moment of the muon (corresponding to a spatial scale $\approx 4\times10^{-15}$ m)[13]. Now, from the Mössbauer Effect we can probe a characteristic nuclear length $\approx 10^{-15}$ m, and due to the high accuracy it can be measured, this effect could be used to impose a more stringent limit on ε for a spatial scale $\approx 10^{-15}$ m.

Considering the macro-cosmos, it would be important to have data from stars within the aforementioned unexplored range $10^{10} \div 10^{20}$ m.

Let us now discuss these two possibilities in more detail. The structure of the atomic spectra in remote stars allows us to probe space dimensionality at cosmic scale. To our purpose, let us assume that there are stars that in a good approximation can be considered as blackbodies[20,21]. Clearly they are not perfect blackbodies, but, in any case, it can be shown that the Stefan-Boltzmann law is a reasonably good approximation to their radiation. Indeed, if one expands the radiation intensity in the series

$$I = I_0 + I \cos\theta + I \cos^2\theta + \ldots$$

(which converges quickly) and takes just the first two terms, "the relation between the energy density of the radiation and the temperature of matter [in the interior of the star] is exactly that which is obtained for perfect thermal equilibrium"[21], *i.e.*, the Stefan-Boltzmann law, $E = aT^4$, is still valid. It is straightforward to generalize this law for n-dimensional space, yielding $E = AT^n$; in this case, Wien's law becomes $\lambda_n T = B$, where A and B are unknown constants. If now we choose two stars (i and j), distants $10^{10} \div 10^{20}$ m from the Earth, and assume that space dimensionality is the same in both spatial regions, a combined measurement of the ratios

$$\lambda_i/\lambda_j = T_j/T_i \quad \text{and} \quad E_i/E_j = (T_i/T_j)^{n+1}$$

can be used to determine the integer value of n. This assumption is more plausible if we choose stars having the same red shift due to the isotropy of space. The advantage of this method is that it does not depend on the explicit form of the function $F(\nu/T)$ as is the case for background cosmic radiation. However, it raises naturally the question of how can we test the assumption that two stars equidistant from the Earth (same red shift) are in spaces of the same (fractal) dimensionality and what would be the precision of this statement (the bound on $|\varepsilon|$). Of course, we can experimentally compare two blackbodies on Earth. However, if we intend to perform a more accurate measurement, we are led to the Mössbauer laboratory.

We envisage at least two Mössbauer experiments to determine $|\varepsilon|$, which will be sketched here and treated in detail elsewhere. The first, more obvious, involves the measurement of the so-called Lamb-Mössbauer or Debye-Waller factor, which gives the fraction of gamma rays emitted or absorbed without energy loss as a function of the temperature T. Of course, this factor depends on the space dimensionality through the phonon spectra. In practice, we could use the variation with temperature of the energy of recoil-free gamma rays from a homogeneous solid[22], given by

$$\partial\nu/\partial T = -\, \nu\, C_L/(2Mc^2),$$

where C_L is the specific heat of the lattice and M is the gram atomic weight of the Mössbauer element (iron, for instance); C_L is given by the Debye model and its expression for n-dimensions is well known[23].

The second possibility, which seems more feasible, is to measure the isomer shift between two identical atoms, in an experimental arrangement similar, in a certain sense, to that of Pound and Rebka experiment[24], but eliminating the gravitational effect by leveling source and absorber. If there is a difference ε between the space dimensionality in the vicinity of the source and that of the absorber it would appear as an isomer shift-like effect, and could be detected with good accuracy. In this case, we should take great care with the temperature control, as pointed out by Josephson[25]. Note that in experiments involving two bodies (in a certain length scale), we can invoke the homogeneity and isotropy of space to interpret ε as a limit to the difference between the fractal dimension of local space and the nearest topological dimension, 3, no matter the distance separating the experimental objects (stars, blackbodies or Mössbauer nuclei). We intend to present a detailed numerical analysis of the available data in a forthcoming paper.

Here we must only note that the interpretation of the suggested Mössbauer experiment depends on the interaction between s-electrons and nuclei, *i.e.*, on the Coulomb electrostatic potential $\propto \alpha_C/r^{1+\delta}$. For macroscopic bodies, from Cavendish-like experiments, it was established[12] that $|\varepsilon-\delta| < 10^{-16}$, which does not give a bound on δ, but supports the assumption $\delta = \varepsilon$. However, we cannot exclude the possibility that δ arises from a broken symmetry, as shown to be a possible case for the Newtonian gravitational potential[26].

Let us make now some final remarks concerning the *time scale* of the arguments discussed here.

Ehrenfest's stability argument applied to planetary motion is valid for distances of the order of the solar system and in a time scale large enough to make the evolution of life possible on Earth, as mentioned by Whitrow[17]. One may add to Whitrow's biophysical argument that it is not sufficient that the intensity of solar radiation on Earth's surface should not have fluctuated greatly for the blossoming of life on Earth; the fact that the radiation spectra of the Sun did not strongly fluctuate is also required.

On the other hand, Tangherlini's work about the stability of hydrogen atoms[18] should be invoked here to suggest the validity of Chemistry in the same time and spatial scale as a necessary condition, although not sufficient, since at least Chemical Thermodynamics of irreversible processes should also be valid to guarantee life on Earth.

Going beyond the solar system, informations from stellar radiation and from the cosmic background radiation are unique in the sense that they have the advantage of telling us that space dimensionality has been the same (up to $|\varepsilon|$) for an even bigger time scale.

As a final conclusion, we would say that the analysis of Mössbauer Experiments and of Stellar Spectra can be used in a complemantary way to shed light on the pervasive problem of space dimensionality in Physics at the extremes of the length scale: from nuclei to stars.

This work, in which Mössbauer Effect and Stellar Radiation are put together for the same scope, *i.e.*, to experimentally determine space dimensionality, is dedicated to the memory of Professor Jacques Danon, a man of wide scientific and cultural interests, who introduced the study of the Mössbauer Effect in Latin America[27] and later became Director of the

Brazilian National Observatory, in the hope that he would appreciate its reading.

ACKNOWLEDGMENTS

We would like to thank Ximenes A. da Silva for fruitful discussion and the Organizers of the book in honour of Prof. Jacques Danon for their kind invitation to write this paper. One of us (F.C.) thanks CNPq of Brazil for financial support.

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