ISSN 0029-3865

CBPF-NF-034/84

VECTOR MESON AND AXIAL-VECTOR DIQUARK DECAY CONSTANTS

by

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ABSTRACT

We propose a natural generalization to the case of unequal constituent masses of the formula which gives the correct decay constants for vector mesons of the quarkonium type. Within the philosophy where diquarks are used as baryonic constituents, this allows us to evaluate the decay widths for all vector particles both mesons and diquarks to be used later on the calculate the baryon lifetimes.

Key-words: Vector-mesons; Diquark; Baryon decay.

In a recent paper[1], we have evaluated the decay constants of pseudoscalar mesons and of scalar diquarks considered as *bona fide* particles. This is done following a philosophy where baryons are considered as quark-diquark systems much in the same way as mesons are quark-antiquark objects. The actual realization[2] of this scheme has shown its viability in providing an accurate description of the hadronic spectrum both in the mesonic and in the baryonic sector from light to heavy masses using a relativistic equation of the Todorov type[3]. To complete the proof that diquarks are convenient tools in describing the baryons, one needs also to estimate their decay constants and the result of ref. 1 is a first step in this direction. The next step would be the discussion of axial-vector diquarks. Unfortunately, the method used in ref. 1 does not lend itself to an immediate generalization to the case of vector particles owing to the spin complications of the problem. As it is well known, these complications are quite substantial and we will proceed in a heuristic way to bypass them. As we will see this can be done in a rather simple way be introducing an Ansatz consistent both with sum rule calculations[4,5] and with the heuristic expression for the wave function at the origin obtained from the Van Royen-Weisskopff formula[6]. The Ansatz relies on the use of relativistic variables appropriate to the case of quark-quark and quark-antiquark states with unequal masses. This will allow us to evaluate the decay constants of all vector particles both mesons and diquarks in terms of the meson masses using the diquark masses calculated in ref. 2. The knowledge of the axial-vector diquark decay constants is particularly relevant for getting the lifetime of particles such as the Λ_c^+

which in a quark-diquark scheme[7] is dominated by the contribution of axial-vector diquarks. This point will be considered in a subsequent paper[8].

The leptonic width of vector mesons of the quarkonium kind is given by the Van Royen-Weisskopff formula as[6,9]

$$\Gamma_\ell(n^3S_1 \to \ell^+\ell^-) \simeq 16\pi\,\alpha^2\,Q_j^2\,|\psi_n(o)|^2/M_V^2 \tag{1}$$

where a colour factor of 3 has been properly inserted in eq. (1) as compared with the derivation of ref. 6. In eq. (1), α is the fine structure constant, M_V is the mass of the vector particle in the state n^3S_1 and Q_j is the charge of the j-constituent quarks. $\psi_n(o)$ is the wave function of the bound $q\bar{q}$ system at the origin.

The experimental data for all vector mesons of the quarkonium type are known to be consistent with the empirical rule[9]

$$\Gamma_e(V \to e^+e^-)/Q_j^2 \simeq \text{const} \simeq 12\ \text{KeV} \tag{2}$$

which follows from the data as one can see from Table I. The latter shows that although the values for Γ_e vary over well one order of magnitude, the ratios Γ_e/Q_j^2 present very small variations as compared with the mass differences involved.

The above considerations imply for the case at hand

$$|\psi_n(o)| = K\,M_V \tag{3}$$

Inserting eq. (3) in the formula[10] $f_V = 2\sqrt{3}\;|\psi(o)|/\sqrt{M_V}$ for the decay constant of vector mesons, one has

$$f_V \simeq 2\sqrt{3}\ K\ M_V^{1/2} \tag{4}$$

where K is the proportionality constant in eq. (3).

The above result, however, is only valid in the case of vector mesons of the quarkonium type. In the case of quarks of unequal masses $m_1 \neq m_2$, sum rule calculations differ from the above estimate (eq.(4)). For instance, sum rules with U(6) wave functions[4] give the relation

$$\frac{f_\rho}{f_{K^*}} \simeq 14.3\ \frac{m_{K^*}}{m_\rho}\ \frac{m_\pi^2}{m_K^2} > 1 \tag{5}$$

(contrary to what one would expect if eq. (4) were valid). Similarly, QCD sum rule calculations give[5]

$$\frac{f_{D^*}}{f_{K^*}} \simeq \frac{f_D}{f_K} \quad , \quad \frac{f_{D^*}}{f_\rho} \simeq \frac{f_D}{f_\pi} \tag{6}$$

where both f_D/f_K and f_D/f_π are of order unity[1] in striking contrast, again with eq. (4) if applicable.

These considerations suggest that a generalization of eq. (4) is needed in the case of vector mesons made of quarks with unequal masses.

In the quarkonium case, the masses of the constituent quark and antiquark being the same, the c.m. of the system coincides with its origin and the effective bound state mass seems an appropriate variable to use: the heavier this mass, the larger the probability that the quark-antiquark system be found in its center of mass (i.e., the larger the wave function at the origin). When the constituents have unequal masses, however, the c.m. does not coincide with the origin and the effect becomes more and more important with the increase of the mass difference of the constituents. We correspondingly expect the probability of finding the system at the origin (i.e. $|\psi_n(o)|^2$) to decrease as $|m_1-m_2|$ increases. This leads us to conjecture that in the case $m_1 \neq m_2$, the mass of the bound state M_V is not the proper quantity to use in (4) but should be replaced by an appropriate relativistic variable which reduces to $M_V/2$ in the quarkonium case. We want to argue that the proper generalization of eq. (3) to the case of unequal constituents is

$$|\psi(o)| = 2K(E_R + m_R) \tag{7}$$

where E_R and m_R are the effective energy and effective mass (or the relativistic reduced mass) which enter into Todorov's equation[3] and which are defined as

$$E_R = \frac{W^2-(m_1+m_2)^2}{2W}$$
$$m_R = \frac{m_1 m_2}{W} \tag{8}$$

where W is the c.m. total energy and m_1, m_2 are the constituent masses. K is the same factor as in eq. (3).

It has already been stressed in previous papers[1,2] the importance of using the above variables when dealing with a relativistic equation in order to obtain both the proper hadronic mass spectrum within a quark-diquark scheme[2] and the correct estimates for the decay constants of pseudoscalar mesons[1].

From (8) we notice that

$$E_R + m_R = \frac{W^2 - (m_1 - m_2)^2}{2W} \tag{9}$$

has exactly the property we expect $|\psi(o)|$ to have on the grounds of our previous intuitive discussion, namely to decrease at given W when $|m_1 - m_2|$ becomes large and to reduce to $M_v/2$ in the case $m_1 = m_2$. We are therefore led to generalize eq. (3) to eq. (7) which we take to be our main Ansatz. Inserting it into Kraseman's equation[10] $f_v = 2\sqrt{3}\,|\psi(o)|/\sqrt{M_v}$ we are then led to replace eq. (4) by

$$f_v = 2\sqrt{3}\ 2K\ \frac{E_R + m_R}{\sqrt{M_v}} \tag{10}$$

which reduces to (4) in the quarkonium case.

To proceed, we now fix first of all the proportionality factor K in eq. (10) by using the experimental value for the leptonic decay width of the ϕ meson. This gives

$$K \simeq 2.12\ (\mathrm{MeV})^{1/2} \tag{11}$$

Using now the experimental mass values[11]: $M_\rho = 770$ MeV, $M_{K^*} = 892$ MeV, $M_{D^*} = 2010$ MeV and $M_{F^*} = 2040$ MeV and the current quark masses[12] $m_u = 5.1$ MeV, $m_d = 8.9$ MeV, $m_s = 175$ MeV and $m_c = 1270$ MeV, we get the following decay constants

$$f_\rho \simeq 204 \text{ MeV} \quad ; \quad f_{D^*} \simeq 200 \text{ MeV}$$
$$f_{K^*} \simeq 212 \text{ MeV} \quad ; \quad f_{F^*} \simeq 236 \text{ MeV} \qquad (12)$$

These results are, for what said above, in automatic agreement with the data for vector mesons of the quarkonium type but represent a generalization of the naive eq. (4) to the case of unequal constituent masses.

We can now proceed to evaluate the axial-vector diquark decay constants by just replacing the vector meson masses in the previous calculation with the diquark masses evaluated previously by Lichtenberg et al[2]. (i.e., with $M^{(1)}_{uu} \simeq M^{(1)}_{dd} \simeq 820$ MeV, $M^{(1)}_{sd} \simeq M^{(1)}_{su} \simeq 892$ MeV and $M^{(1)}_{cu} \simeq M^{(1)}_{cd} \simeq 1910$ MeV to obtain

$$f^{(1)}_{ud} \simeq 208 \text{ MeV}$$
$$f^{(1)}_{sd} \simeq 212 \text{ MeV} \qquad (13)$$
$$f^{(1)}_{cd} \simeq 181 \text{ MeV}$$

With the above results, we could now obtain the decay amplitudes of axial-vector diquarks which would have the same expression as that well known for $\rho^+ \to \ell^+ + \nu$ ($\Gamma_{\rho^+ \to \ell\nu} = \frac{G^2}{8\pi} f^2_\rho M^3_\rho$). This amplitude in the case of heavy diquarks is presumably the leading contribution of the process known as W-exchange for baryons decay. We shall come back to this problem in a forthcoming paper[8] discussing the lifetime of the charmed baryon Λ^+_c.

TABLE I

Empirical rule Γ_e/Q_j^2

meson	Γ_e (KeV)	Quark content	Q_j^2	Γ_e/Q_j^2 (KeV)
ρ	6.5 ± 0.5	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	1/2	13 ± 1
ω	0.76 ± 0.17	$\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$	1/18	13.6 ± 2.2
ϕ	1.34 ± 0.08	$s\bar{s}$	1/9	12 ± 0.72
J/ψ	4.9 ± 0.6	$c\bar{c}$	4/9	11.1 ± 1.3
Υ	1.26 ± 0.21	$b\bar{b}$	1/9	11.4 ± 1.9

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