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TIME DEPENDENT DELAYED β-RAY SPECTRA FROM FISSION FRAGMENTS

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ABSTRACT

The time-dependent spectra of electrons from fission fragments are theoretically studied. The β-decay constants and electron spectra are calculated from the gross theory of β-decay. Theoretical and experimental curves are in a good agreement.

1 - INTRODUCTION

In these several years, studies of nuclear β-decay properties in the region far from the β-stability line have made a considerable progress. In particular the gross theory proposed by Yamada and Takahashi[1] is found to be a powerful tool for analyzing the strength function phenomena as well as for predicting unknown half-lives. The recent experiments[2] on β-strenght function

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seem to suport the theory.

In this paper, we apply the gross theory to the calculation of the electron spectra from fission fragments. Since the electron spectra from fission fragments are quantities averaged over a bulk of nuclei, the gross theory is suitably applied. Some preliminary results were reported earlier[3].

2 - CALCULATION METHOD

The most recent experiment on the electron spectra from fission fragments was given by Tsoulfanidis et al.[4]. They measured three different types of spectra: the building up spectra after initiation of a constant fission rate in a ^{235}U foil (Start-up), the decay of the spectrum after termination of a constant fission rate (Shut-down), and the spectrum after sudden burst of fission produced by a reactor power pulse (Pulse). In this paper, we discuss the shut-down and pulse espectra only, since the start-up spectra depend on the history to acquire the constant fission rate.

2.1 - Electron Spectrum

The abundances of fragment nuclei $n_{ZA}(t)$ at time t after fission satisfy the following coupled differential equations

$$\frac{d\, n_{ZA}(t)}{d\, t} = - \lambda_{ZA}(t) + \lambda^{\beta}_{Z-1\ A}\ n_{Z-1\ A}(t) + \lambda^{n}_{Z-1\ A+1}\ n_{Z-1\ A+1}(t) \qquad (1)$$

where Z and A are, respectively, the atomic number and the mass number, λ^{β}_{ZA} the decay constant of β-decay, λ^{n}_{ZA} the rate of delayed neutron emission, and $\lambda^{\beta}_{ZA} = \lambda_{ZA} - \lambda^{n}_{ZA}$, the rate of produc tion of daughter nuclei via pure β-decay. The pulse spectrum is given by

$$P_{pulse}(E_e,t) = \sum_A \sum_Z n_{ZA}(t)\, P_{ZA}(E_e) \qquad (2)$$

where E_e is the kinetic energy of electron and $P_{ZA}(E_e)$ stands for β-decay electron spectrum of a nucleus, which will be calculated later. The shut-down spectrum is expressed as

$$P_{shut\text{-}down}(E_e,t) = \sum_A \sum_Z N_{ZA}(t)\, P_{ZA}(E_e) \qquad (3)$$

where

$$N_{ZA}(t) = \int_0^\infty n_{ZA}(t)\, dt - \int_0^t n_{ZA}(t')dt' \qquad (4)$$

As the initial condition for Eq. (1) we assume the following simple mass and charge distribution of fragment nuclei[5]

$$Y_{ZA} = N(A)\, \exp(-(Z-Z_p(A))^2/C)/S \qquad (5)$$

where S is the normalization factor

$$S = \sum_Z \exp(-(Z-Z_p(A))^2/C \qquad (6)$$

N(A) is the total chain yield, and $Z_p(A)$ the most probable Z value in an A-chain. In the present work, we discuss only the thermal neutron induced fission of ^{235}U. The values of $Z_p(A)$ are taken from the empirical tables of Whal et al.[5] and c is fixed to 0.8.

2.2 - Decay Constants

According to the gross theory, the total β-decay constant in units of $m_e = \hbar = c = 1$, is given by[7]

$$\lambda_{ZA} = \frac{1}{2\pi^3} \int_{-Q}^{0} \sum_{\Omega} |G_\Omega|^2 \, |M_\Omega(\varepsilon)|^2 \, f_\Omega(-\varepsilon) \, d\varepsilon \qquad (7)$$

where $|M_\Omega(\varepsilon)|$ is the strength function and Ω denotes the type of β-decay operator, G_Ω the coupling constant, and f_Ω the f-function.

The emission rate of delayed neutron is calculated as[8]

$$\lambda^n_{ZA} = 4.88 \times 10^{-6} \, (Q - S_n) \qquad (8)$$

where S_n is the neutron separation energy, S_n and Q given in mc^2 units.

The electron spectrum is given as

$$P_{ZA}(E_e) = \int_{-Q}^{E_e} \sum_{\Omega} |G_\Omega|^2 |M_\Omega(\varepsilon)|^2 \, F_o(Z,W) C_\Omega(Z,-\varepsilon,W) \, P \, W(\varepsilon+E)^2 \, d\varepsilon \qquad (9)$$

where $F_o(Z,W)$ is the Fermi function, $C_\Omega(Z,-\varepsilon,W)$ is the so-calle shape factor, and $W = E_e + 1$.

The fundamental quantity necessary to evaluate the equation (7) is the nuclear masses. We take the mass values given by Mattauch et al.[9], when the experimental error is less than 500 KeV. For nuclei whose masses are not known or experimental error is greater than 500 KeV, we evaluate them from the mass formula of Garvey et al.[10]. For the sake of comparison the mass formula of Myers Swiatecki[11] is also tried.

3 - RESULTS AND DISCUSSION

In the Fig. 1, we show the time-dependent electron spectra for the shut-down case, where the modified-Lorentz type of β-strength function is used. The agreement with the experiment is quite satisfactory except in the low energy region ($E_e \lesssim 1\ mc^2$). This discrepancy is due partly to the break-down of the statistical treatment, and partly to the contribution from the long-lived nuclei near the β-stability line. The experimental shut-down measurement was done after only 8 hours of irradiation and hence the saturation might not be complete in the low energy region. For $t \geq 900$ sec the calculated curves fluctuate since very few nuclides contributes to the spectrum. In the experimental curve, this effect is smoothed out after the unfolding procedure[4].

The calculated pulse spectra, Fig. 2, also show a reasonable agreement with the experiment, although the calculated curves are systematically higher than the experimental ones in the high energy region. The theoretical curves are calculated as the time average of the spectra between the interval indicated.

Heller calculated the pulse spectra basing on his β-decay half-life systematics and a simple β-trength function[12]. For the purpose of comparison his results are shown in the Fig. 2 by dashed lines. Note that the time interval does not correspond to the experiment taken here.

We calculated the shut-down and pulse spectra for other different sets of parameters of fission fragment distributions. It is found that the pulse spectra change a little but the shut-down spectra are quite insensitive to the change of fission fragment distribution.

In the Fig. 3, we show the mean energy of electrons emitted from fission fragments together with the experimental values. Fig. 3-a is for the shut-down case and 3-b for the pulse case. The calculated curve for the pulse case is in a very good agreement with the experiment, but the shut-down curve is shifted up. This may be explained by the incomplete satuartion of the constant fission rate in the shut-down experiment.

CONCLUDING REMARKS

The gross theory is found to be a suitable tool for calculate the main feature of β-decay properties of fission fragment nuclei. The calculation method present here may be useful for estimating the decay heat problem in nuclear reactor engineering[15].

The similar method is now being applied to some astro physical problems, such as the r-process nucleosynthesis[16,17] and cooling of supernova remnant[18].

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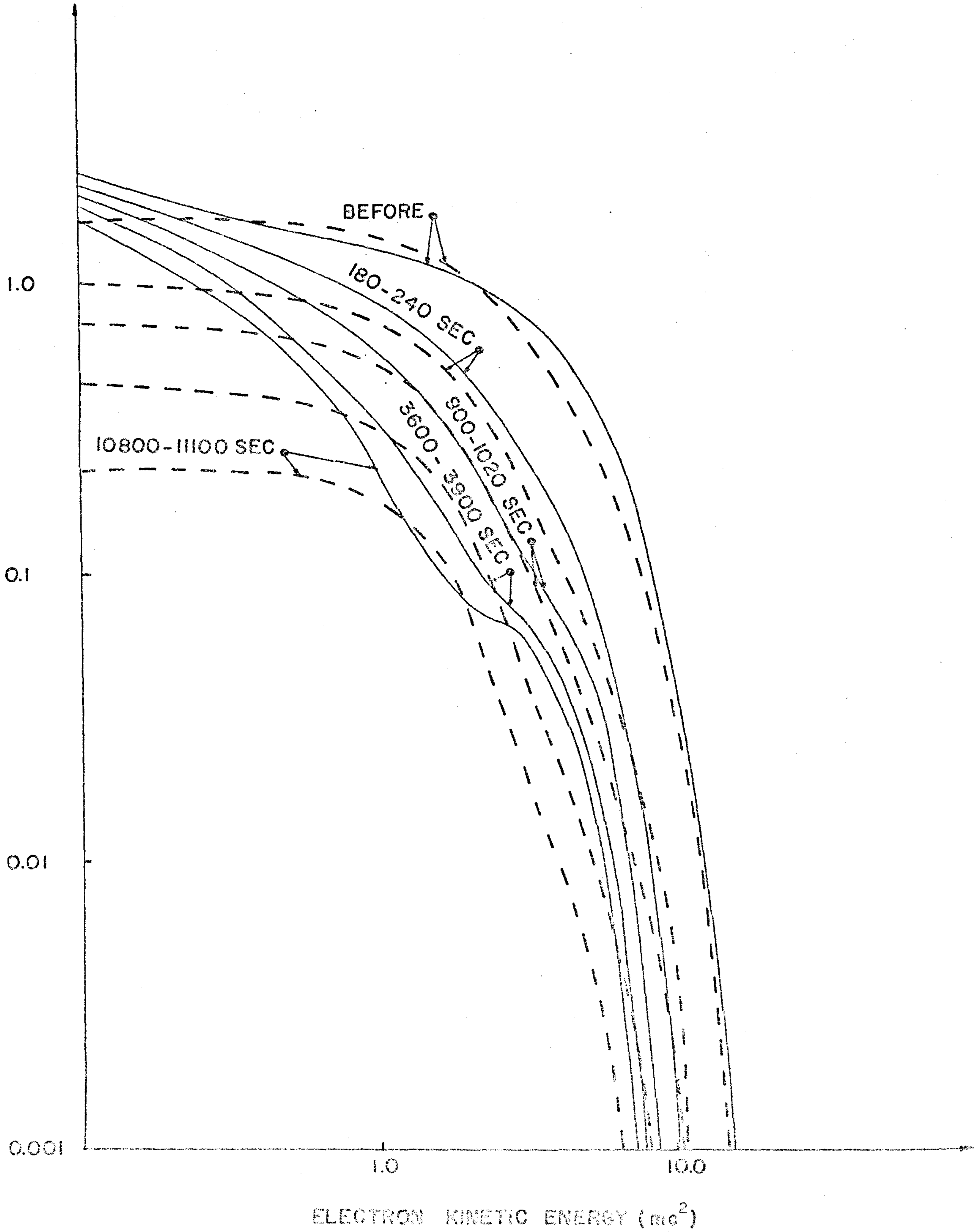
The numerical culations was done on the IBM 370 at Centro Brasileiro de Pesquisas Físicas.



BEFORE
180-240 SEC
900-1020 SEC
3600-3900 SEC
10800-11100 SEC
1.0
0.1
0.01
0.001
1.0
10.0
ELECTRON KINETIC ENERGY (mc^2)

FIG. 1

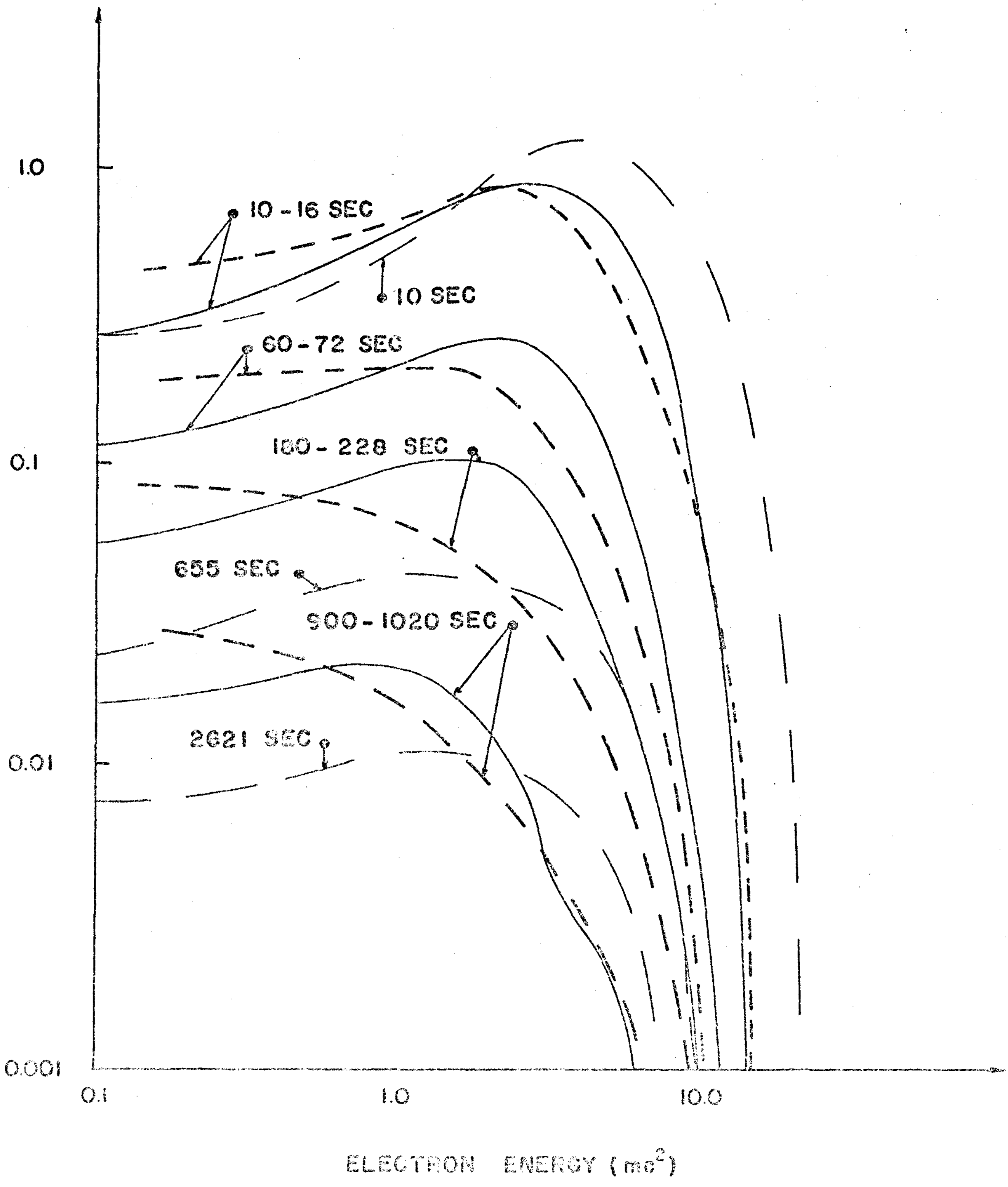
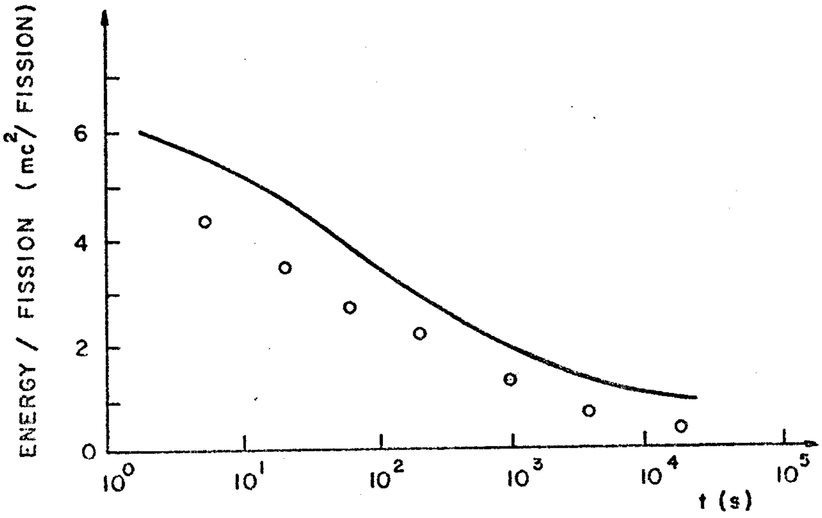
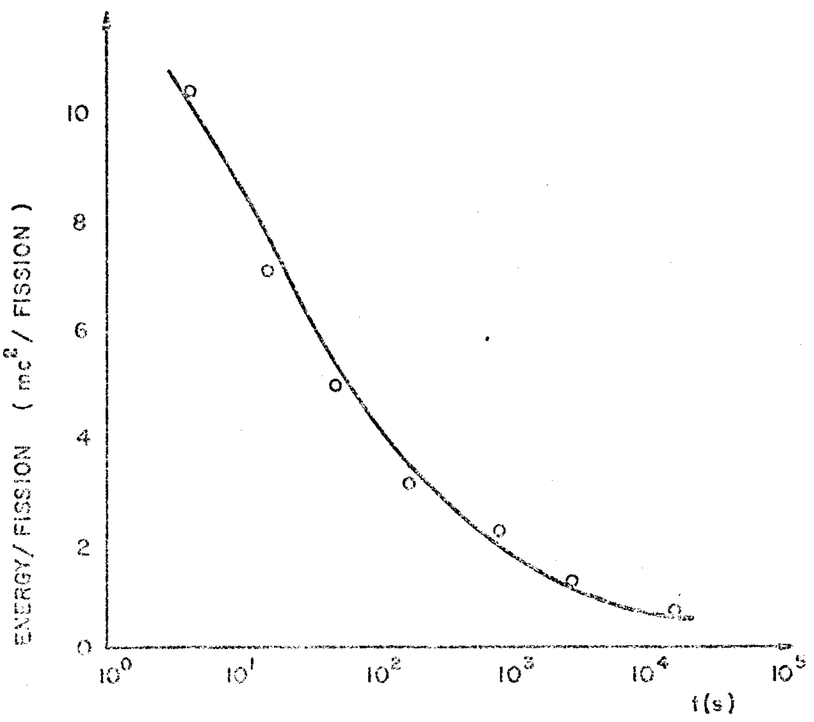

1.0
0.1
0.01
0.001
10 - 16 SEC
10 SEC
60 - 72 SEC
180 - 228 SEC
655 SEC
900 - 1020 SEC
2621 SEC
0.1
1.0
10.0
ELECTRON ENERGY (mc^2)

FIG. 2



ENERGY / FISSION (mc2/ FISSION)
0
2
4
6
10^0
10^1
10^2
10^3
10^4
10^5
t (s)

FIG. 3a



ENERGY/ FISSION (mc2 / FISSION)
0
2
4
6
8
10
10^0
10^1
10^2
10^3
10^4
10^5
t(s)

FIG. 3b

FIGURE CAPTIONS

Figure 1 - Time dependent spectra from fission fragments of ^{235}U for shut-down case. The curves are averaged in the indicated time interval.

——————— this work

– – – – – – – – experiment

Figure 2 - Time dependent spectra from fission fragments of ^{235}U for pulse case. The curves are averaged in the indicated time interval. Note that the time in Heller's result is different from the others.

——————— this work

– – – – – – – – experiment

------------- Heller's

Figure 3a - Time dependence of the electron mean energy for the shut-down case.

Figure 3b - Time dependence of the electron mean energy for the pulse case.

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