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Supersymmetric Extension of the Lorentz and CPT-violating Maxwell-Chern-Simons Model

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Abstract

Focusing on gauge degrees of freedom specified by a 1+3 dimensions model hosting a Maxwell term plus a Lorentz and CPT non-invariant Chern-Simons-like contribution, we obtain a minimal extension of such a system to a supersymmetric environment. We comment on resulting peculiar self-couplings for the gauge sector, as well as on background contribution for gaugino masses. Furthermore, a non-polynomial generalization is presented.

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I. INTRODUCTION

Lorentz and CPT invariances are cornerstones in modern Quantum Field Theory, both symmetries being respected by the Standard Model for Particle Physics. Nevertheless, nowadays one faces the possibility that this scenario is only an effective theoretical description of a low-energy regime, an assumption that leads to the idea that these fundamental symmetries could be violated when one deals with energies close to the Planck scale [1]. Taking this viewpoint, several approaches to analyze the violation of Lorentz symmetry have been proposed in the literature. Eventually a common feature arises: the violation is implemented by keeping either a four-vector (in a CPT-odd term [1]) or a traceless symmetric tensor (CPT-preserving term [2]) unchanged by particle inertial frame transformations [3] which is generally called spontaneous violation. Furthermore, the issue of preserving supersymmetry (*Susy*) while violating Lorentz symmetry is addressed to [4]. This breaking of Lorentz symmetry is also phenomenologically motivated as a candidate to explain the patterns observed in the detection of ultra-high energy cosmic rays, concerning the events with energy above the GZK ($E_{GZK} \simeq 4 \times 10^{19}\ eV.T$) cutoff [5]. Moreover, measurements of radio emission from distant galaxies and quasars verify that the polarization vectors of these radiations are not randomly oriented as naturally expected. This peculiar phenomenon suggests that the space-time intervening between the source and observer may be exhibiting some sort of optical activity, the origin of which is not known.

In a Theoretical Field proposal where this breaking of Lorentz invariance is taken into account, an analysis of the unitarity, causality, and vortex-like solutions had been carried out in Ref. [6]. Another focus of interest points to planar gauge systems, which play a relevant role in Condensed Matter descriptions, as they happen to be related to issues like high-Tc superconductivity and fractionary quantum Hall effect. Possible contributions from Lorentz-violating terms to the appearance of anisotropy in planar systems had been investigated in Refs. [7] and [8].

A first proposal of Supersymmetry-Preserving Lorentz Violation was carried out in the work of Ref.[4]. The aim of that work was to investigate whether one could maintain desired properties of supersymmetric systems, namely, cancellation of divergences and the patterns of spontaneous breaking schemes, while violating Lorentz symmetry. A Lorentz breaking tensor with constants entries has been adopted following an original suggestion given by Colladay [3]. Working upon a modified Wess-Zumino model, the authors of Ref. [4] had demonstrated that convenient changes of the Susy-algebra of fermionic charges and of Susy-covariant derivatives expressions were enough to define a Susy-like invariance for the Lorentz violating starting theory. As a matter of fact the modification of the algebra was achieved by adding a particular tensor-dependent central term, of the $k_{\mu\upsilon}\partial^{\nu}$ type, where $k_{\mu\upsilon}$ exhibits real symmetric traceless tensor properties.

As a net result, it was shown that a model for a modified-Susy invariant but Lorentz non-invariant *matter* system can be built. Moved by a different perspective, we now present an analysis on Lorentz and Susy breakings concerning degrees of freedom in the *gauge* field sector. We start off by establishing the supersymmetric minimal extension for the Chern-Simons-like term [1],

$$\Sigma_{CS} = -\frac{1}{4}\int dx^4 \epsilon^{\mu\nu\alpha\beta} c_\mu A_\nu F_{\alpha\beta}, \tag{1}$$

preserving the usual $(1+3)$-dimensional Susy algebra. The breaking of Susy will follow the very same route to Lorentz breaking: the statement that c_μ is a constant (in the active sense) vector triggers both Lorentz and, as we shall comment on, Susy breakings. Handling proper superfield extensions for the background shall prevent the model from displaying higher spin excitations, and interesting self-couplings for the gauge sector as well as background contribution for the gaugino masses come up naturally as a consequence of the (initially) supersymmetric structure.

In the next section, we present the Susy minimal extension for 1. In Section 2, a first generalization, with non-polynomial couplings, shows up. Finally, we comment on conclusions and perspectives in Section 4.

II. THE SUPERSYMMETRIC EXTENSION OF THE MAXWELL-CHERN-SIMONS MODEL.

Adopting covariant superspace-superfield formulation, we propose the following minimal extension for 1:

$$A = \int d^4x d^2\theta d^2\bar{\theta} \left\{ W^a (D_a V) S + \overline{W}_{\dot{a}} (\overline{D}^{\dot{a}} V) \overline{S} \right\}, \tag{2}$$

where the superfields W_a, V, S and the Susy-covariant derivatives D_a, $\overline{D}_{\dot{a}}$ hold the definitions:

$$\begin{aligned} D_a &= \frac{\partial}{\partial\theta^a} + i\sigma^\mu{}_{a\dot{a}}\bar{\theta}^{\dot{a}}\partial_\mu \\ \overline{D}_{\dot{a}} &= -\frac{\partial}{\partial\bar{\theta}^{\dot{a}}} - i\theta^a\sigma^\mu{}_{a\dot{a}}\partial_\mu; \end{aligned}$$

from $\overline{D}_{\dot{b}}W_a\left(x,\theta,\bar{\theta}\right)=0$ and $D^aW_a\left(x,\theta,\bar{\theta}\right)=\overline{D}_{\dot{a}}\overline{W}^{\dot{a}}\left(x,\theta,\bar{\theta}\right)$, it follows that

$$W_a(x,\theta,\bar{\theta}) = -\frac{1}{4}\overline{D}^2 D_a V :$$

Its θ-expansion reads as below:

$$\begin{aligned} W_a(x,\theta,\bar{\theta}) &= \lambda_a(x) + i\theta^b\sigma^\mu{}_{b\dot{a}}\bar{\theta}^{\dot{a}}\partial_\mu\lambda_a(x) - \frac{1}{4}\bar{\theta}^2\theta^2\Box\lambda_a(x) \\ &\quad +2\theta_a D(x) - i\theta^2\bar{\theta}^{\dot{a}}\sigma^\mu{}_{a\dot{a}}\partial_\mu D(x) \\ &\quad +\sigma^{\mu\nu}{}_a{}^b\theta_b F_{\mu\nu}(x) - \frac{i}{2}\sigma^{\mu\nu}{}_a{}^b\sigma^\alpha{}_{b\dot{a}}\theta^2\overline{\theta}^{\dot{a}}\partial_\alpha F_{\mu\nu}(x) \\ &\quad -i\sigma^\mu_{a\dot{a}}\partial_\mu\,\overline{\lambda}^{\dot{a}}(x)\,\theta^2 \end{aligned}$$

and $V=V^\dagger$. The Wess-Zumino gauge choice is taken as usually done:

$$V_{WZ} = \theta\sigma^\mu\bar{\theta}A_\mu(x) + \theta^2\bar{\theta}\overline{\lambda}(x) + \bar{\theta}^2\theta\lambda(x) + \theta^2\bar{\theta}^2 D,$$

so the action (2) is gauge-invariant. The background superfield is so chosen to be a chiral one. Such a constraint restricts the maximum spin component of the background to be an $s=\frac{1}{2}$ component-field, showing up as a Susy-partner for a spinless dimensionless scalar field. Also, one should notice that S happens to be dimensionless. The superfield expansion for S then reads:

$$\begin{aligned} \overline{D}_{\dot{a}}S(x) = 0 \text{ and } S(x) &= s(x) + i\theta\sigma^\mu\overline{\theta}\partial_\mu s(x) - \frac{1}{4}\bar{\theta}^2\theta^2\Box s(x) \\ &\quad +\sqrt{2}\theta\psi(x) + \frac{i}{\sqrt{2}}\theta^2\overline{\theta}\overline{\sigma}_\mu\partial_\mu\psi(x) + \theta^2F(x). \end{aligned}$$

The component-wise counterpart for the action (2) is as follows:

$$\begin{aligned} A_{comp.} &= \int d^4x\,\Big\{-\frac{1}{2}(s+s^*)F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\partial_\mu(s-s^*)\varepsilon^{\mu\alpha\beta\nu}F_{\alpha\beta}A_\nu + 4D^2(s+s^*) \\ &- 2is\,\lambda\,\sigma^\mu\partial_\mu\overline{\lambda} - 2is^*\,\overline{\lambda}\,\overline{\sigma}^\mu\partial_\mu\lambda - \sqrt{2}\lambda(\sigma^{\mu\nu})F_{\mu\nu}\psi + \sqrt{2}\overline{\lambda}(\overline{\sigma}^{\mu\nu})F_{\mu\nu}\overline{\psi} + \\ &+ \lambda\,\lambda F + \overline{\lambda}\,\overline{\lambda}F^* - 2\sqrt{2}\lambda\,\psi D - 2\sqrt{2}\overline{\lambda}\,\overline{\psi}D\Big\} \end{aligned} \quad (3)$$

As one can easily recognize, the first line displays the 4D Chern-Simons-like term (1), where the vector c_μ is expressed as the gradient of a real background scalar: $c_\mu=\partial_\mu\sigma$, for $s=\xi+i\sigma$. Such a reduction of the vector into a gradient of a scalar field stems directly from the simultaneous requirements of both gauge[1] and supersymmetry invariances.

Another interesting feature of this model concerns the presence of self-couplings for the gauge sector: the fermionic background field, ψ, triggers the coupling of the gauge boson (through the field-strength) to the gaugino. Moreover, using the field equation for the gauge auxiliary field D one arrives at a quartic fermionic

[1] The gauge invariance of action 2 will become clearly manifest in the next section, where we rephrase the supersymmetrization of the 4D Chern-Simons-like term in a formulation restricted to the chiral (anti-chiral for the h.c. counterpart) sector of superspace.

fields coupling - $\lambda\lambda\psi\psi$ -, and the background nature of ψ indicates a background contribution for the gaugino mass[2].

Concerning the breaking of Lorentz symmetry, realized by assuming $c_\mu = \partial_\mu \sigma$ to be constant under the action of particle inertial frame transformations, one should observe that such an assumption implies that the scalar component-field σ must be linear in the coordinates, $\sigma = c_\mu x^\mu$. As a matter of fact, a linear dependence on x^μ cannot be implemented by means of a Susy-covariant constraint (i.e., Susy-covariant derivatives acting on S), and, in that sense, the choice of a rigid $\partial_\mu\sigma$ breaks Susy in exact analogy to the Lorentz breaking scheme adopted. To better establish such a correspondence, one can consider the choice for constant $\partial_\mu\sigma$ to be accompanied by a constant ψ requirement (and a constant auxiliary field, F, as well[3]). In this context, a (passive) Susy-transformation keeps the status of all component-fields unchanged.

In the next section, we provide the model with a non-polynomial generalization, which brings about the possibility of understanding the 4D C.S.-like term as a first order correction in a complete exponential scenario.

III. NON-POLYNOMIAL GENERALIZATION

Let us note that the integration defined through the Grassmanian measure $d^2\bar{\theta}$ (or $d^2\theta$) can be represented by the action of a squared Susy-covariant derivative (up to a normalization factor), $\overline{D}^2$ (or D^2), on the super-Lagrangian $W^a(D_aV)S + h.c.$, if one neglects boundary terms, and that the only sector of the superfield product $W(DV)S$ (or $\overline{W}(\overline{D}V)\overline{S}$) that admits a non-null action of $\overline{D}^2$ (or D^2) is the factor DV (or $\overline{D}V$). Such a manipulation leads to the Lagrangian $d^4x(\, d^2\theta\; W^a(\overline{D}^2 D_aV)S + d^2\bar{\theta}\; \overline{W}_{\dot{a}}(D^2\overline{D}^{\dot{a}}V)\overline{S})$, and one can rephrase (2) through such a parametrization:

$$A = h\int d^4x\left\{d^2\theta[W^aW_aS] + d^2\bar{\theta}\left[\overline{W}_{\dot{a}}\overline{W}^{\dot{a}}\overline{S}\right]\right\},$$

where a suitable dimensionless (perturbation) parameter h is inserted. We remark that such an inclusion does not spoil any power-counting renormalization property of the model. Moreover, as we aim at a Susy version for a model hosting both regular Maxwell kinetic term and the 4D C.S.-like term [6], we end up with the following combination:

$$A_{Max.+C.S.} = \frac{1}{4}\int d^4x\left\{d^2\theta[W^aW_a] + d^2\bar{\theta}\left[\overline{W}_{\dot{a}}\overline{W}^{\dot{a}}\right]\right\} + \frac{h}{4}\int d^4x\left\{d^2\theta[W^aW_aS] + d^2\bar{\theta}\left[\overline{W}_{\dot{a}}\overline{W}^{\dot{a}}\overline{S}\right]\right\}.$$

Such an expression induces a straightforward non-polynomial generalization:

$$A_{non-pol.} = \frac{1}{4}\int d^4x\left\{d^2\theta\,[W^aW_a\exp(hS)] + d^2\bar{\theta}\left[\overline{W}_{\dot{a}}\overline{W}^{\dot{a}}\exp(h\overline{S})\right]\right\}, \tag{4}$$

leaving room for a perturbative approach parametrized by orders of h. In fact, the action (4) includes a zero order supersymmetric Maxwell theory, a first-order Susy-extended 4D C.S.-like term (reproducing the action of the eq. (3)), and higher orders contributions. In component-field parametrization, action (4) reads:

$$\begin{aligned} A_{non-pol.} \;=\; & \frac{1}{4}\int d^4x\left\{\exp(hs)\left[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{i}{2}\widetilde{F}_{\mu\nu}F^{\mu\nu} - 2i\lambda^a\sigma^\mu{}_{a\dot{a}}\partial_\mu\,\overline{\lambda}^{\dot{a}} + 4D^2 + \right.\right.\\ & \left.\left. +\; h\left(-2\sqrt{2}\lambda^a\psi_aD + \lambda^a\lambda_aF - \sqrt{2}\lambda^a\,(\sigma^{\mu\nu})_a{}^b\,F_{\mu\nu}\psi_b\right) - \frac{h^2}{2}\lambda^a\lambda_a\psi^b\psi_b\right] + h.c..\right\} \end{aligned}$$

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[2] We shall analyze the propagator structure for the gauge component-fields in a forthcoming communication. We anticipate that a constant ψ component-field configuration is compatible with the supersymmetry algebra.

[3] In fact, a constant auxiliary field F is singled out as a susy-invariant parameter, as far as one deals with a constant ψ.

The exponential version brings about the 4D C.S.-like term in the form $-\frac{i}{8}\exp(hs)\widetilde{F}_{\mu\nu}F^{\mu\nu}+h.c.$, demanding an integration by parts to reproduce the expression $i\partial_\mu(s-s^*)\varepsilon^{\mu\alpha\beta\nu}F_{\alpha\beta}A_\nu$. One should also realize that a quartic fermion-fields coupling is already present at order h^2 , even if the field equation for the auxiliary field D is not used to eliminate it. It is also interesting to observe how the background components s, ψ and F influence on the gaugino physical mass.

IV. CONCLUDING COMMENTS

Working on the *gauge*-field sector of a system with a Lorentz breaking 4D-Chern-Simons-like term, we have been able to derive its minimal supersymmetric extension and a peculiar nom-polynomial generalizations has been proposed that is compatible with $N = 1$-Susy. Focusing on the minimal Susy-extension, one should already realize the presence of new couplings induced by the background (passive-)superfield components. The assumption that the Lorentz breaking is implemented by means of a constant vector, regarded as a background input, finds its as a Susy-counterpart in a set of requirements on the space-time dependence of each component-field of the background superfield, S. A scalar field, s, linearly dependent on x^μ, as well as a constant spinor field, ψ, arise as a consequence of gauge invariance, and these results impose that, eventually, coupling terms are to be regarded as mass terms. A complete analysis of the propagator structure for the gauge supermultiplet, both in superspace and in component-fields, is mandatory, including an interesting study of the gaugino (background-)induced mass. We shall very soon report our efforts in this matter elsewhere.

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