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TWO POTENTIAL FIELDS IN THE SAME GROUP

by

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Abstract

Two potential fields undergo gauge transformations belonging to the same group. In this case the concepts of gauge fixing and ghosts are extended. A mixed propagator is obtained. A model that is renormalizable but non-unitary is studied.

Key-words: Gauge fields in the same group.

I INTRODUCTION

Nature's flux manifest a continuous changing state. It moves through interactions. We see it as a reborning act. From the point of view of particle physics this continuous nature reborning should be based on the interaction between its building blocks. In modern science this process is being described through field theory. Therefore we think that the term in a Lagrangian with physical meaning is the one which symbolizes the interaction. For instance, in QED it would be

$$e\, A_\mu\, \psi \tag{1}$$

Nevertheless the Maxwell equations are the only case where experiment guide the field equations. The common tendency is symmetry dictated interactions. The proposition of this work is to build up a dynamics with a symmetry that includes two fields in the same group. The physical insight must be contained in the interacting part. Then, there are possibilities to associate these fields either to same or to different matter fields. Consider the case

$$g_A\, A_\mu\, \psi \quad \text{and} \quad g_B\, B_\mu\, \phi \tag{2}$$

where the gauge fields A_μ^a and B_μ^a belong to the same group U, with $U = e^{iw^a t_a}$. (2) appears just as an attitude. In order to give life to these fields it is necessary to know how they propagate. Gauge symmetry is a tool for that. Gauge theories appear as a servomechanism in order to represent nature forces. The presence of two fields in the same group gives the transformations,

$$A_\mu \rightarrow UA_\mu U^{-1} + \frac{i}{g_A} (\partial_\mu U) U^{-1} \qquad (3)$$

$$B_\mu \rightarrow UB_\mu U^{-1} + \frac{i}{g_B} (\partial_\mu U) U^{-1} \qquad (4)$$

The gauge symmetry is extended through (3) and (4). The respective covariant derivatives are

$$D_\mu(A) = \partial_\mu + ig_A A_\mu \qquad (5)$$

$$D_\nu(B) = \partial_\nu + ig_B B_\nu \qquad (6)$$

They define a covariant system as in Fig. 1. It is called an extended gauge symmetry [1]. QCD is generated through one of the basis. Different strenght tensors can be build up by associating different basis. For instance,

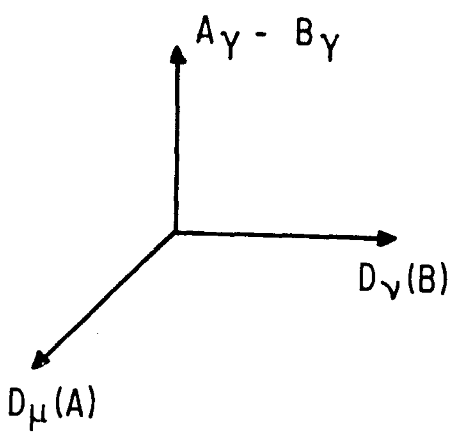

Fig. 1

A Covariant basis

(3) and (4) transformations define a covariant system. Strength tensors will be obtained by combining the basis. Depending also on the choice that trace is taken, different gauge invariant Lagrangians can be generated.

$$G_{\mu\nu} = [D_\mu(B), D_\nu(A)] \tag{7}$$

$$H_{\mu\nu} = [D_\mu(A), D_\nu(B)] \tag{8}$$

$$M_{\mu\nu} = (A_\mu - B_\mu)\ (A_\nu - B_\nu) \tag{9}$$

yielding

$$G^a_{\mu\nu} = g_A\, \partial_\mu A^a_\nu - g_B\, \partial_\nu B^a_\mu + i g_A g_B\ [B_\mu, A_\nu]^a \tag{10}$$

$$H^a_{\mu\nu} = - G^a_{\nu\mu} \tag{11}$$

and also the Yang-Mills case

$$A^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i g_A\ [A_\mu, A_\nu]^a \tag{12}$$

$$B^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + i g_B\ [B_\mu, B_\nu]^a \tag{13}$$

(10)-(12) give the possibility for different combinations to build up gauge invariant Lagrangians. A second aspect that defines the extended gauge symmetry is in how the trace is taken.

The motivation of this work is to study the Lagrangian

$$\mathcal{L}_G = -\frac{1}{4}\ G^{\mu\nu}_a\ G^a_{\mu\nu} \tag{14}$$

that involves two fields. Although they are independent each one influences the dynamics of the other. A different aspect is introduced by a mixed propagator defined between different gauge fields. (3) and (4) allows a mass term in (14). This term will have physical influence. For instance, from dimensional analysis a gauge propagator must have the form $\frac{1}{k^2}$. The mass term generates a hope of obtaining a propagator of the type $\frac{m^2}{k^4}$. Adopting a static view of confinement the Fourier transform of the massive propagator can give a linear potential.

Section II discusses the presence in the same group of more than one gauge field. Section III presents, in order to quantize, the consequences of these fields in the gauge fixing and ghosts terms. Section IV investigates the particles spectrum for (14). Feynman rules and a non-absolute confinement discussion are left for the Appendix.

II THE GAUGE FIELDS A_μ^a AND B_ν^a

Group theory allows the presence of more than one field under a group algebra. Consider the matter fields case. An example would be the fermionic fields ψ^i and χ^i associated to different lepton or quark families. Physically they would belong to the same group. Other case would be in the twelve colourful stones [1]. The theory requires bosons and fermions to have the same colour. However this work intends to study (14) as a pure gauge theory. There is no matter interaction.

In a first approach we have to understand the relations between A_μ^a and B_μ^a. It is going to be explored through three different methods. In order to simplify arguments we are going to adopt just one coupling constant. The first aspect is based on gauge invariance. This means that the gauge transformations incorporate the fields identities. The non-abelian transformations generate a tensor with the form

$$y^a_{\rho\sigma} = \partial_\rho y^e_\sigma - \partial_\sigma y^a_\rho + ig\ [y_\rho, y_\sigma]^a \tag{1 }$$

From (3) and (4), the question is how to write the fields y^a_σ and y^a_ρ in terms of $A^a_{\mu(\nu)}$ and $B^a_{\mu(\nu)}$ yielding for (15) a covariant transformation. Thus consider that (3) and (4) carry in truth just a semantic difference. Consequently the tensor

$$L^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + ig\ [B_\mu, A_\nu]^a \tag{16}$$

should be covariant. Calculating $\delta L^a_{\mu\nu}$ this property is not verified. Another method to distinguish these fields is through conserved currents. It gives,

$$j^a_\mu(A) = \frac{1}{2}\, c^a{}_{bc}\ A^b_\nu\, G^c_{\mu\nu} \tag{17}$$

$$j^a_\mu(B) = -\frac{1}{2}\ c^a{}_{bc}\, B^b_\nu\, G^c_{\mu\nu} \tag{18}$$

and

$$\partial^\mu [j^a_\mu(A) + j_\mu(B)] = 0 \tag{19}$$

Observe that in (19) the conservation is not isolatedely. It is consistent with the presence of two fields in the same group. It yields,

$$\Box(A^a_\mu + B^a_\mu) - \partial_\mu \partial.(A+B)^a + ig\,\partial^\nu [B_\mu, A_\nu]^a + h^a_\mu = 0$$

where h^a_μ is such that $\partial_\mu h^{\mu a} = 0$ (20)

(20) can be satisfied with $A^a_\mu \neq B^a_\mu$. An example with group U(1) is given in the Appendix A.

Another aspect it would be to look for gauge transformations between these fields. Consider the relation,

$$\phi = U_\Gamma \psi \quad \text{and} \quad D_\mu(B)\phi = U_\Gamma D_\mu(A)\psi \tag{21}$$

it gives the non abelian relation

$$B_\mu = U_\Gamma A_\mu U_\Gamma^{-1} + \frac{i}{g}(\partial_\mu U_\Gamma)\,U^{-1} \tag{22}$$

(22) in (10) gives a gauge dependent tensor. Consider an abelian case

$$B^a_\mu = A^a_\mu - \partial_\mu \Lambda^a \tag{23}$$

(10) is invariant with the condition

$$\partial_\mu \partial_\nu \Lambda^a - c^a{}_{bc}\,\partial_\mu \Lambda^b A^c_\nu = 0 \tag{24}$$

In gauge theories the strength tensor is the contact with reality. Thus it might happen a necessity of having this tensor with some property. Therefore it can be useful to separate (10) in its symmetric and antisymmetric pieces,

$$G^a_{\mu\nu} = G^a_{(\mu\nu)} + G^a_{[\mu\nu]}$$

where

$$G^a_{[\mu\nu]} \doteq \frac{1}{2}\,[G^a_{\mu\nu} + H^a_{\mu\nu}]$$

$$G^a_{(\mu\nu)} = \frac{1}{2}\,[G^a_{\mu\nu} - H^a_{\mu\nu}] \tag{25}$$

Take for instance, a case with some charge distribution where the currents conserve separatedely. It will be described by $G_{[\mu\nu]}$. Another possibility is to consider the dual tensor $\tilde{G}_{\mu\nu}$.

III GAUGE FIXING AND GHOST TERMS

The gauge symmetry is realized by choosing some gauge reference system. Physics is constructed through the numbers obtained from this framework - the gauge fixing. It leads to a better defined partition function and reduces by one the number of degrees of freedom. However in our case there are two gauge-field families associated to the same group. Thus they will share a common family of group parameters, $\omega^a(x)$. Consequently, one can expect the existence of a correlation between the gauge fixing terms considering each family either separately or not.

In giving a separate treatment for the fields we will have the gauge fixing functionals $g[A_\mu] = 0$ or $g[B_\mu] = 0$. Another case which deserves our attention is the gauge fixing functional mixing A_μ and B_μ, $g[A_\mu;B_\mu] = 0$. We should however bear in mind that the criterium for choosing a good gauge fixing is that it must lead to the presence of just one set of group parameters $\omega^a(x)$.

Take the cases,

$$\text{i)}\quad g[A_\mu] = \partial^\mu A_\mu \,,\; g[B_\mu] = \partial^\mu B_\mu \tag{26}$$

$$\text{ii)}\quad g[A_\mu;B_\mu] = \partial^\mu(A_\mu + B_\mu) \tag{27}$$

$$\text{iii)}\quad g[A_\mu;B_\mu] = \partial^\mu(A_\mu - B_\mu) \tag{28}$$

Working out (26)

$$\omega^a(y) = -g^2\int d^4x\; G_o(x-y)\; c^a{}_{bc}\,(\partial^\mu\omega^b(x))\; A^c_\mu(x) \tag{29}$$

with a similar equation for B^c_μ. It seems to be very restrictive to impose both conditions at same time. The field configurations would not be independent due to the relation $\omega^a[A_\mu;y] = \omega^a[B_\mu;y]$. For instance, for SU(2) we would have six equations and three variables (the parameters ω^i). However either $g[A_\mu]$ or $g[B_\mu]$ may very well be used as the gauge fixing functionals. (27) also selects a family of group parameters $\omega^a(y)$ given by

$$\omega^a(y) = -g^2\int d^4x\, G_o(x-y)\; c^a{}_{bc}\,(\partial^\mu\omega^b(x))\,(A^a_\mu(x) + B^c_\mu(x)) \tag{30}$$

(30) satisfies the gauge fixing condition because the univoque relation established between the gauge fields and the group

parameters. This does not happen for the case (28). It yields the relation

$$c^{a}_{\ bc}(\partial^{\mu}\omega^{b}(x))(A^{c}_{\mu}-B^{c}_{\mu}) = 0 \qquad (31)$$

Another way to observe the gauge fixing condition is through the following criteria for a field χ_{μ}.

i) If χ_{μ} satisfies the equation $g[\chi_{\mu}] = 0$, then there is no non-trivial U for which $g[\chi^{U}] = 0$

ii) Given any χ_{μ} which does not satisfy $g[\chi_{\mu}] = 0$ then there exists an U such that $g[\chi^{\mu}] = 0$.

Let us apply it for (28). Suppose that $A_{\mu}(x)$ and $B_{\mu}(x)$ are given such that $A_{\mu}-B_{\mu}$ has a compact support. Further that it satisfies the gauge condition

$$\partial^{\mu}(A_{\mu}-B_{\mu}) = 0 \qquad (32)$$

Then from $\partial^{\mu}(A^{U}_{\mu}-B^{U}_{\mu})$,

$$(\partial^{\mu}U^{-1})(A_{\mu}-B_{\mu})U + U^{-1}(A_{\mu}-B_{\mu})\partial^{\mu}U = 0 \qquad (33)$$

(33) does not satisfy the condition i), because U can be different from one.

In order to quantize non-abelian gauge theories is being necessary to introduce the Faddeev-Popov fields. Introducing two gauge fields in the same group does not change qualitatively in the functional the ghost factor. It is given by

$$\mathcal{L}_{Ghost} = \int d^4x \; \eta^*(x) \; \frac{\delta g(x)}{\delta\omega(y)} \; \eta(y) \tag{34}$$

In the covariant gauge (26), ghosts would interact with just one gauge field. For (27), both fields will interact with ghosts. In the abelian case and for axial gauge they are decoupled.

IV HAMILTONIAN AND DEGREES OF FREEDOM

It is necessary to understand the spectrum corresponding to (14). Therefore we are going start by studying the dynamical variables in the theory. The canonical momenta are

$$\pi_\mu(A) = G_{o\mu} \tag{35}$$

$$\pi_\mu(B) = -G_{o\mu} \tag{36}$$

There are seven degrees of freedom. The momentum-energy tensor is given by

$$T_{\mu\nu} = \frac{1}{4} g_{\mu\nu} G^2 - \frac{1}{2} g^{\rho\zeta} \left(G^a_{\mu\rho} G_{\nu\zeta a} + G^a_{\rho\mu} G_{\zeta\nu a}\right) \tag{37}$$

Defining

$$G^{ko} = E^k \quad , \quad G^{ok} = \tilde{E}^k \quad , \quad G^{ij} = -\varepsilon^{ijk} B^k \tag{38}$$

yields

$$T_{00} = \frac{1}{4}\,(-3G_{00}^2 + \vec{E}^{a\,2} + \tilde{\vec{E}}^{a\,2} + \vec{B}^{a\,2}) \tag{39}$$

For $A_0 = B_0 = 0$ the negative term disappears satisfying (3) and (4). In a further work the conditions for to be bounded by below will be studied. Therefore Feynman rules are developed in Appendix B. The supersymmetric version avoids the negative hamiltonian discussion.

Nevertheless it is in terms of the physical fields C_μ and D_ν that the spectrum of the theory is better understood. Rewriting (14) for abelian case gives,

$$\mathcal{L}_G = -\frac{1}{4}(\partial_\mu C_\nu + \partial_\nu C_\mu)^2 - \frac{1}{4}(\partial_\mu D_\nu - \partial_\nu D_\mu)^2 + m^2 C_\mu^2$$

where

$$\begin{aligned} D_\mu^a &= A_\mu^a + B_\mu^a \\ C_\mu^a &= A_\mu^a - B_\mu^a \end{aligned} \tag{40}$$

The field C_μ carries four degrees of freedom. In order to interpret them observe the Klein Gordon equation

$$(\Box + \frac{m^2}{2})\,\partial_\mu C^\mu = 0 \tag{41}$$

and the propagator

$$\langle T(C_\mu C_\nu)\rangle = \frac{-g_{\mu\nu} + \dfrac{k^\mu k^\nu}{2k^2 - m^2}}{k^2 - m^2} \tag{42}$$

It appears that one degree of freedom is given by a scalar particle. In order to observe this consideration it is propitious to separate the field C_μ

$$C_\mu = C_\mu^T + C_\mu^L \tag{43}$$

The transversal field will have three degrees of freedom and there is no problem with negative hamiltonian. The longitudinal part carries the other degree of freedom. The respective propagators are

$$<T(C^T_\mu C^T_\nu)> = -\frac{1}{k^2-m^2}\,\eta_{\mu\nu} \tag{44}$$

$$<T(C^T_\mu C^T_\nu)> = \frac{2}{m^2}\,(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2-\frac{m^2}{2}}) \tag{45}$$

Observe the propagators pole. They are different. The longitudinal case shows a scalar particle with mass $\frac{m}{\sqrt{2}}$. It is consistent with what was obtained in (41).

A solution for the case could be in freezing the longitudinal part. Consider it in the following piece of the Lagrangian

$$C^{L\mu}\;(\partial_\mu\partial_\nu + m^2\eta_{\mu\nu})\;C^{L\nu} + \frac{1}{\alpha}(\partial.C)^2 \tag{46}$$

where (26) or (27) contributes to the gauge fixing term. For the value $\alpha = -1$ the longitudinal part will not propagate. Thus the contribution of longitudinal part would be only in the mass term. Then it can be integrated over and (46) just contributes to the infinite constant.

V CONCLUSION

Physically the experimental gauge bosons are free to be understood with any approach. The motivation of this work is to incorporate two gauge fields in the same group. However it is necessary to show that these fields belong to the same group and that they are different. Three criteria were selected to characterize the presence of a same group in the theory. They are the gauge fixing condition, the presence of an interacting term as in (49) and the mass term as in (B9). Each of them is sufficient to determine the presence of a same group. Let us analyse them

Gauge theories bring a non-well-defined gauge field. However, in order to perform calculations it is necessary to select a particular field configuration among the whole class of infinite gauge equivalent configurations. It is just the same as having Poincaré invariance and being forced to choose a particular reference frame to do physics. Thus being forced to select a certain field configuration to do calculations, a gauge-fixing term must be added which explicitly breaks the gauge invariance and satisfies certain basic requirements, namely:

(i) it allows the determination of the group parameters, $\omega^a(x)$, as functionals of the chosen field configurations

$$\omega^a(x) = \omega^a[\chi_\mu;x] \tag{47}$$

(47) means that the path integral defining the generating functional of the Green function of the theory is better defined.

(ii) it permits the calculation of propagators for the gauge fields. This means that the operator $K^{ab}_{\mu\nu}$ appearing in the bilinear piece of the Lagrangian is invertable after the gauge fixing term has been introduced.

In our case the condition (47) must be generalized to two gauge fields. As we saw (26) and (27) establish this condition univocally. The condition (ii) has appeared when we tried to invert the block matrices in the text. Physically the gauge fixing importance is in terms that it is associated to the longitudinal part propagation. Therefore the mechanism that controls the longitudinal part will be enlarged through (26) and (27) Take the minimum gauge invariant massive Lagrangian studied in ref. [1]. It is

$$\mathcal{L}_G = -\frac{1}{4}\{\partial_\mu C^a_\nu + g[\chi_\mu, C_\nu]^a\}^2 + \frac{1}{2} m^2 C^a_\mu C^{\mu a} \tag{48}$$

where χ_μ is any gauge field that transforms like (3). Conditions (26) and (27) can avoid the propagation of C^L_μ. Thus if it is associated to a conserved current the longitudinal field can be integrated over.

The second criteria concerns the gauge field relations. When each gauge field is associated to a different group, these fields are generated independently. Therefore they do not interact between themselves. However (14) yields the three-gauge-boson vertex

$$-\frac{g}{2}\ \partial^\mu A^\nu_a\ [B_\mu, A_\nu]^a \tag{49}$$

where the fields are interacting. Feynman rules are in Appendix B. Other aspect for this model is in the of a mixing propagator between the gauge fields as in Fig. 3. The origin for such propagator is in transformations in the same group. Observe in Appendix B that in general $P_{\mu\nu}^{A\to A} \neq P_{\mu\nu}^{B\to B}$.

Group theory does not restrict the number of fields that can participate on its transformations. However in order to associate particles it is necessary to understand the distinguibility of these fields. The massless case is studied in section II. The third condition is related to the mass term. As we know, a gauge theory just based on one field does not allow us to introduce mass. Our motivation is to realize it through (B9). Moreover to the gauge symmetry to be preserved these fields must belong to the same group. The presence of a mass term defines the physical fields. Then (A_μ, B_ν) should be identified as current fields while (D_μ, C_ν) are the physical fields.

In terms of gauge theories one must specify yet which type of vector field C_μ^a is. From equations (40) one sees that C_μ^a transforms like

$$C_\mu^{a\,'} = C_\mu^a + c^a{}_{bc}\,\omega^b\, C_\mu^c \tag{50}$$

Rigourously (50) does not represent the usual gauge transformation. However a definition must be based in terms of physical and mathematical properties. Observe that (50) brings the context of Bohm-Aharanov discussion for C_μ^a field, although it is not valid for the U(1) case. There the transformation is static, therefore the first tendency is to identify C_μ^a just as a Proca field. However here it contributes to gauge theories properties : it has consequences in the gauge fixing terms (26)-(27), and it can interact with ghosts. The Slavnov-Taylor identities also depend on it. Another aspect is that it is not isolated from the D^a_μ field, i.e, a mixing propagator with this fields can be calculated.

It appears that the question if (50) represents or not a gauge field depends of a nomenclature extension. Considering that by gauge field should be any field taking values in the Lie algebra of the group we would call D^a_μ as a pure gauge field and C^a_μ as a gauge field.

Other observation to be understood is the mapping between current fields (A_μ, B_ν) and the physical fields (C_μ, D_ν). Take for instance, the following gauge invariant Lagrangians

$$\mathcal{L}_G = A^{\mu\nu a} A_{\mu\nu a} + B^{\mu\nu a} B_{\mu\nu a} = C^{\mu\nu a} C_{\mu\nu a} + D^{\mu\nu a} D_{\mu\nu a} \tag{51}$$

and $$\mathcal{L}_G = A^{\mu\nu a} B_{\mu\nu a} = C^{\mu\nu a} C_{\mu\nu a} - D^{\mu\nu a} D_{\mu\nu a} \tag{52}$$

Adding and subtracting (51) and (52) yields that either C^a_μ or D^a_μ will not propagate. Parallely it is not the same with A^a_μ and B^a_ν. There both propagate and there is also a mixing propagator contributing. This shows that there is no direct mapping between the propagations. It is not trivial that some physical conditions as renormalizability and unitarity are independent of the set choice. In a further work these aspects will be studied.

In ref.[1] a gauge model for families is motivated. Take the fermionic example. There the electron and muon with ist neufield. However here it contributes to gauge theories properties : it has consequences in the gauge fixing terms (26)-(27), and it can interact with ghosts. The Slavnov-Taylor identities also depend on it. Another aspect is that it is not isolated from the D_μ^a field, i.e, a mixing propagator with this fields can be calculated.

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In ref.[1] a gauge model for families is motivated. Take the fermionic example. There the electron and muon with ist neutrinos are associated to the gauge fields A_μ and B_ν respectively, Other case is the twelve colourful stones model where gauge fields are associated to spin half and zero families respectively. For this context, the physical meaning for the mixing propagator is to represent a type of interaction between the families as in Fig. 2. The Fourier transform of this propagator gives the potential between families for the static case. A we know the asymptotic freedom property is motivated from the three-gauge-boson vertex. Thus its origin is also from the interaction between different families as in Fig. 4. This common fact creates a relationship between such potential and asymptotic freedom. Although the graphs have an origin in different terms of the Lagrangian their physical insight is the same. It is to relate families with different nature. If the potential calculated is linear, this model relates confinement and asymptotic freedom.

One could have thought why not to associate a quantum member to the vector fields A_μ^a and B_μ^a in such a way to distinguish them

Fig. 2

Mixing Propagator

In (a) it connects electron and muon families. For the twelve colourful stones case, fermions and bosons can interact through (b).

from each other explicitly. In principle this can be achieved by introducing the group Z_2 where A_μ^a and B_μ^a would be its two elements. We have not considered this case here. Anyway one can also think that radiative corrections could break this invariance.

The motivation of this work is to develop the extended gauge symmetry [1]. (14) had basic situation to be explored. Other situations propagating different massive fields will be developed. Our propose is to add other types of graphs through gauge theories. For instance, the evolution from abelian to non-abelian theories is in the appearance of a three-gauge boson vertex. The covariant basis as in Fig. 1 allows us to extend different combinations for this vertex. Cases where one (two) massless field interacts with two (one) massive fields, or when three massive fields interact between themselves, can be extended.

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APPENDIX A: GROUP U(1)

The two fields A_μ and B_ν transfom like

$$A_\mu \rightarrow A_\mu - \partial_\mu \lambda$$

$$B_\mu \rightarrow B_\mu - \partial_\mu \lambda \qquad (A1)$$

yielding that the tensor

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu B_\mu \qquad (A2)$$

is invariant under (A1). Suppose the electron and its neutrino belonging to the same group. They are written by the fermionic fields ψ and χ respectively. Using the condition (2) and (A2) for the gauge field Lagrangian we get

$$\partial^\mu [j_\mu(\psi) + j_\mu(\chi)] = 0 \qquad (A3)$$

and that the fields A_μ and B_μ are independent.

APPENDIX B: FEYNMAN RULES

Propagators and vertices for (14) are studied. (26) and (27) give different possibilities for the gauge fixing term. A massive term can be included. Consider

$$\mathcal{L}_G = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2\alpha} (\partial . A^a)^2 \tag{B1}$$

and observing that the fields are zero at infinity, it gives for the Fourier transform of the action free part,

$$S^{FREE} = \frac{1}{4} \int d^4k \, [A_\mu(-k), B_\mu(-k)] \, M^{\mu\nu} \begin{bmatrix} A_\nu(k) \\ B_\nu(k) \end{bmatrix}$$

where

$$M^{\mu\nu} = \begin{bmatrix} -k^2\eta^{\mu\nu} - \frac{1}{2\alpha} k^\mu k^\nu & k^\mu k^\nu \\ k^\mu k^\nu & -k^2\eta^{\mu\nu} - \frac{1}{2\alpha} k^\mu k^\nu \end{bmatrix} \tag{B2}$$

Propagators will be obtained by calculating the inverse of $M^{\mu\nu}$. A mixed propagator coming from the non-diagonal terms appears as in Fig. 3. It gives the general form

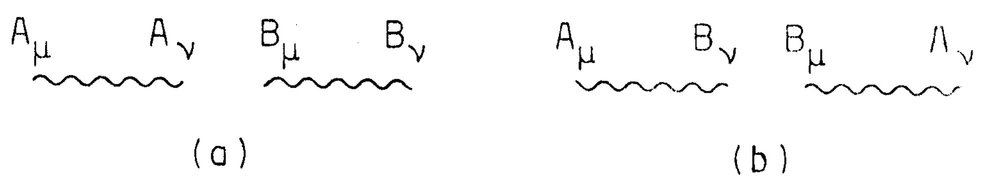


Fig.3. (14) brings two types of propagators. They are between the same fields (a) or different fields (b).

$$P^{\mu\nu}_{A\to A} = -\frac{1}{k^2}[\eta_{\mu\nu} - r\,\frac{k_\mu k_\nu}{k^2}] \quad (B3)$$

$$P^{\mu\nu}_{B\to B} = -\frac{1}{k^2}[\eta_{\mu\nu} - s\,\frac{k_\mu k_\nu}{k^2}] \quad (B4)$$

$$P^{\mu\nu}_{A\to B} = t\,\frac{k^\mu k^\nu}{k^2} \quad (B5)$$

and

$$P^{\mu\nu}_{B\to A} = P^{\mu\nu}_{A\to B} \quad (B6)$$

For (B1),

$$r = 1-2\alpha \quad , \quad s = -2\alpha \quad , \quad t = -2\alpha \quad (B7)$$

For $\frac{1}{2\beta}(\partial.B^a)^2$ condition,

$$r = -2\beta \quad , \quad s = 1-2\beta \quad , \quad t = -2\beta \quad (B8)$$

A MASSIVE PROPAGATOR

It was observed in [1] that (3) and (4) yield an expression that transforms covariantly,

$$A_\mu - B_\mu \to U(A_\mu - B_\mu)\; U^{-1} \quad (B9)$$

It yields for (27),

$$S = \frac{1}{4}\int d^4x \, [A^\mu(x)\, ,\, B^\mu(x)] \; N_{\mu\nu}(x) \begin{bmatrix} A_\nu(x) \\ B_\nu(x) \end{bmatrix}$$

where

$$N_{\mu\nu}(x) = \begin{bmatrix} (\Box + m^2)\,\eta_{\mu\nu} + \frac{1}{2\alpha}\,\partial_\nu\partial_\mu & \lambda'\partial_\mu\partial_\nu - m^2\,\eta_{\mu\nu} \\ \lambda'\partial_\mu\partial_\nu - m^2\,\eta_{\mu\nu} & (\lambda^2\Box + m^2)\,\eta_{\mu\nu} + \frac{1}{2\alpha}\,\partial_\mu\partial_\nu \end{bmatrix} \qquad \text{(B10)}$$

$\lambda = g_B/g_A$; $\lambda' = \sigma$ and $\sigma = \dfrac{1}{2\alpha}$

A method to calculate the inverse of (B10) is developed. It is a case where all elements are invertible. It gives

$$P^{M,\mu\nu}_{A\to A} = A_1\,\eta_{\mu\nu} + B_1\,\frac{k_\mu k_\nu}{k^2} \qquad \text{(B11)}$$

$$P^{M,\mu\nu}_{B\to B} = A_2\,\eta_{\mu\nu} + B_2\,\frac{k_\mu k_\nu}{k^2} \qquad \text{(B12)}$$

$$P^{M,\mu\nu}_{A\to B} = A_3\,\eta_{\mu\nu} + B_3\,\frac{k_\mu k_\nu}{k^2} \qquad \text{(B13)}$$

and

$$P^{M,\mu\nu}_{B\to A} = P^{M,\mu\nu}_{A\to B} \qquad \text{(B14)}$$

where

$$A_1 = \frac{-\lambda^2 k^2 + m^2}{\lambda^2 k^4 - m^2 k^2(\lambda^2+1)} \qquad \text{(B15)}$$

$$A_2 = \frac{-k^2 + m^2}{\lambda^2 k^4 - m^2 k^2(\lambda^2+1)} \qquad \text{(B16)}$$

$$A_3 = \frac{m^2}{\lambda^2 k^4 - (\lambda^2+1)m^2k^2} \tag{B17}$$

The gauge depend term for $-\frac{1}{8\alpha}[\partial.(A+B)]^2$ is

$$B_1 = \frac{a_1k^6 + x_1m^2k^4 + y_1m^4k^2 + z_1m^6}{p_1k^8 + q_1k^6 + r_1k^4 + s_1k^2} \tag{B18}$$

$$B_2 = \frac{a_2k^6 + x_2m^2k^4 + y_2m^4k^2 + z_2m^6}{p_2k^8 + q_2m^2k^6 + r_2m^4k^4 + s_2m^6k^2} \tag{B19}$$

$$B_3 = \frac{x_3k^4 + y_3m^2k^2 + z_3m^4}{p_3k^6 + q_3m^2k^4 + r_3m^4k^2} \tag{B20}$$

where

$$a_1 = (1-\sigma)\lambda^6 - 2\sigma\lambda^5 \quad ; \quad p_1 = -(\sigma\lambda^6 + 2\sigma\lambda^5 + \sigma\lambda^4)$$

$$x_1 = 2\lambda^5 + (\sigma-2)\lambda^4 + 4\sigma\lambda^3 \quad ; \quad q_1 = \lambda^6 + 2(\sigma+1)\lambda^5 + (3\sigma+1)\lambda^4 + 4\sigma\lambda^3 + 2\sigma\lambda^2$$

$$y_1 = -4\lambda^3 + \lambda^2 - 2\sigma\lambda \quad ; \quad r_1 = \lambda^6 + 2\lambda^5 + (\sigma+3)\lambda^4 + (2\sigma+2)\lambda^3 + (2\sigma+2)\lambda^2 + 2\sigma\lambda + \sigma$$

$$z_1 = 2\lambda \quad ; \quad s_1 = (\lambda^2+1)^2(\lambda^2+2\lambda+1) \tag{B21}$$

$$a_2 = -(3\sigma-1) \quad ; \quad p_2 = -[(1+\sigma)\lambda^4 + (3\sigma-1)\lambda^2$$

$$x_2 = 5\sigma \quad ; \quad p_2 = (3+2\sigma)\lambda^4 + (7\sigma+2)\lambda^2 + 3\sigma - 1$$

$$y_2 = -(2\sigma+3) \quad ; \quad r_2 = (\lambda^2+1)^2(1+\sigma) + (\lambda^2+1)(2\sigma+1+2\lambda^2) + 2\lambda^2$$

$$z_2 = 2 \quad ; \quad s_2 = 4(\lambda^2+1) \tag{B22}$$

$$x_3 = -(\sigma-\lambda)\lambda^2 \quad ; \quad p_3 = (\sigma+1)\lambda^4 + \lambda^3 + \sigma^2\lambda^2$$

$$y_3 = -(\lambda^3+\lambda^2+(2\sigma+1)\lambda) \quad ; \quad p_3 = -[(1+\sigma)\lambda^4 + 2(1+\sigma)\lambda^3 + (2\sigma+3)\lambda^2 + \sigma]$$

$$z_3 = 2\lambda \quad ; \quad r_3 = \lambda^4 + 2\lambda^3 + 2\lambda^2 + 2\lambda + 1 \tag{B23}$$

For $-\frac{1}{8\alpha}[\partial.A]^2$,

$$B_1 = \frac{x_1k^4 + y_1m^2k^2 + z_1m^4}{q_1k^6 + r_1m^2k^4 + s_1m^4k^2} \tag{B24}$$

$$B_2 = \frac{a_2k^6 + x_2m^2k^4 + y_2m^4k^2 + z_2}{p_2k^8 + q_2m^2k^6 + r_2m^4k^4 + s_2m^6k^2} \tag{B25}$$

$$B_3 = \frac{x_3m^2k^4 + y_3m^4k^2 + z_3m^6}{p_3k^8 + q_3m^2k^6 + r_3m^4k^4 + s_3m^6k^2} \tag{B26}$$

$$x_1 = (\sigma-1)\lambda^4 \quad ; \quad q_1 = \sigma\lambda^4$$

$$y_1 = -[2\lambda^3 + (2\sigma-1)\lambda^2] \quad ; \quad r_1 = -[(\sigma+1)\lambda^4 + 2\lambda^3 + (2\sigma+1)\lambda^2]$$

$$z_1 = \sigma + 2\lambda \quad ; \quad s_1 = \lambda^4 + 2\lambda^3 + (\sigma+2)\lambda^2 + 2\lambda + \sigma + 1 \tag{B27}$$

$$a_2 = \lambda^6 \quad ; \quad p_2 = -[\lambda^{10}+(\sigma+1)\lambda^8]$$

$$x_2 = 2(\lambda^5-\lambda^4) \quad ; \quad q_2 = 5\lambda^8 - 2\lambda^7 + (4\sigma^2+3)\lambda^6$$

$$y_2 = -(4\lambda^3-\lambda^2) \quad ; \quad r_2 = -8\lambda^6 - (4\sigma^2+2)\lambda^4$$

$$z_2 = 2\lambda \quad ; \quad s_2 = 4(\lambda^4-\lambda^3) \qquad \text{(B28)}$$

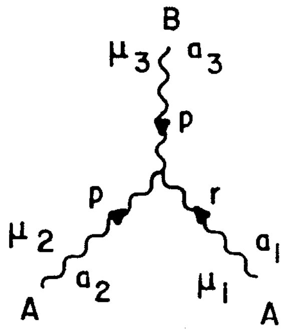
$$x_3 = \lambda^3 \quad ; \quad p_3 = \lambda^6 - \lambda^5$$

$$y_3 = \lambda^3+(\sigma-1)\lambda^2-\lambda \quad ; \quad p_3 = -[2\lambda^5+(2-\sigma)\lambda^4 - 2\lambda^3]$$

$$z_3 = \sigma + 2\lambda \quad ; \quad r_3 = [(\lambda^2+1)(2\lambda^4-2\lambda^3+(3-\sigma)\lambda^2-2\lambda) + 2\lambda^3+\sigma\lambda^2]$$

$$s_3 = -(\lambda^2+1)[(2\lambda+\sigma) + (\lambda^2+1)] \qquad \text{(B29)}$$

(14) yields two type of vertice. The first one is



$$i\Gamma^{\mu_1\mu_2\mu_3}_{a_1a_2a_3} = \frac{g}{2}\, c_{abc}(q-r)_{\mu_3}\, g^{\mu_1\mu_2} \qquad \text{(B30)}$$

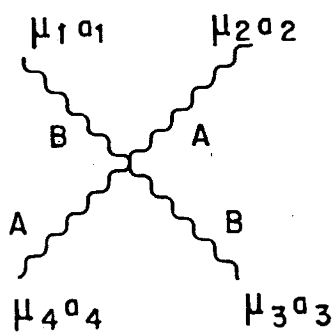
The other yields

$$\langle 0|T(B^a_\mu(x)\, B^b_\nu(y)\, A^c_\lambda(z)|0\rangle = -\langle 0|T(A^a_\mu(x)\, A^b_\nu(y)\, B^c_\lambda(z)|0\rangle \qquad \text{(B31)}$$

The four gauge-boson-vertex gives

$$i\Gamma^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4} = -\frac{ig^2}{2}\,(c_{a_1a_2f}\,c_{a_3a_4f} + c_{a_1a_4f}\,c_{a_2a_3f})\,g^{\mu_1\mu_3}\,g^{\mu_2\mu_4} \qquad \text{(B32)}$$

corresponding to the graph



APPENDIX C: POWER COUNTING AND PRIMITIVE DIVERGENCE

In order to determine whether a certain graph, in a theory with a cutoff, depends strongly or weakly on the cutoff the asymptotic behaviour of the integrand has to be determined. Thus power counting is a necessary criterium for (14) renormalizability. The primitive divergences are expected to depend on the number of space dimensions, on the type of vertices and on the order in perturbation theory. Consider a primitively divergent graph with E external legs, I internal lines, and I_{AB} mixing propagators. The total number of vertices is V and V_3 is the number of three gauge boson vertices. The superficial divergence will be given by

$$\delta^{\text{SUPERFICIAL}}_{\text{GRAPH}} = 2I - 2I_{AB} - 4(V-1) + V_3 \qquad \text{(C1)}$$

The topological relation between legs and vertices is

$$E + 2I = 3V_3 + 4V_4 \qquad \text{(C2)}$$

(C1) in (C2) gives

$$\delta^{\text{SUPERFICIAL}}_{\text{GRAPH}} = 4 - E - 2I_{AB} \qquad \text{(C3)}$$

We can therefore see that there is only a finite number of kinds of primitively divergent graphs as the above formula does not depend on the order in perturbation theory. The conclusion is that the present theory is renormalizable. As an example, some graphs are analysed in Fig. 4. The first case behave as $\ell n\ \Lambda$, the others are convergent.

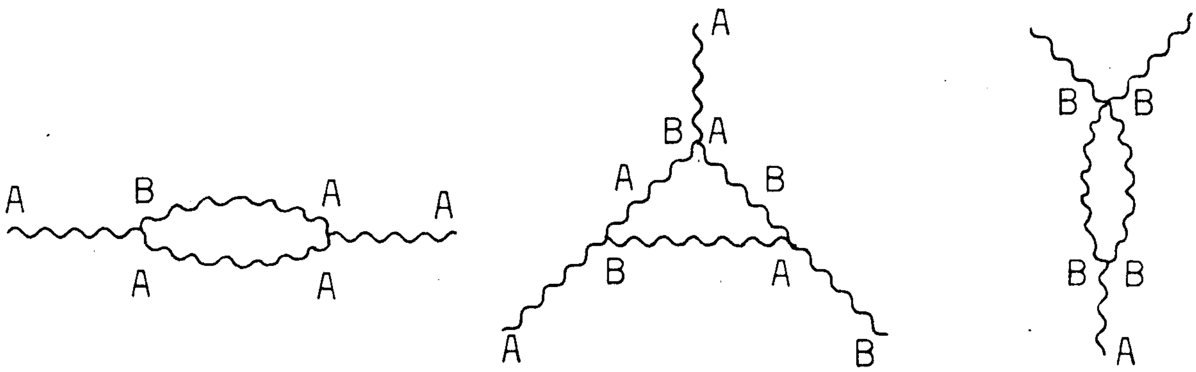


Fig. 4. Superficial divergence analysis. In Fig. (a), E=2, $I_{AB} = 1$ and so, $\delta = 0$. In Fig. (b), $\delta = -5$. In Fig. (c), $\delta = -1$.

APPENDIX D: A MODEL FOR THE TWELVE COLOURFUL STONES

A colourful model should have the confinement property. A building block model the asymptotic freedom property. Therefore a Lagrangian with colour must show such situations. (14) yields three gauge boson graphs for asymptotic freedom as in Fig. 5. Consider confinement in terms of static potential. Following [2] the separation energy is given by the difference between the self energy and the interquark potential

$$W(r) = \frac{1}{(2\pi)^3} \int d\vec{k}\,(1-e^{i\vec{k}.\vec{r}})\ P_{00}(k^2) \tag{D1}$$

where P_{00} is the propagator time-time dependent part. Substituing (B15) in (D1) give

$$W_1(r) = \frac{1}{2}\frac{1}{r}\,[1+e^{-mr}] \tag{D2}$$

(B16) in (D1),

$$W_2(r) = \frac{1}{2}\frac{1}{r}\,[1-e^{-mr}] \tag{D3}$$

Expanding for $mr < 1$ the energy separation is

$$W_1(r) \rightarrow \frac{1}{r}\Big|_{r=0} - \frac{1}{4} - \frac{m^2 r}{4} + \sigma(r^2) \tag{D4}$$

$$W_2(r) \rightarrow \frac{m^2}{4} r + \sigma(r^2) \tag{D5}$$

In terms of the proton radius this expansion is valid for a gauge boson mass $m < 0.2\,\text{GeV}$. Observe that (D5) confines, but (D4) does not. It is a naive approach. We notice it just as a first representation. Our intention is to observe how gauge boson mass becomes a parameter for linear potential. In the above case confinement has a non-absolute aspect.

The main motivation is to understand the asymmetry between fermions and bosons. Expressions as in (D4) and (D5) should work as an initial laboratory. Consider the twelve colourful stones model as building blocks for bosonic and fermionic quarks and leptons [3]. There gauge theories are developed in terms of bosonic and fermionic families. (D4) would represent same families interaction. This means that the bosonic case would not be a bound state made by a linear potential. However fermionic quarks and leptons would be composed by a confining potential due to (D5). Therefore, inside of this naive and restrict model, a motivation for having more fermionic than bosonic composite structures appears.

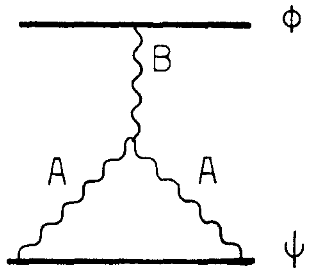


Fig. 5. Three-gauge-boson vertex generated by (14). It will contribute for the asymptotic freedom property of the theory.

APPENDIX E: THE PROPAGATOR DIAGONIZATION

In order to avoid mixed propagators we are going to redefine the fields,

$$A_\mu \to r\,A'_\mu + s\,B'_\mu \quad ; \quad B_\mu \to t\,A'_\mu + \mu\,B'_\mu \tag{E1}$$

where r, s, t, and u are variables to be fixed by the diagonalization procedure. Consider the ortogonal form

$$\begin{pmatrix} A_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix} \begin{pmatrix} A'_\mu \\ B'_\mu \end{pmatrix} \tag{E2}$$

The diagonalization method seeks a Lagrangian

$$\mathcal{L}_G = (A'_\mu, B'_\mu) \quad (Y)^{\mu\nu} \begin{pmatrix} A'_\nu \\ B'_\nu \end{pmatrix}$$

where $(Y)^{\mu\nu}$ is diagonal (E3)

A list with the Lagrangians studied in terms of the new fields A'_μ and B'_ν is written below:

(a) For $\mathcal{L}_G = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{8\alpha} (\partial^\mu A_\mu)^2 + \frac{1}{8\beta} (\partial^\mu B_\mu)^2$ (E4)

yields,

$$P^{\mu\nu}_{A'\to A'} = \frac{1}{k^2}\,[\eta^{\mu\nu} + \frac{\Lambda}{1-\Lambda}\,\frac{k^\mu k^\nu}{k^2}]$$

$$P^{\mu\nu}_{B'\to B'} = -\frac{1}{k^2}\,[\eta^{\mu\nu} + \frac{\Lambda'}{1-\lambda'}\,\frac{k^\mu k^\nu}{k^2}]$$

where $\Lambda = \frac{1}{2\alpha}\cos^2\theta + \frac{1}{2\beta}\,\mathrm{sen}^2\theta + \mathrm{sen}^2\theta$

$$\Lambda' = \frac{1}{2\beta}\,\mathrm{sen}^2\theta + \frac{1}{2\alpha}\,\mathrm{sen}^2\theta - \mathrm{sen}^2\theta \qquad \text{(E5)}$$

and $\cot^2 g\; 2\theta = \frac{1}{4}\,(\frac{1}{\alpha} - \frac{1}{\beta})$

(b) Including in (E4) the mass term $\frac{1}{2}\,m^2(A^a_\mu - B^a_\mu)^2$ gives,

$$P^{\mu\nu}_{A'\to A'} = \frac{1}{\cos^2\theta\,(k^2-m^2)}\,[\eta^{\mu\nu} - \rho\,\frac{k_\mu k_\nu}{[(\cos^2\theta+\rho)k^2 - \cos^2\theta\, m^2]}$$

$$P^{\mu\nu}_{B'\to B'} = \frac{1}{\cos^2\theta\,(k^2-m^2)}\,[\eta^{\mu\nu} - \rho'\,\frac{k_\mu k_\nu}{[(\cos^2\theta+\rho')k^2 - \cos^2\theta\, m^2]}]$$

where $\rho = \cos^2\theta + (\cos^2\theta - \mathrm{sen}^2\theta)\,\frac{1}{4\alpha} + (\cos^2\theta + \mathrm{sen}^2\theta)\,\frac{1}{4\alpha}$

$$\rho' = \cos^2\theta + (\cos^2\theta + \mathrm{sen}^2\theta)\,\frac{1}{4\alpha} + (\cos^2\theta - 2\,\mathrm{sen}^2\theta)\,\frac{1}{4\beta}$$

and $(\frac{1}{\alpha} - \frac{1}{\beta})\,\mathrm{sen}^2\theta = 0$ (E6)

As discussed in the text, (E5) and (E6) get a physical meaning either making $\alpha\to\infty$ or $\beta\to\infty$.

(c) For $\mathcal{L}_G = -\frac{1}{4}\,G^a_{\mu\nu}\,G^{\mu\nu}_a + \frac{1}{8\alpha}\,[\partial^\mu(A^a_\mu + B^a_\mu)]^2$ (E7) yields,

$$P^{\mu\nu}_{A'\to A'} = -\frac{1}{k^2}\left[\eta^{\mu\nu} + \frac{1+\mathrm{sen}^2\theta(2\alpha-1)}{(1-2\alpha)[1-\mathrm{sen}2\theta]}\frac{k^\mu k^\nu}{k^2}\right]$$

$$P^{\mu\nu}_{B'\to B'} = -\frac{1}{k^2}\left[\eta^{\mu\nu} + \frac{1-\mathrm{sen}^2\theta(2\alpha-1)}{(1-2\alpha)(1+\mathrm{sen}^2\theta)}\frac{k^\mu k^\nu}{k^2}\right]$$

with $$\left(\frac{2\alpha-1}{2\alpha}\right)(\cos^2\theta-\mathrm{sen}^2\theta) = 0 \qquad \text{(E8)}$$

(d) Including in (E7) the mass term as (E6)

$$P^{\mu\nu}_{A'\to A'} = \frac{1}{-k^2+m^2(1+\mathrm{sen}^2\theta)}\left[\eta^{\mu\nu} + \frac{k_\mu k_\nu}{-[(1+\frac{1}{2\alpha})+(1-\frac{1}{2\alpha})\mathrm{sen}^2\theta]k^2+m^2(1+\mathrm{sen}^2\theta)}\right]$$

$$P^{\mu\nu}_{B'\to B'} = \frac{1}{-k^2+m^2(1-\mathrm{sen}^2\theta)}\left[\eta^{\mu\nu} + \frac{k_\mu k_\nu}{-[(1-\frac{1}{2\alpha})+(1-\frac{1}{2\alpha})\mathrm{sen}^2\theta]k^2+m^2(1-\mathrm{sen}^2\theta)}\right]$$

with $$(1-2\alpha)\cos^2\theta = 0 \qquad \text{(E9)}$$

The diagonalization procedure will avoid the mixing propaga tor but changes the interacting part. Observe that physically a massive field can not be transformed as in (E2).

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