

CBPF-NF-032/81
THEORETICAL AND PHENOMENOLOGICAL
PROBLEMS CONCERNING $J^P=1^+$ MESONS

by

I. Bediaga*, J.M. Pires*[†]

and A.F.S. Santoro

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Av. Wenceslau Braz, 71, fundos
22290 - Rio de Janeiro - R.J. - BRASIL

*Fellow of the Conselho Nacional de Desenvolvimento Científico e Tecnológico

[†]Universidade Federal do Espírito Santo - Departamento de Física e Química.

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ABSTRACT

From both the experimental and theoretical points of view, the researches on $J^P=1^+$ mesons have been very important for hadronic physics. The complex problems involving these states (particularly in regard the determination of resonances and other non-resonant effects) have occupied a great part of the hadron-hadron phenomenology literature in the last decade. We give a description of the principal views of the subject with a particular emphasis on the case of the $A_1(\rightarrow\rho\pi)$ meson because it is the most important source of papers on $J^P=1^+$ objects and presents still now, open problems. An almost exhaustive of references on the subject and correlated topics is given.

1. INTRODUCTION

Our principal motivation here was to describe the complexity of problems involving spin-parity $J^P = 1^+$ states and, at the same time, to make a critical review, of the models proposed up to now. To promote a status of bona fide resonances for these states (predicted by SU(3)), has required to overcome enormous experimental difficulties on the one hand and has suggested a great quantity of theoretical models on the other hand. As an example of the problems connected with the dilemma "resonance" versus "Deck effect" (false proposition) have triggered a large production of papers as we will see in the following. In spite of many problems being still completely open, some of them have become obsolete or have been forgotten due to the coming of new a phenomenology - using the language of Quarks and Partons -. But what are those problems and what suggestions could we give to solve them? We intend here to answer at least partially these questions. There are many theoretical models to describe particular views of the subject but we feel that one lacks a global view to take into account all particular facts in a self-contained form.

First of all, we describe the experimental results -(section 2) - and next we discuss the theoretical and phenomenological points of view - (Section 3). These two parts of the paper can be read independently. The reader can go directly at any one of them. We finish with a conclusion (Sec.4) - that summarizes all points discussed in the text. As a gener-

al remark closing this Introduction, it is gratifying that the SU(3) prediction for these $J^P = 1^+$ objects have been now almost fulfilled by the new experimental results, although some difficulties still exist like the observation of the A_1^0 that continues to be a problem, (specially in charge exchange reactions) as we will see in the following and the detection of H and H'.

2. EXPERIMENTAL RESULTS

2.1 GENERALITIES

We begin with a general view of the experimental results obtained in the last years. The aim of some of these experiments was the identification of a number of resonances B, (H, H'), (D, D' \equiv E) (Q_A, Q_B) and A_1 as $J^P = 1^+$ meson states [1], belonging to two SU(3) nonets given in Table (I) [2]. We have chosen a certain number of distributions, (invariant mass, transfer momentum, angular) partial wave analysis (PWA) and relative or phase-shift analysis, to give an idea about the experimental situation. In fact we display only a little number of results, the most significative and recent ones. Reactions of the diffractive dissociation type (see Fig.(1)) have been preferentially chosen due to the great number of diffractive productions. As it is evident from Fig.(1), by diffractive production for $2 \rightarrow 3$ (particles) ($a + b \rightarrow 1 + 2 + 3$) we mean those reactions for which the squared center of mass energy $S = (p_a + p_b)^2$ and the sub-energy $S_2 = (p_2 + p_3)^2$ are large while $S_1 = (p_1 + p_2)^2 = M_{12}^2$ (squared invariant mass of 1+2) is small. In these reactions we also observe a strong concentration of events at small t_2 (momentum transfer between b and 3). The high value of S_2 and small values of t_2 justify the assumption

tion of Pomeron (\mathbb{P}) exchange for the (b3) vertex. We report also others experimental results, as non-diffractive reactions, forward and backward productions, charge exchange reactions. In these cases for reasons related to searches for A_1^0 and H , we favour the charge exchange reactions. By contrast, we go hastily through the experimentally well establish results (e.g. B and D). When it will be necessary we make some theoretical comments in this section.

2.2 B - (Budha)

The constituent of the SU(3) nonet, $J^{PC} = 1^{+-}$ called Budha (B) [3] (Table I) is now well established as a good resonance. We give below, a number of distributions with the purpose of pointing out the existence of this object among others with the same spin-parity. Its main properties are [1]:

- a) $I^G J^{PC} = 1^+ 1^{+-} (B^+, B^- \text{ and } B^0)$ [4], where I^G refers to the isospin and G-parity.
- b) Mass and Width, $M_B = 1231 \pm 10$ (MeV), $\Gamma_B = 129 \pm 10$ (MeV).
- c) Decay mode [5], $B \rightarrow \omega \pi$ (only seen).
- d) Studied reactions [1], πN , KN and $p\bar{p}$ at different energies.
- e) Cross-sections [6]:
 - i) $\pi^- p \rightarrow \pi^- \omega^0 p$ [6] at 6.7 (GeV/c) 160 ± 22 (μb)
 - ii) $\pi^- p \rightarrow p \omega^0 \pi^-$ [7] at 9.1 (GeV/c) 123 ± 22 (μb)
 - iii) $\pi^- p \rightarrow p \pi^+ \pi^0 \pi^- \pi^-$ [8] $\left\{ \begin{array}{l} 3.2 \text{ (GeV/c)} \quad 108 \pm 30 (\mu\text{b}) \\ 4.2 \text{ (GeV/c)} \quad 67 \pm 20 (\mu\text{b}) \end{array} \right.$
 - iv) $K^- p \rightarrow \Sigma^- B^+$ [9] (Backward Production)
at 4.2 (GeV/c) 3.2 ± 0.5 (μb)

We show in Fig.(2) the invariant mass distributions for two different reactions and energies, the πN and KN backward production of B . In both reactions the B production is clear. The mass and width values obtained for the B from the reaction πN (Fig.(2a)), are: $M_B = 1242 \pm 10$ (MeV) and $\Gamma_B = 140 \pm 40$ (MeV).

From Fig.(2b), the backward production of B in KN reaction is estimated at $M_B = 1208 \pm 18$ (MeV) and $\Gamma_B = 163 \pm 50$ (MeV).

Both experiments [7,9] fit the data with a Breit-Wigner formula and backgrounds of different types. In Fig.(3) we show a transfer momentum distribution for πN interaction ($t_{p_t p_f} = t_2$, $p_t =$ proton target and $p_f =$ final proton).

A strong concentration of events occurs at small values of t_2 . This situation is not exceptional and characterizes the contribution of peripheral mechanisms present in B production. In spite of the fact that the B is a well established resonance [1b], other peripheral mechanisms (non-resonant background) can be very important in an exact determination of production cross-sections in each specific reaction. All results from the published literature [1a] confirm the resonance hypothesis for B in a clearer way than for most of the other $J^P = 1^+$ objects as we will see in the following.

2.3 H and H'

These two states seem to be particularly difficult to detect experimentally [10a]. Only now [10b] after almost ten years, does a reasonable evidence about the H existence emerge and even now its parameters are not yet very conclusively estab-

lished: a) $J^{PC} = 1^{+-}$, $M_H \approx 1.0$ (GeV); b) possible decays: $K\bar{K}\pi$ and $\rho\pi$. One of the difficulties is that the region of mass of this resonance overlaps with that of the A_1 . Although the isospin values are different for $A_1(I=1)$ and $H(I=0)$, it is very difficult to make a clear separation in the global mass spectrum of $\pi^+\pi^-\pi^0$. Some authors [11] argue that the $\rho^-\pi^+$ charge states are more suitable to search for the H and A_1^0 . This is due to the different $\rho^-\pi^+$ mass spectrum obtained in comparison with $\rho^+\pi^-$ and $\rho^0\pi^0$ in the small masses region. Together with others [12], however, we think that the $\rho^0\pi^0$ is the best case to search for the H, since in this case, the A_1^0 contribution is obviously absent. In comparison with the $K\bar{K}\pi$ decay, we have in this case only one neutral particle [13] (π^0) since the ρ^0 is identified by $\pi^+\pi^-$ in the final state. We call attention to the fact that the difficulty in the experimental identification of the H and A_1^0 are very similar. We believe also that if the resonance H exists, it is enhanced (similarly A_1) by strong contributions coming from kinematical [14] threshold Deck like effects. From a theoretical point of view the situation is not less confused. There are contradictory predictions [15] coming from the naive Quark Model and duality schemes respectively. In section (2.6), particularly in the A_1^0 subsection we return to many of these comments [10b].

2.4 D and D' (\equiv E)

While the D(1285) meson is well established as a good resonance [16] the E(1420) meson is not yet definitely identi

fied as a resonant state although recent results^[17] obtained from π^-p at 3.95 (GeV/c) confirm the previous quantum numbers assignement^[1a] for the E meson. The main characteristics for these D and E states are:

- a) Both have been seen initially in $p\bar{p}$ annihilations^[1,18] and have afterwards been produced in other reactions (πN and KN).
- b) The mass spectra are compatible with a Breit-Wigner formula.
- c) $I^G, J^{PC} = 0^+, 1^{++}$
- d) ^[1a] $M_D = 1284 \pm 10$ (MeV), $\Gamma_D = 27 \pm 10$ (MeV)
 $M_E = 1418 \pm 10$ (MeV), $\Gamma_E = 50 \pm 10$ (MeV)
- e) Decays:

$$D \rightarrow 4\pi, K\bar{K}\pi, \eta\pi\pi, \delta\pi$$

$$E \rightarrow K\bar{K}\pi, (K^*\bar{K}+K\bar{K}^*), \eta\pi\pi, \delta\pi$$

We show in Fig.(4a) the mass spectrum of the ($\eta\pi\pi$) final state obtained from the reaction $\pi^-p \rightarrow \eta\pi^+\pi^-n$ at 8.45 (GeV/c) where the D is well seen. The plot of the relative phase versus effective mass $m_{\eta\pi\pi}$ is shown in Fig.(4b) and we note that ($\delta\pi$) decay is preferred. In Fig.(5) we show the effective mass of $K^\pm K^0 K^\mp$ final states for E(1420) production. Finally to complete these information about D and E we present in Table (II) some branching-ratios, cross-sections and decay modes. From a theoretical point of view the D and E mesons are predicted by the Quark model^[2,22] although we should mention a recent controversial interpretation of the E(1420) as an object related to the existence of glueball^[22d-d].

2.5 Q_A and Q_B

Each meson Q (Q_A, Q_B) is a constituent of one different no net (see Table I). They are produced mainly from reactions in itiated by K . These two mesons have motivated a great amount of theoretical and experimental [1a] work due to the difficulty in determinating these states as two resonances. Their main properties are [1a]:

a) $I = 1/2, J_{(Q_A)}^{PC} = 1^{++}, J_{(Q_B)}^{PC} = 1^{+-}$

b) $M_{Q_A} = 1280(\text{MeV}); \Gamma_{Q_A} = 120(\text{MeV})$

$M_{Q_B} = 1400(\text{MeV}); \Gamma_{Q_B} = 150(\text{MeV})$

c) Decay:

$$\begin{array}{l}
 \left. \begin{array}{l}
 Q_A \\
 \\
 \\
 \\
 \end{array} \right\} \begin{array}{l}
 K\pi\pi \text{ dominant} \\
 K\omega \text{ recently seen [23,1a]} \\
 K\rho \text{ favoured} \\
 K^*\pi
 \end{array} \\
 \\
 \left. \begin{array}{l}
 Q_B \\
 \\
 \end{array} \right\} \begin{array}{l}
 K^*\pi \text{ favoured} \\
 K\rho
 \end{array}
 \end{array}$$

d) Cross-sections [24c]

i) $\sigma(Q_A \rightarrow K\rho) = 6.2 \pm 0.6 (\mu\text{b})$

ii) $\sigma(Q_A \rightarrow K^*\pi) = 1.7 \pm 0.5 (\mu\text{b})$

iii) $\sigma(Q_B \rightarrow K\rho) \geq 0.2 (\mu\text{b})$

iv) $\sigma(Q_B \rightarrow K^*\pi) < 0.5 (\mu\text{b})$

e) Helicity Conservation:

s - channel - (mode $K\rho$)

t - channel - (mode $K^*\pi$)

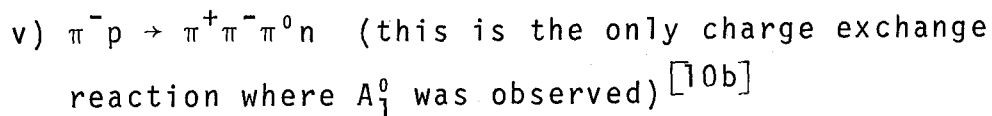
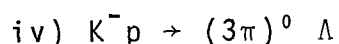
We choose a certain number of distributions to characterize them, in forward and backward production and for different reactions and energies. The effective mass of the $(K\pi\pi)$ system is shown in figure (6) [24]. For $K^-p \rightarrow K^-\pi^+\pi^-p$ at 10, 14 and 16 (GeV/c) it is shown in figure (6a) and in figure (6b) for $K^-p \rightarrow K^0\pi^-\pi^0p$. From the latter figure, we identify two peaks corresponding to the $Q_A(1.27)$ and to the $Q_B(1.37)$ respectively. Other data are shown in Fig.(6c) for the reactions $K^+d \rightarrow K^+\pi^+\pi^-d$ at 12(GeV/c) and in Fig.(6d) for backward productions. The (PWA) and relative phases are given in Fig.(7) (for $K^-p \rightarrow K^-\pi^+\pi^-p$ and $K^-p \rightarrow \bar{K}^0\pi^-\pi^0p$ reactions) and in Fig.(8) (for $K^\pm p \rightarrow K^\pm\pi^+\pi^-p$ at 13 (GeV/c). Two peaks are seen in the s-wave ($\ell=0$) associated to the Q_A and Q_B states. These data support the interpretation of the meson $Q_A \rightarrow K\rho$ as a good resonance but the evidence seems less conclusive in favour of $Q_B \rightarrow K^*\pi$. Similar results coming from other experiments and favouring the existence of two resonant states are given in ref. [25]. Other more recent analysis [26] - (PWA)-obtained from the reactions $K^-p \rightarrow K^-\pi^-\pi^+p$ and $K^-p \rightarrow \bar{K}^0\pi^-\pi^0p$ at 4.2 (GeV/c) are consistent with two s-wave resonances. For the study of possible mechanisms at work, one may turn now to angular and momentum transfer (t_2) distributions. In Fig. (9) [27] we show the t_2 -distribution and we note that the $K^-\pi^+\pi^-$ system has a slope greater than the $\bar{K}^0\pi^-\pi^+$ system.

This exponential behaviour is typical of diffractive productions. These $(d\sigma/dt_2)$ distributions present a well known cross-over^[27] for K^- , K^+ that was a motivation for the phenomenological models presented in the section 3. The mass-slope correlation parameters are given in Fig.(10) and Table III. In conclusion we think that the situation regarding these two $J^P = 1^+$ objects is not yet completely well established. By analogy with all states of this set we believe that there are two resonances, but that other mechanisms (such as e.g., the Deck effect) are also contributing.

2.6 A_1

The axial vector meson A_1 ^[30] was surely the subject of the greatest number of theoretical experimental papers among those of the $J^P = 1^+$ family. For example it is at the origin of the Deck Model (see section 3), and still nowadays we have a number of interesting problems not completely solved associated with this object^[1]. Its principal characteristics are:

- a) $I^G = 1^-$; $J^{PC} = 1^{++}$
- b) $M_{A_1} \approx 1.1$ (GeV); $\Gamma_{A_1} \approx 300$ (MeV)
- c) principal decay mode: $\rho\pi$
- d) different reactions studied:
 - i) $\pi^\pm p \rightarrow (3\pi)^\pm p$ (forward and backward production). Favoured reaction for observing the A_1^0 - in charge and hypercharge exchange reactions -.
 - ii) $\pi^+ n \rightarrow (3\pi)^0 p$



e) No s-channel helicity conservation [31].

f) Until very recently, all searches for A_1^0 production in charge exchange reactions gave basically negative results. Recently however strong evidences has been given for an A_1^0 resonant state in reaction (v) above 8.45 (GeV/c).

We try now to illustrate the various aspects of the problem for and against a resonant interpretation and due to the important literature associated with the subject we will try to be fairly complete. The reactions chosen are backward and forward productions, diffractive and non diffractive interactions and others. We recall first of all that while the diffractive reactions favoured the non-resonant interpretation of the $(\rho\pi)$ enhancements, the others reactions favoured a resonant interpretation via a Breit-Wigner formula. In Fig.(11) we show the total mass spectrum of (3π) from $\pi^- p \rightarrow (\pi^- \pi^+ \pi^-) p$ at 11 and 25 (GeV/c) where we see some evidence for two peaks associated with the A_1 and A_2 states where the latter is the well known $J^P = 2^+$ resonance at 1310 MeV. The solid curve is the result of a fit made by the authors of ref. [32] with the Deck-Model. An example of background productions [33,34] in πN reactions at 9(GeV/c) is shown in Fig.(12) where the solid line represents the result obtained from a fit with two Breit-Wigner

formulas for the A_1 [34] with $m_{A_1} = 1050 \pm 11$ (MeV) and $\Gamma_{A_1} = 195 \pm 32$ (MeV) and for the A_2 respectively. To compare with the experimental data we calculate the $(\rho\pi)$ mass distribution, using a double-Regge Model without the optimization of the Regge parameters (see ref. [35]). The result, shown by a dashed line in fig.(12), is very large and centered at $M_{A_1} = 1.18$ (GeV). This result can be improved by small variations of the parameters used. In Fig.(13) we show a backward production from the reaction $K^-p \rightarrow \Sigma^- \pi^+ \pi^- \pi^+$ at 4.15 (GeV/c) [35,36]. The (PWA) results are shown in Fig.(14) [36] and support the evidence of the A_1 as a $1^+S(\rho^0\pi^+)$ wave. While the combined results obtained from $\pi^-p \rightarrow \pi^- \pi^- \pi^+ p$ at 25 (GeV/c) and 40 (GeV/c) do not show any significant variations of the relative phases (see Fig.(15)), other more recent results [39] - from $\pi^-p \rightarrow \pi^- \pi^+ \pi^- p$ at 63 and 94 (GeV/c) provide the strongest piece of evidence in favour of the resonance interpretation of the A_1 (see Fig.16). The solid lines (Fig.16a-d) are the result of the analysis after the A_2 contribution has been subtracted out in the form of a Breit-Wigner and take into account also a Deck contribution. The mass and width found for the A_1 are $m_{A_1} = 1280$ (MeV) and $\Gamma_{A_1} = 300$ (MeV), values that do not agree with others of the current literature [1a] ($m_{A_1} \approx 1.1$ (GeV)). It is quite possible that threshold phenomena [40] may be responsible for the different values found. The authors of ref. [39] claim that only a resonance or a Deck amplitude separately could not account for the effects observed in these global spectra [41]. However, we call attention to the fact that the Deck contribution used in these fits takes into account only the π -exchange

term. We return to this point in Section 3.

A_1^0 Observation

As we have already mentioned, the observation of the A_1^0 has been particularly difficult in charge exchange reactions^[43] and only recently^[10b] a pronounced relative phase variation for the A_1^0 state has been observed confirming the resonant interpretation of this object. Also from the theoretical point of view the situation is quite confused as we can see in Table (IV) where several predictions of the cross-sections for A_1^0 production are reported. These predictions turn out to be very dependent on the mass and the approach employed.

There are other experiments^[42] - K^-p at 4-5 (GeV/c) and K^+p at 12.7 (GeV/c) - that identify the A_1^0 in $M(\pi^+\pi^-\pi^0)$ mass spectrum. For both reactions the effective mass distribution (Fig.(17)) shows a peak around 1.05 (GeV) associated with the A_1^0 . The reactions where the resonant interpretation is favoured^[43] ($\pi^+n \rightarrow \pi^+\pi^-\pi^0p$ at 4. (GeV/c)^[43a] $\pi^-p \rightarrow \pi^+\pi^-\pi^0n$ at 12 and 15 (GeV/c)^[43b] $\pi^+p \rightarrow \pi^+\pi^-\pi^0\Delta^{++}$ at 7. and 15 (GeV/c)^[43c], $K^-p \rightarrow \pi^+\pi^-\pi^0\Lambda^0$. at 4.2 (GeV/c)^[43d]) exhibit a strong cancellation responsible for not observing the A_1^0 in these reactions. An example is given in Fig.(18) where we show data from the same experiment at 15 (GeV/c)^[43c]: the A_1^+ signal is absolutely in the channel $\pi^+p \rightarrow p\pi^+\pi^-\pi^0$ in $1^+(\rho\pi)$ S wave, whereas no structure is seen in the $\pi^+p \rightarrow \Delta^{++}(\pi^+\pi^-\pi^0)$ results. As we have already pointed out in Section 2.3 in the cases of H and H'^[11] also in the mass region of the A_1^0 there are other competing resonant states, and this makes very difficult the analy-

sis and it is only in one charge exchange reaction^[10b], that a clear signal has been observed recently. Fig. (18c-e) show these results for A_1^0 as well as for the H mesons. The analysis for these states is made simultaneously since they are very close in mass ($m \approx 1.13(\text{GeV})$ in this experiment) exhibiting analogous difficulties. More data and analysis are necessary to make consistent the finding of the various charge exchange experiments^[43,10a].

$$\tau \rightarrow A_1 \nu$$

New experimental results^[44] coming also from lepton-hadrons interactions provide supplementary support in favour of the resonant interpretation of the A_1 meson. In spite of the small number of observed events, these experiments show that the heavy lepton τ ($J=1/2$, $m \approx 1784 \pm 4(\text{MeV})$) decays into $A_1(\rightarrow \rho\pi) \nu_\tau$. Fig.(19) shows the mass spectrum from e^+e^- interactions with A_1 identification. We return to this point in the next Section.

The interested reader may find many other results and angular distributions^[36,45] in the listed here.

3. THEORETICAL APPROACHES, SCHEMES AND MODELS

3.1 GENERALITIES

An evidence of the importance of the subject for particle phenomenology is the number of papers about $J^P=1^+$ mesons including papers dealing with theoretical schemes and models. We give here a short description of each of the main approaches. If, on the one hand $SU(3)$ predicts easily these resonances on

the other hand, their experimental detection has been very difficult. Experimentally a great step in improving this analysis has been the Partial Wave Analysis [37,43] of Ascoli and Collaborators. These analysis are now determinant in the identification of a resonance. For the sake of simplification, we can classify in three main categories the various theoretical schemes proposed so far, according to which mechanism they make responsible for the enhancements observed in the different reactions and which we call 1^+ mesons:

- I) pure resonant states described by Breit-Wigner formula;
- II) pure non-resonant states interpreted kinematically via Deck-like models.
- III) composite models where it is assumed that resonances exists but a Breit-Wigner formula is unable to account for all the spectrum, since these objects are produced the threshold of a new channel and other effects do also contribute. Thus the kinematical effects which give rise to Drell-Hiida-Deck-like models must also be taken into account in the complete amplitude.

Approaches (I) and (II) are too simple minded to provide a realistic description of the data. We believe like everybody else that approach (III) is the correct one. The difficulty is at a technical level in the sense of taking into account all contributions without incurring in the sin of double counting. The development of this subject occurred in parallel in diffractive dissociation reactions. We do not intend here to give an exhaustive description of each approach

but to give a good idea of the main ones and some information about the others.

3.2 RESONANT APPROACH

Usually, in the same reactions, 1^+ mesons are produced together with some well identified resonances such as A_2 , K_{1420}^* etc., but the former are much more difficult to detect.

A well known approach consists in interpreting the enhancements observed in the invariant mass distributions of a reaction like $a+b \rightarrow a^*b$ where $a^* \rightarrow 1+2$ (see Fig.(1)), as objects described by a Breit-Wigner (B.W.)^[47] formula which we write here for pedagogical purposes including threshold effects:

$$BW = \frac{pq}{m_R^2 - S_1 - im_R \Gamma}$$

where

$$\Gamma = \Gamma_R \left(\frac{q}{q_R}\right)^{2S+1} \frac{M_R}{\sqrt{S_1}}, \quad S_1 = M_R^2$$

(see ref. [46h] for the notation and definitions of variables). In a more complete analysis, we examine also the phase-shifts ($\delta_\ell(S_1)$) associated with each partial wave produced to verify which of them, if any, goes through $\pi/2$ around $S_1 = M_R^2$, (M_R = mass of the resonance). These relative phase variations (PWA)^[46] give an enhancement the status of a good resonance or not.

Many ambiguities are inherently present in the defini-

tion of a resonance like tail effects, background contamination, superposition of closed-by resonances which all make the BW formula somewhat unreliable beyond a certain level. We also mention that the symmetric curve produced by a (B.W.) - like formula is not always in agreement with experimental spectra. In general, near the threshold one finds an asymmetry that is well described by other mechanisms, and this is an indication that a pure (B.W.) formula does not describe completely the effect observed experimentally. In the particular case of the A_1^+ (seen in $\rho^0\pi^\pm \rightarrow \pi^+\pi^-\pi^\pm$) other effects coming from Bose symmetry^[47d] must be taken into account. It was shown^[47d] that this symmetrization increases the enhancement due to the A_1 resonance. (Note that this is not the case for A_1^0).

A special consideration deserves some classes of models, the so called dual models, whereby the full amplitude is constructed as a never ending superposition of resonances. A complete discussion of this class of models is, however, outside the scope of our present review and we refer the interested reader to the large literature existing on the subject.

Finally, the resonance approach with or without ambiguities is very simple, perhaps too simple and the $J^P = 1^+$ mesons are the proof that it does not always possible to use it to fully describe the physical reality.

3.3 KINEMATICAL EFFECTS AND DRELL-HIIDA-DECK (D.H.D.)

APPROACHES

The main points about the (D.H.D.)^[48] model are given in

the following with a somewhat more detailed description. In the original form, the (D.H.D.) model consists in considering the dissociation of the beam of particle into two virtual others that interact with the target (in general a nucleon or nucleus), diffractively (high energy and small transfer of momentum). For example, take the $\pi N \rightarrow \rho \pi N$ reaction shown in Fig.(20). This is given by the product of a pion-exchange and an off-mass-shell elastic subreaction characterized by a Pomeron exchange in the Regge language, i.e., the diffractive part of the global process. The cross-section^[48] that this mechanism gives for the diagram of the Fig.(20) is:

$$d\sigma = G \frac{|M_{\pi N}|^2}{(t_1 - \mu^2)^2} \delta^4(p_1 + p_2 + p_3 - p_a - p_b) \frac{dp_1}{E_1} \frac{dp_2}{E_2} \frac{dp_3}{E_3}$$

where G summarizes all numerical constants-flux factor and eventual off-on-mass-shell corrections for the elastic subreaction $M_{\pi N \rightarrow \pi N} = M_{\pi N}$ which is parametrized as

$$|M_{\pi N}|^2 = (8\pi S_2)^2 \left(\frac{d\sigma}{dt_2} \right)_{t_2=0} e^{bt_2}$$

where b is the slope of diffraction peak from $\frac{d\sigma}{dt_2}$ distribution and $\left(\frac{d\sigma}{dt_2} \right)_{t_2=0}$ is the differential forward cross-section.

The principal points associated with the development of the (D.H.D.) model are:

- i) Reggeization of the (D.H.D.) amplitude^[49],
- ii) Dualization^[50]
- iii) Considerations about others components^[51].

Since their first version^[48], the (D.H.D.) model had a great success for the mass spectrum description of $A_1 \rightarrow \rho\pi$, and looked initially as a competitive possibility for the resonant approach. It is easily understood why a D.H.D. amplitude produces a non symmetric enhancement. We recall the well known kinematical relation $\frac{S_1 S_2}{S} \approx \text{const.}$ and we look for the phase space in the invariants S_1 and S_2 . Since S and S_2 are great ($M_{\pi N}$ is dominated by Pomeron exchange) S_1 is necessarily small by energy-momentum conservation ($S_1 + S_2 + S_3 = S + m_1^2 + m_2^2 + m_3^2$). We see also that the amplitude is directly proportional to S_2 and $e^{bt_2} (t_2 < 0)$. Let us look to the Chew-Low plot (S_1 versus t_2) in the mass spectrum of Fig.(21); a small increase in S_1 corresponds to a rapid variation in t_2 arising from the exponential e^{bt_2} ; consequently we have rapid decrease of the curve in $S_1 (=m_{\rho\pi}^2)$ in spite of the peripheral π - exchange term ($1/(t_1 - \mu^2)$).

However, this naive Deck model does not describe other aspects of this reaction since so far we have taken into account only one of the three possible contributions: π - exchange, ρ - exchange and direct π -pole-exchange (for $\pi N \rightarrow \rho\pi N$ e.g.). There are many reasons^[51b] proving that besides considering these three components it is also necessary to take into account all phase space, mass-slope-cos $\theta^{G.J.}$ correlations, angular distributions, S and t channels helicity conservation^[52] etc. Finally, this model can be considered as a particular case of the Double Regge Model, and we return to this point in section 3.5.

3.4 CONTRIBUTIONS FROM RESCATTERING COMPONENTS

A model with final particles rescattering corrections is presented by some authors^[53] (see Fig.(22)) to take into account A_1 resonance effects as well as Deck like contributions. This model adds coherently the three following terms:

- i) one π -exchange Deck type,
- ii) one term representing the rescattering correction from final $\rho\pi$ states, and
- iii) one term representing the direct resonance production (via a Breit-Wigner formula) of the A_1 decaying into $\rho\pi$.

The results^[53b] of this model, are claimed to give support to the existence of the A_1 resonance in spite of the small phase variation found. The physical interpretation appear to be that the phase shifts due to the resonance term and those coming from the rescattering term would cancel each other leaving only those due to the Deck non-resonant term. But it is not obvious to us that the way in which the resonant and Deck components are added in this approach does not lead to double counting by duality arguments. If there is double counting we can ask what would it be the results with in a more complete model where the three Deck terms, π and ρ exchange and π -direct-pole-exchange corresponding to the t,u and S channels of the subreaction $\pi \rightarrow \rho\pi$ are all taken into account. An advantage of this approach with a rescattering term is that it can take into account directly the phase shifts of

the $\rho\pi$ elastic system via the final state (Watson)theorem [54]. Other authors [55] take into account all effects produced with resonant and Deck effects - with rescattering component - considering the corrections coming from unitarity in the Deck amplitude and keeping also the coupled channel contribution at the resonance (e.g. A_1 with $\rho\pi$ and $K^*\bar{K}$; Q with $K\rho$ and $K^*\pi$) via the K matrix formalism [56]. In principle, double counting should be absent from these approaches, in practice, however, the parameters used for the A_1 resonance ($M_{A_1} \approx 1300$ (MeV), $\Gamma_{A_1} \approx 400 \pm 100$ (MeV)) [55], ($M_{A_1} \approx 1450$ (MeV), $\Gamma_{A_1} \approx 380$ (MeV)) [57] are in contradiction with the current experimental results [43c]. Another problem is the great number of free parameters [58] used to take into account rescattering corrections, Deck effects, resonances, coupled channels in a unique amplitude; but it is well possible that this is the price to pay to take into account all these components at the same time. On the other hand, the parameters obtained for $Q_1 (\rightarrow K\rho)$ and $Q_2 (\rightarrow K^*\pi)$ [55a] are compatible with the experimental masses and width. Concerning the coupled channel resonances in the first case, (A_1), the two threshold ($K^*\bar{K}$ and $\rho\pi$) are very far away in comparison with the second case ($K\rho$ and $K^*\pi$). Perhaps this is responsible for the mass shift obtained for the A_1 meson. These approaches retain only two components of the Born term for the Deck amplitude corresponding to the t and u channels of the subreaction $a P \rightarrow l+2$. It is clear that if we take into account also the third Born term, the direct pole diagram - see Fig.(23) - we must be careful with the double counting Duality problem. In a way, the contribution coming from rescattering (see Fig.

(22b)) is justified in the context of the Landau singularities (triangle singularities^[59] in the present case) since the authors of ref. [59] show how the $J^P = 1^+$ mesons A_1 and Q_1 were an evidence for these singularities (the peaks corresponding to the A_1 and Q_1 taking place for $m_{3\pi} \approx 1.1$ (GeV) and $m_{K^*\pi} \approx 1.2$ (GeV) respectively). This point may suggest that the shift from 1.2 to 1.3 (GeV) for m_{A_1} is not due of rescattering term. More information about this interesting subject is in ref. [59] and there in.

3.5 DOUBLE REGGE APPROACH

As we said above in Section 3.3, the $2 \rightarrow 3$ reactions and particularly the Diffractive Dissociation reactions, interpreted via (D.H.D.) model can be considered in general via a Double Regge (D.R.) exchange model^[49,60] that present a great flexibility of applications. If we take the Regge trajectory $\alpha_2 = \alpha_{\mathbb{P}}$ (see Fig.(25)), we obtain a simple (D.H.D.) model. A great advantage of this Double Regge exchange mechanism is that it can be applied without difficulties to forward as well backward production^[61] with a small number of parameters. And as it was pointed out recently^[35], a Dual amplitude like Double Regge, corresponding to the diagram shown in Fig.(25),

$$A_5(S, S_1, S_2, t_1, t_2) = \xi_1(t_1) \xi_{21}(t_1, t_2) S^{\alpha_1(t_1)} S_2^{\alpha_2(t_2) - \alpha_1(t_1)} V_{12}(t_1, t_2) + \xi_2(t_2) \xi_{12}(t_1, t_2) S^{\alpha_2(t_2)} S_1^{\alpha_1(t_1) - \alpha_2(t_2)} V_{21}(t_1, t_2)$$

where

$$\begin{aligned}\alpha_i(t_i) &= \alpha_i^!(t_1 - m^2) \\ \xi_i(t_i) &= \tau_i + \exp[-i\pi\alpha_i(t_i)] \\ \xi_{ij}(t_i, t_j) &= \tau_i \tau_j + \exp\{-i\pi[\alpha_i(t_i) - \alpha_j(t_j)]\} \\ V_{ij} &= V_0 / \{\alpha_i(t_i) [\alpha_j(t_j) - \alpha_i(t_i)]\} \\ & \quad i = 1, 2\end{aligned}$$

contains implicitly resonances and background. Due to the general properties [62] of this amplitude, we also have a good result in forward as well in backward production, in spite of the arbitrariness in the parametrization. We know that this was the case with many problems connected with the A_1 , Q and other $J^P = 1^+$ mesons. This model however, is not able to account for the strong suppression of the A_1^0 in charge exchange reactions. For the cross-sections of this special case A_1^0 , we give in Table (IV) a list of theoretical predictions according to the model used. A strong variation is observed with the mass attributed to the A_1 , and with the approach taken.

3.6 NEW INTERACTIONS AND RESULTS

Interactions like [65] lepton-hadron, $\ell + N \rightarrow \ell + V(A) + N$ where ℓ =lepton, N =nucleon and $V(A)$ is a vector (axial-vector) particle, are used for the observation of vector and axial mesons. More specifically for the diffractive physical region these reactions have been studied in the sense of A_1 productions. The study of these lepton-hadron reactions is very important for the subsequent information which can be obtained for

charged and neutral currents and is of particular relevance for our purpose of clarifying the problems connected with $J^P = 1^+$ mesons. These reactions have also a possible Deck like background see Fig.(26) - which contributes. Figure (26) shows the two possible components, resonance and Deck to interpret the results of these reactions. Here we think that it is particularly necessary work in the scheme III mentioned at the beginning of section 3.1. To investigate the system $A_1(\rightarrow \rho\pi)$ in the semi-leptonic decay mode $\tau \rightarrow A_1 \nu_\tau \rightarrow (\rho\pi) \nu_\tau$ different techniques have been employed (some of which well knowns [66]), in an attempt to clarify the problems of purely hadronic reactions. The matrix element [66b] for this decay can be written (Fig.(27)),

$$M = \ell_\mu H_\mu$$

where ℓ_μ and H_μ represent the semileptonic and hadronic vertex shown in Fig.(27) and have the following form,

$$\ell_\mu = \bar{u}_{\nu_\tau} \gamma_\mu (1-\gamma_5) u_\tau$$

and

$$H_\mu = \left[\epsilon_\mu^{(\rho)} - Q_\mu \frac{\epsilon^{(\rho)} \cdot p^{(\pi)}}{M^2} \right] F_1(M^2) + \\ + \epsilon^{(\rho)} \cdot p^{(\pi)} \left[\Delta_\mu - Q_\mu \frac{m_\rho^2 - m_\pi^2}{M^2} \right] F_2(M^2)$$

where

$$Q = p^{(\rho)} + p^{(\pi)}, \quad \Delta = p^{(\rho)} - p^{(\pi)}$$

and $M^2 = Q^2$. F_1 and F_2 are the form factors used in many theoretical speculations [66b,d]. We can see in Fig.(28,29) that the approach of ref. [66b] applied to these reactions is compatible with those of Diffractive Dissociation. Some solutions [66b,c] are compared with the data.

Another type of reactions that begin now to produce interesting results in the context of the problems we are discussing are those with polarized target. Recently [68] measurements of a 3π system diffractively produced in the reaction $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$ have been made at 17 (GeV/c) confirming the results obtained previously in other experiments. The axial-vector A_1 meson is observed in this reaction with a mass $m_{A_1} = 1.2 - 1.3$ (GeV) and a width $\Gamma_{A_1} \approx 300$ (MeV). The interested reader will find more information in ref. [68]. Here we just call attention to these "new" reactions and hope that new results will emerge from pp annihilation at intermediary energy.

4. CONCLUSIONS

We summarize now the main points discussed above. We have shown that the study of $J^P = 1^+$ objects is interesting both from the experimental as well as from the theoretical point of view since they have been a good "theoretical laboratory" for the development of many issues of hadron spectroscopy. Many experiments that were realized to search for these mesons have yielded valuable information about hadron interactions in general. Many problems were solved in the last ten years and the improvement of the experiments with the increase of statistics (number of events) and accuracy of the techniques used is quite evident.

From a strictly phenomenological point of view, some of these $J^P = 1^+$ states are now well established as good resonances (B(1235), D(1285)); others can be considered as almost definitively established ($A_1(1.1)$; $Q_A(1.24-1.29)$; $Q_B(1.3-1.4)$; $D'\equiv E(1420)$; while H(1.1)) and the H' rest today without a clear determination. From the experimental point of view it still remains necessary to make compatible the several existing results. For example for a set of experiments the A_1 mass is ≈ 1.1 (GeV) and for another 1.2-1.3 (GeV). The results obtained from charge exchange reactions concerning A_1^0 production gave good determination in ref. [10b] whereas no evidence was found in ref. [43]. Thus, we would need a good compatible set of experimental results which could be the principal tool to exclude some theoretical models. In this sense, the recent results about A_1^0 production in charge exchange reaction [10b] is to be considered an important experimental step. Next, we would need a deeper theoretical study of the several existing models to state all the problems, and compare the virtues and failures of the different approaches. A work that would explain clearly all these experimental and theoretical problems would surely be wellcome to give a final answer to all the contradictions pointed out in this paper.

Acknowledgments

We would like to acknowledge Dr. Enrico Predazzi for many fruitful discussions and for a critical reading of the manuscript.

REFERENCES

- [1] a) The complete definitions of these objects is found in Particle Data Group - Review of particles properties - Rev. of Mod. Phys. 52, n°2, part II - April (1980).
b) A recent experimental review is given in Ph. Gavillet - Thèse d'Etat - n° 2147 - Université Paris XI - Orsay (1979).
- [2] G. Fox and A.J.G. Hey - N. Ohys. 56B, 386 (1973).
- [3] The B meson was discovered by M. Abolins, R.L. Lauder, W.A.W. Mehlhop, N.H. Xuong and P.M. Yager Phys. Rev. Lett. 11, 381 (1963).
- [4] For spin-parity analysis see, G. Ascoli, H.B. Crawley, D.W. Montara and A. Shapiro, Phys. Rev. Lett. 20, 1411 (1968); S.U. Chung et al. Phys. Rev. D11, 2426 (1975); id. Phys. Lett. 47B, 526 (1973) and ref. [1].
- [5] For decay properties see V. Chaloupka, A Ferrando, M.J. Lorty and L. Montanet, Phys. Lett. 51B, 407 (1974) and ref. [1].
- [6] A set of cross-sections for $B \rightarrow \omega^0 \pi^-$ in different reactions and energies are given in table IX from S. Migashita, J. Von Krogh, J.B. Kopelman and L.M. Libby, Phys. Rev. D1, 771(1970).
- [7] N. Armenise et al. N. Cim. Lett. 8, 425 (1973).
- [8] S.U. Chung, O.I. Dahl, J. Kirz and D.H. Miller, Ohys. Rev. 165, 1491 (1968).
- [9] Ph. Gavillet et al. Phys. Lett. 78B, 158 (1978) and references therein.

- [10] a) A strong experimental evidence for the H meson decaying into $\rho\pi$ is found for π^+d reaction at 5.1 (Gev/c) with $m_H = 998 \pm 10$ (Mev) and $\Gamma_H \approx 45 \pm 30$ (Mev) in G. Benson, E. Marquit, B. Roe, D. Sinclair and J.V. Veld, Phys. Rev. Lett. 17, 1234 (1966). See also other comments about an interpretation of this reaction Phys. Rev. D2, 2110 (1970).
- b) For the more recent evidence of the axial-vector resonance H(I=0) in forward charge exchange reaction ($\pi^-p \rightarrow \pi^+\pi^-\pi^0n$) at 8.45 (Gev/c) see J.A. Dankowych et al., Phys. Rev. Lett. 46, 580 (1981). In the A_1^0 - section (2.6) we comment further about this object.
- [11] N. Armenise et al. Phys. Lett. 26B, 336 (1968).
- [12] Rosenfeld et al. - Data on Particles and Resonant States - Rev. of Mod. Phys. 39, 1 (1967).
- [13] See other arguments about this subject in ref. [1b].
- [14] S.Y. Fung, W. Jackson and R.T. Pu, Phys. Rev. Lett. 21, 47 (1968).
- [15] See discussion in ref. [2].
- [16] a) This object was discovered simultaneously in $p\bar{p}$ annihilation at 1.2 (Gev/c) and in π^-p reaction between 1.5 and 4.2 (Gev/c), Ch. D'Andlan et al., Phys. Lett 17, 367 (1965); id., N.Phys. B5, 693 (1968); D.H. Miller et al. Phys. Rev. Lett. 14, 1074 (1965), O.I. Dahl et al. Phys. Rev. 163 1377 (1967). See also a particular discussion in ref. [1b].
- b) For a recent phase-shift analysis see N.R. Stanton et al. Phys. Rev. Lett. 42, 346 (1979).

- [17] C. Dionisi et al. N. Phys. B169, 1 (1980).
- [18] A. Gurtu et al. N. Phys. B151, 181 (1979).
- [19] M.J. Corden et al. N. Phys. B144, 253 (1978).
- [20] H. Grassler et al. N. Phys. B121, 189 (1977).
- [21] R. Nacash et al. N. Phys. B135, 203 (1978).
- [22] a) J. Mandula, J. Weyers and G. Zweig, Ann. Rev. Nucl. Sci. 20, 289 (1970);
b) C.E. Carlson, J.J. Cogne, P.M. Fishbane, F. Gross and S. Meshkov, Phys. Lett. 98B, 110 (1981);
c) K. Ishikawa, Phys. Rev. Lett. 46, 978 (1981);
d) M. Chanowitz, Phys. Rev. Lett. 46, 981 (1981).
- [23] C.G. Wohl et al., N. Phys. B132, 401 (1978).
- [24] a) G. Otter et al. N. Phys. B106, 77 (1976);
b) A. Firestone et al., Phys. Rev. D5, 505 (1972);
c) Ph. Gavillet et al., Phys. Lett. 76B, 517 (1978);
d) A. Firestone, N. Phys. B47, 348 (1972);
e) For other energies see ref. [1a] and H.H. Bingham et al. N. Phys. B48, 589 (1972).
- [25] R.K. Carnegie et al. N. Phys. B127, 509 (1977) see also other results in G.W. Brandenburg et al. Phys. Rev. Lett. 36, 703 (1976); S.N. Tovey et al. N. Phys. B95, 109 (1975).
- [26] J.S.M. Vergeest et al. N. Phys. B158, 265 (1979).
- [27] R. Barloutaud et al. N. Phys. B59, 374 (1973).
- [28] G. Otter et al. N. Phys. B96, 29 (1975).
- [29] G.W. Brandenburg et al. N. Phys. B45, 397 (1972).

- [30] The axial-vector A_1 meson was discovered by G. Bellini , N. Cim. 29, 896 (1963) in reaction initiated by π at 6.1 and 18.1 (Gev/c).
- [31] G. Ascoli et al. Phys. Rev. Lett. 26, 929 (1971).
- [32] G. Ascoli et al. Phys. Rev. D9, 1963 (1974).
- [33] The first observation of the A_1 in backward productions is due at E.W. Anderson et al., Phys. Rev.Lett. 22, 1390 (1969).
- [34] A. Ferrer et al., N. Phys. B142, 77 (1978); id., L.A.L. - Orsay preprint 77/10 (1977).
- [35] Calculated from a Double-Regge-Model with an arbitrary global normalization, similarly to what done in J.Anjos, A. Endler, A. Santoro and F.R.A. Simão, N. Cim. 57A, 70 (1980). In the present case (πN backward production of A_1) the result can be improved with a more convenient choice of the set of parameters.
- [36] Ph. Gavillet et al., Phys. Lett. 69B, 119 (1977) and ref. [1b]. Similar results are obtained for A_1 coherently produced on nuclei by J. Pernegr et al., N. Phys. B134, 436 (1978).
- [37] J.B. Dainton and A.J.G. Hey - "Three Particle Phase shift Analysis and Meson Resonance Production: "Proceedings of the Daresbury Study Weekend series n98 (1975).
- [38] Yu. M. Antipov et al. N. Phys. B63, 153 (1973).
- [39] C. Damn et al. Phys. Lett. 89B, 281 (1980) These authors claim an A_1 resonance with a relative phase changing going through $\approx 90^\circ$ at $m_{A_1} \approx 1280$ (MeV) with $\Gamma_{A_1} \approx 300$ (MeV).

- [40] P.R. Graves-Morris, Ann. of Phys. 41, 477 (1967); A.M. Badalyam, M.I. Polikarpov, Yu.A. Simonov, ITEP-102 Moscow (1980).
- [41] Similar conclusions are given by the authors of ref. [35] above. We are convinced that a resonant plus a Deck back ground like contribution should be fully capable of explaining these effects. The difficulty is in the evaluation of each component separately.
- [42] a) R.E. Juhala et al. Phys. Rev. Lett. 19, 1355 (1967);
b) J.C. Berlinghieri et al. Phys. Rev. Lett., 23, 42 (1969).
- [43] a) M.J. Emms et al., Phys. Lett. 60B, 109 (1975);
b) M.J. Corden et al., N. Phys. B136, 77 (1978);
c) F. Wagner, M. Tabak and D.M. Chew, Phys. Lett. 58B, 201 (1975); C. Baltay, C.V. Cautis and M. Kalelkar. Phys. Rev. Lett. 39, 591 (1977) see in this paper many interesting critical comments about the A_1^0 production and models;
d) M. Cerrada et al., N. Phys. B126, 241 (1977).
- [44] a) G. Alexander et al. Phys. Lett. 73B, 99 (1978);
b) J.A. Jaros et al. Phys. Rev. Lett. 40, 1120 (1978).
- [45] J. Ballam et al. Phys. Rev. D4, 1946 (1971) and also ref. [1a].
- [46] a) D.V. Brockway - Ph.D. Thesis - C00-1195-197 - University of Illinois - USA - (1968);
b) G. Ascoli et al. Phys. Rev. Lett. 20, 1321 (1968);
c) id. Phys. Rev. Lett. 21, 1712 (1968);
d) id. Phys. Rev. Lett. 25, 962 (1970);
e) see ref. [38] above;
f) Yu.M. Antipov et al. N. Phys. B86, 381 (1975);

- g) G. Ascoli and H.W. Wyld, Phys. Rev. D12, 43 (1975) and references therein;
- h) B. Weinstein, G. Ascoli and L.M. Jones, Phys. Rev. D8, 2904 (1973);
- i) see also ref. [32] .
- [47] a) J.D. Jackson - N. Cim. 34, 1644 (1964) and references therein;
- b) R. Balian, J. Bros, M. Froissart, A. Krzywicki , "Quest-ce qu'une Resonance?" - École D'Eté de Physique des Particules - Gif-sur-Yvette - France (1971);
- c) There is a great list of papers and books about resonance subject. The interested reader is referred for example to R.G. Newton "Scattering Theory of Waves and Particles"- page 505 - Mc Graw - Hill Inc. (1966) page 505 where a large literature is quoted;
- d) N.P. Chang - Phys. Rev. Lett. 14, 806 (1965).
- [48] a) S.D. Drell, Phys. Rev. Lett. 5, 342 (1960);
- b) S.D. Drell, K. Hiida, Phys. Rev. Lett. 7, 199 (1961);
- c) R.T. Deck, Phys. Rev. Lett. 13, 169 (1964).
- [49] E.L. Berger - Phys. Rev. 166, 1525 (1968); id., Phys. Rev. Lett. 21, 701 (1968).
- [50] A. Santoro - Thèse D'État - Université de Paris VII-France (1977) - "La Dualité dans les Dissociations Diffractives".
- [51] a) M. Ross and Y.Y. Yam - Phys. Rev. Lett. 19, 546 (1967);
- b) G. Cohen - Tannoudji, A. Santoro and M. Souza, N. Phys. B125, 445 (1977).
- [52] E.L. Berger, J.I. Donohue, Phys. Rev. D15, 790 (1977).

- [53] a) M.G. Bowler and M.A.V. Game - Oxford University 38/74 (1974);
- b) M.G. Bowler, M.A.V. Game, I.J.R. Aitchison and J.B. Dainton, N. Phys. B97, 227 (1975).
- c) I.J.R. Aitchison and M.G. Bowler - Oxford University preprint 24/77 (1977);
- d) For a short review about this subject see R. Aaron , R.S. Longacre and J.E. Sacco - Ann. of Phys. 117, 56 (1979).
- [54] a) K.M. Watson, Phys. Rev. 88, 1163 (1952);
- b) see also G. Fox - "Peripheral production of resonances"- Proceedings of the III^d Philadelphia Conference on Experimental Meson Spectroscopy pag. 271 (1972).
- [55] a) J.L. Basdevant and E.L. Berger - Phys. Rev. D19, 246 (1979);
- b) id. Phys. Rev. D16, 657 (1977);
- c) id. Phys. Rev. Lett. 37, 977 (1976);
- d) O. Babelon, J.L. Basdevant, D. Caillerie and G. Menessier-N. Phys. B113, 445 (1976).
- [56] A similar theoretical approach, via Fadeev-equations, is used by:
- a) G. Menessier, J.Y. Pasquier and R. Pasquier - Phys. Rev. D6, 1351 (1972);
- b) R.L. Schult and H.W. Wyld Jr., Phys. Rev. D16, 62, (1977).
- c) see ref. [46h] also.
- [57] R.S. Longacre and R. Aaron, Phys. Rev. Lett. 38, 1509 (1977).
- [58] E.L. Berger - ANL-HEP-CP-77-31- (1977) - "Production and decay systematics of the A_1 resonance".

- [59] a) M. Month - Phys. Lett. 18, 357 (1965);
 b) I.J.R. Aitchison - Phys. Rev. 133, B1257 (1964);
 c) L.D. Landau - N. Phys. 13, 181 (1959).
- [60] a) G. Cohen-Tannoudji, A.Santoro and M. Souza, N. Phys. B95, 445 (1975);
- [61] a) C. Chang Shih and B. Lin Young - Phys. Rev. D1, 2631(1970).
 b) A. Santoro, Rev. Bras. Fis. Special Vol. En. Int. abril (1979) - see also ref. [35] .
- [62] There are a great number of papers about Double Regge Model. See for a review, R.C. Brower, C.E. Detar and J.H. Weis , Phys. Reports 14C, 257 (1974). For another Dual approach for Diffractive Dissociation reactions see J.P. Ader and L. Clavelli - N. Phys. B139, 507 (1978).
- [63] The Three components (D.H.D.) model give a good mass-slope- $\cos\theta^{G.J.}$ - (see ref. [51b]) in comparison with data. For the Cross-Over of the Q see ref. [60]. Both papers can be considered as an application of a Double-Regge-Model in particular situations.
- [64] a) H.E. Haber and G.L. Kane, N. Phys. B129, 429 (1977);
 b) see also A.C. Irving and H.R. Sepangi, N. Phys. B139 , 327 (1978).
- [65] a) M.S. Chen, F.S. Henyey and G.L. Kane, N. Phys. B118, 345 (1977);
 b) A. Bartl, H. Fraas, W. Majerotto, Phys. Rev. D16, 2124 (1977); id. Phys. Rev. D19, 3222 (1979);
 c) D.A. Geffen and W.J. Wilson, Phys. Rev. D18, 2488 (1978);
 d) J. Bell et al. Phys. Rev. Lett. 40, 1226 (1978);

- [66] a) "The hard-pion approach" for example in A. Bartl and N. Paver, N. Cim. 55A, 475 (1980) and references therein.
- b) J.L. Basdevant and E.L. Berger - Phys. Rev. Lett. 40, 994 (1978). These authors use the same approach applied before to coupled channel resonances;
- c) see also ref. [65c] above.
- d) For general references: F. Gilman, Phys. Reports 4C, 95 (1972); C.H. Llewellyn Smith, Phys. Reports 3C, 261 (1972); C.A. Piketty and L. Stodolsky, N. Phys. B15, 571 (1970).
- [67] a) G. Alexander et al. Phys. Lett. 73B, 99 (1978);
- b) J.A. Jaros et al. SLAC-PUB-2048 (1978);
- c) see ref. [66b];
- d) see ref. [44d].
- [68] V. Chaband et al. N. Phys. B178, 401 (1981).

Tables Captions

Table I

All states $J^P = 1^+$ are classified according to SU(3) (see ref. [2]). J^P is the spin-parity and I is the isospin quantum numbers.

Table II

Decay, cross-sections and Branching-Ratios for some reactions initiated by π , K and N, for the D(1285) and E(1420) production.

Table III

Slopes for several mass ($K\pi\pi$) intervals at different energies.

Table IV

Several approaches for the A_1^0 cross-sections and their different results. (see ref. [64a]).

| $I \backslash J^{PC}$ | 1^{+-} "B-Nonet" | 1^{++} "A ₁ -Nonet" |
|---------------------------------|--------------------------|---|
| 1 | B(1235) | A ₁ (1.07) |
| Strange 1/2 | Q _B (1.3-1.4) | Q _A (\equiv C) (1.24-1.29) |
| 0 (Singlet/octet) Mixing? | H | D(1285) |
| | H' | D'(\equiv E)(1422) |

Table I

| Reactions | Decay Modes | Cross-Section [μb] | Branching-Ratios |
|--|---|---|--|
| (Ref. [18]) $K^- p \rightarrow \Lambda D$ $4.2 [\text{Gev}/c]$ $1.265 \leq M_{\eta\pi\pi} \leq 1.32$ | $\eta\pi\pi$ $K\bar{K}\pi$ $2\pi^+2\pi^-$ 4π | 3.7 ± 1.0 5.5 ± 1.5 2.3 ± 0.5 1.2 ± 0.7 3.6 ± 2.1 | $D \rightarrow \frac{K\bar{K}\pi}{\eta\pi\pi} = 0.42 \pm 0.15$ $D \rightarrow \frac{4\pi}{\eta\pi\pi} = 0.70 \pm 0.50$ $D \rightarrow \frac{\delta^\pm \pi^\mp \rightarrow \eta\pi^+\pi^-}{\eta\pi^+\pi^-} = 0.72 \pm 0.15$ |
| (Ref. [19]) $\pi^- p \rightarrow \begin{cases} \eta\pi^+\pi^- n \\ K^+ K^- \pi^0 n \end{cases}$ $12 \text{ and } 15 [\text{Gev}/c]$ $1.2 \leq M_{\eta\pi\pi} \leq 1.36$ For other results from $\pi^\pm p$ see ref. [20] | $K\bar{K}\pi$ $\eta\pi\pi$ | | $D \rightarrow \frac{\delta\pi \rightarrow \eta\pi^+\pi^-}{\eta\pi^+\pi^-} = 0.6^{+0.3}_{-0.2}$ $D \rightarrow \frac{K\bar{K}\pi}{\eta\pi\pi} = 0.5 \pm 0.2$ $D \rightarrow \frac{\delta\pi \rightarrow K\bar{K}\pi}{K\bar{K}\pi} = 1.0 \pm 0.3$ $E \rightarrow \frac{\eta\pi\pi}{K\bar{K}\pi} \leq 0.5$ $E \rightarrow \frac{\delta\pi \rightarrow \eta\pi\pi}{K\bar{K}\pi} \leq 0.3$ |
| (Ref. [21]) $p\bar{p} \rightarrow K^0 K^\pm \pi^\mp \pi^+ \pi^-$ $0.7 [\text{Gev}/c]$ For ten other final states see ref. [21]. | - | $\sim 100 \pm 12$ | - |

Table II

| $M(K\pi\pi) (\text{Gev})$ | $B(\bar{K}^0\pi^-\pi^0) (\text{Gev}^{-2})$ | $B(K^-\pi^+\pi^-) (\text{Gev}^{-2})$ |
|---------------------------|--|--------------------------------------|
| Ref. [28] 10,16(Gev/c) | | |
| 1.05 - 1.20 | 10.5 ± 1.0 | 12.8 ± 0.7 |
| 1.20 - 1.35 | 9.0 ± 0.8 | 9.0 ± 0.6 |
| 1.35 - 1.50 | 6.9 ± 0.6 | 7.6 ± 0.5 |
| 1.50 - 2.0 | 5.7 ± 0.6 | 6.4 ± 0.5 |
| Ref. [29] 4-12(Gev/c) | $B(K^*\pi^-) (\text{Gev}^{-2})$ | $B(K^*\pi^+) (\text{Gev}^{-2})$ |
| 1.0 - 1.2 | 8.7 ± 1.1 | 13.8 ± 1.4 |
| 1.2 - 1.3 | 6.6 ± 1.1 | 11.6 ± 1.4 |
| 1.3 - 1.4 | 5.5 ± 1.1 | 8.9 ± 1.1 |
| 1.4 - 1.5 | 3.3 ± 1.0 | 6.9 ± 1.1 |
| 1.5 - 1.75 | 2.9 ± 1.0 | 5.4 ± 1.0 |

Table III

| Approach | Plab. (Gev/c) | Reaction | σ_{Tot} (Theo.) [μb] | | | $\sigma_{\text{exp.}}$ |
|--|---------------|---|---|---------------------------|---------------------------|------------------------|
| | | | $M_{A_1}=1.1(\text{Gev})$ | $M_{A_1}=1.3(\text{Gev})$ | $M_{A_1}=1.5(\text{Gev})$ | |
| No D wave in $A_1 \rightarrow \rho\pi$ | 7. | $\pi^+ p \rightarrow A_1^0 \Delta^{++}$ | 1.7 | 0.8 | 0.4 | <2 [43c] |
| | 15 | $\pi^+ p \rightarrow A_1^0 \Delta^{++}$ | 0.7 | 0.3 | 0.2 | <0.5 [43c] |
| | 8.4 | $\pi^- p \rightarrow A_1^0 n$ | 2.0 | 0.9 | 0.6 | |
| Broken $SU(6)_W$ result | 7 | $\pi^+ p \rightarrow A_1^0 \Delta^{++}$ | 16.0 | 2.4 | 0.9 | |
| | 15. | $\pi^+ p \rightarrow A_1^0 \Delta^{++}$ | 6.0 | 1.1 | 0.5 | |
| | 8.4 | $\pi^- p \rightarrow A_1^0 n$ | 16.0 | 2.8 | 1.2 | |
| Current Algebra Result. | 7. | $\pi^+ p \rightarrow A_1^0 \Delta^{++}$ | 72.0 | 2.3 | 9.0 | |
| | 15. | $\pi^+ p \rightarrow A_1^0 \Delta^{++}$ | 28.0 | 10.0 | 4.7 | |
| | 8.4 | $\pi^- p \rightarrow A_1^0 n$ | 69.0 | 26.0 | 12.0 | |

Table IV

Figures Captions

Fig.1

A general diffractive production where the Pomeron is exchanged in the vertex b_3 . Σ_i represents all possible processes
a) $\mathbb{P} \rightarrow 12$.

Fig.2

- a) Mass distribution for $\omega\pi$ from ref. [7]
- b) $\omega\pi^+$ mass distribution for K^-p backward reaction at 4.2 GeV/c from ref [9]

Fig.3

Momentum transfer distribution in the B^- mass region from reference [7]

Fig.4

- a) Histogram of $\eta\pi^+\pi^-$ mass from ref. [16b]
- b) Results of the phase-shift analysis from ref. [16b].

Fig.5

$K^\pm K^0 \pi^\mp$ mass distribution of the reaction $\pi^-p \rightarrow k^0 k^\pm \pi^\mp n$ from ref. [17].

Fig.6

- a,b) $K\pi\pi$ mass distribution for the combined data at 10, 14 and 16 [GeV/c] for the reactions $k^-p \rightarrow k^- \pi^+ \pi^- p$ (a) and $k^-p \rightarrow \bar{k}^0 \pi^- \pi^0 p$ (b) from ref. [24a]

- c) $M(K\pi\pi)$ for all $K^+d \rightarrow K^+\pi^+\pi^-d$ events from ref. [24b]
- d) $(K\pi\pi)^+$ effective mass spectra for the sum of $K^-p \rightarrow \Xi^-K^+\pi^+\pi^-$, $K^-p \rightarrow \Xi^-K^0\pi^+\pi^0$ and $K^-p \rightarrow \Xi^-\pi^+$ + neutrals, from ref. [24c]. For other results on backward reactions see also ref. [24d]

Fig.7

Cross-Sections of $\{J^P_{\mu\nu\lambda}\}$ as function of the $(K\pi\pi)$ mass
See ref. [24a] for notations and other information.

Fig.8

- a) Comparison of the 1^+0^+ and $1^+1^+_{\rho}K$ cross sections and relative phases. See ref. [25]
- b) Comparison of the 1^+0^+ , 1^+1^+ and $2^+1^+K^*\pi$ cross sections and relative phases.

Fig.9

The dN/dt distributions for $K^-p \rightarrow K^-\pi^+\pi^-p$, $K^-p \rightarrow \bar{K}^0\pi^-\pi^0 p$ and $K^-p \rightarrow \bar{K}^0\pi^+\pi^-n$, and for $1.0 \leq M(K\pi\pi) \leq 1.5$ Gev.

Fig.10

Slope-mass correlations for the indicated reactions, from ref. [27]

Fig.11

3π mass distribution of the reaction $\pi^-p \rightarrow (\pi^-\pi^+\pi^-)p$ at 11. and 25 Gev/c. [32]

Fig.12

$(\rho^0\pi^-)$ mass spectrum for (9+12 GeV/c) data. [34] Events with $m(p_f\pi^-) < 1.8$ GeV rejected; only events with $u'_{pf} < 0.5$ GeV² has been retained. The solid curve results from fits explained in ref. [34] where the mass and width of A_1 are $m_{A_1} = 1050 \pm 11$ MeV and $\Gamma_{A_1} = 195 \pm 32$ MeV. The dashed curve is obtained from a Double-Regge model [35]

Fig.13

(3π) mass spectrum for $K^-p \rightarrow \Sigma^-\pi^+\pi^-\pi^+$ at 4.15 GeV/c [36]

Fig.14

(3π) mass spectrum for $K^-p \rightarrow \Sigma^-\pi^+\pi^-\pi^+$ at 4.15 (GeV/c) from partial wave analysis of ref. [36].

Fig.15

(3π) mass spectrum of different partial waves and interference phases in A_1 region for reaction $\pi^-p \rightarrow \pi^-\pi^-\pi^+p$ at 25 (GeV/c) and 40 (GeV/c) combined. [38]

Fig.16

Results obtained for a "Resonant A_1 plus rescattered Deck" fits in l^+S intensity and phase with respect to 2^+D1^+ , A_2 phase subtracted. (a), (b), (c), and (d) indicate different t' intervals; from ref. [39].

Fig.17

The three-pion $(\pi^+\pi^-\pi^0)$ mass spectra for A_1^0 production in KN reactions. (a) events in $K^-p \rightarrow K^-p\pi^+\pi^-\pi^0$ at 4.6 and 5.0

(Gev/c) from ref. [42a] . (b) Events in $K^+p \rightarrow K^+p\pi^+\pi^-\pi^0$ at 12.7 (Gev/c) from ref. [42b].

Fig.18

(3 π) mass spectra for $1^+(\rho\pi)S$ partial wave at 15.(Gev/c).^[43c]

(a) Reaction $\pi^+p \rightarrow p\pi^+\pi^+\pi^-$ where the A_1^+ is seen and fitted (solid curve) by a Breit-Wigner formula with $m = 1.152 \pm 0.009$ (Gev), $\Gamma = 0.264 \pm 0.011$ (Gev) and $\sigma = 129.8 \pm 7.8$ (μ b).

(b) Reaction $\pi^+p \rightarrow \Delta^{++}\pi^+\pi^-\pi^0$ with a complete absence of a resonance structure.

(c) Partial wave analysis results from ref.[10b] for the H and (d) for the A_1 in $(11+\rho S1+)$ spectrum and others for different sets of quantum numbers.

e) Relative phase between different sets of quantum numbers representing A_1 versus H, A_1 versus exotic and H versus exotic respectively. The notation IJP(isobar) $LM\eta$ are given in references [10b].

Fig.19

(3 π) mass spectra from $e^+e^- \rightarrow \tau^+\tau^-$ where $\tau \rightarrow (\rho\pi)\nu$. (a)

$M(\pi^+\pi^+\pi^-)$ distribution for events consistent with $e^+e^- \rightarrow \tau^+\tau^-$ reaction from ref. [44a].

(b) id. from ref. [44b]

Fig.20

The original (D-H-D) model and its interpretation with

the Pomeron exchange (P). $S = (p_\pi + p_n)^2$, $S_1 = (p_\pi + p_\rho)^2$,

$S_2 = (p_\pi + p_{n_f})^2$, $t_1 = (p_\pi - p_\rho)^2$ and $t_2 = (p_{n_i} - p_{n_f})^2$.

Fig.21

Comparison between a typical mass-spectra and a Chew-Low plot for (DHD) like model.

Fig.22

The $\pi p \rightarrow \rho\pi p$ reaction described by (a) a π - Deck exchange, (b) rescattering of $\rho\pi$ final states and (c) Direct resonance production.

Fig.23

The a-direct pole graph for $a+b \rightarrow l+2+b$ reaction.

Fig.24

Rescattering diagram representing a triangle singularity for a $\pi N \rightarrow (\pi\pi) \pi(K)N$ interactions from ref. [47d]

Fig.25

Double Regge Exchange diagram. α_1 and α_2 represent the trajectories exchanged. The kinematical variables S, S_1, S_2, t_1 and t_2 were defined before.

Fig.26

Possible resonant (a) and Deck (b) Diagrams for $\ell N \rightarrow \ell'(\rho\pi)N'$ reactions.

Fig.27

τ decay into $A_1 \nu_\tau \rightarrow (\rho\pi) \nu_\tau$

Fig.28

The (3π) mass spectrum. (a) The continuum and dashed curves are theoretically obtained from ref. [67c] in comparison with the data from ref. [67a] and (b) with the data from ref. [67b]

Fig.29

The $(\rho\pi^\pm)$ mass spectrum. a) The data from ref. [67a] and (b) from ref. [67d]. The theoretical curves are obtained from ref. [65c] in the Current Algebra context.

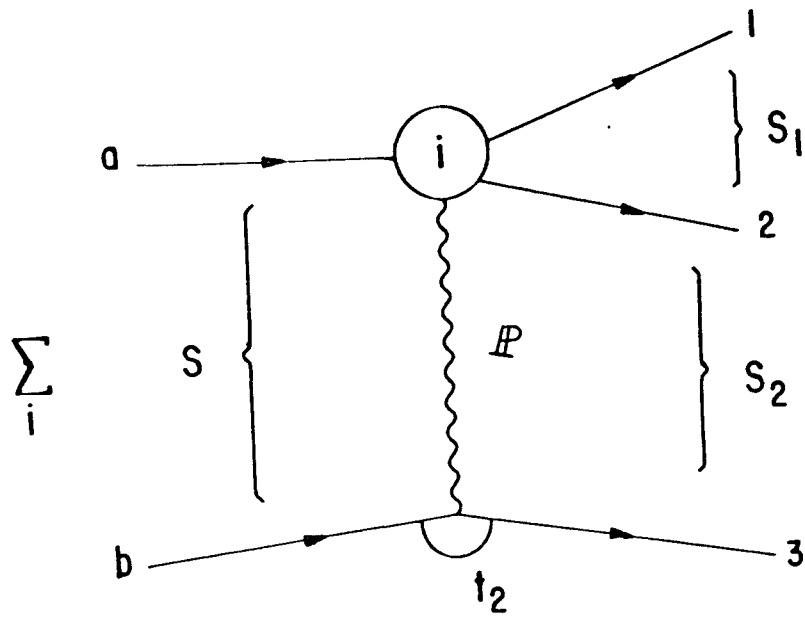


Fig. 1

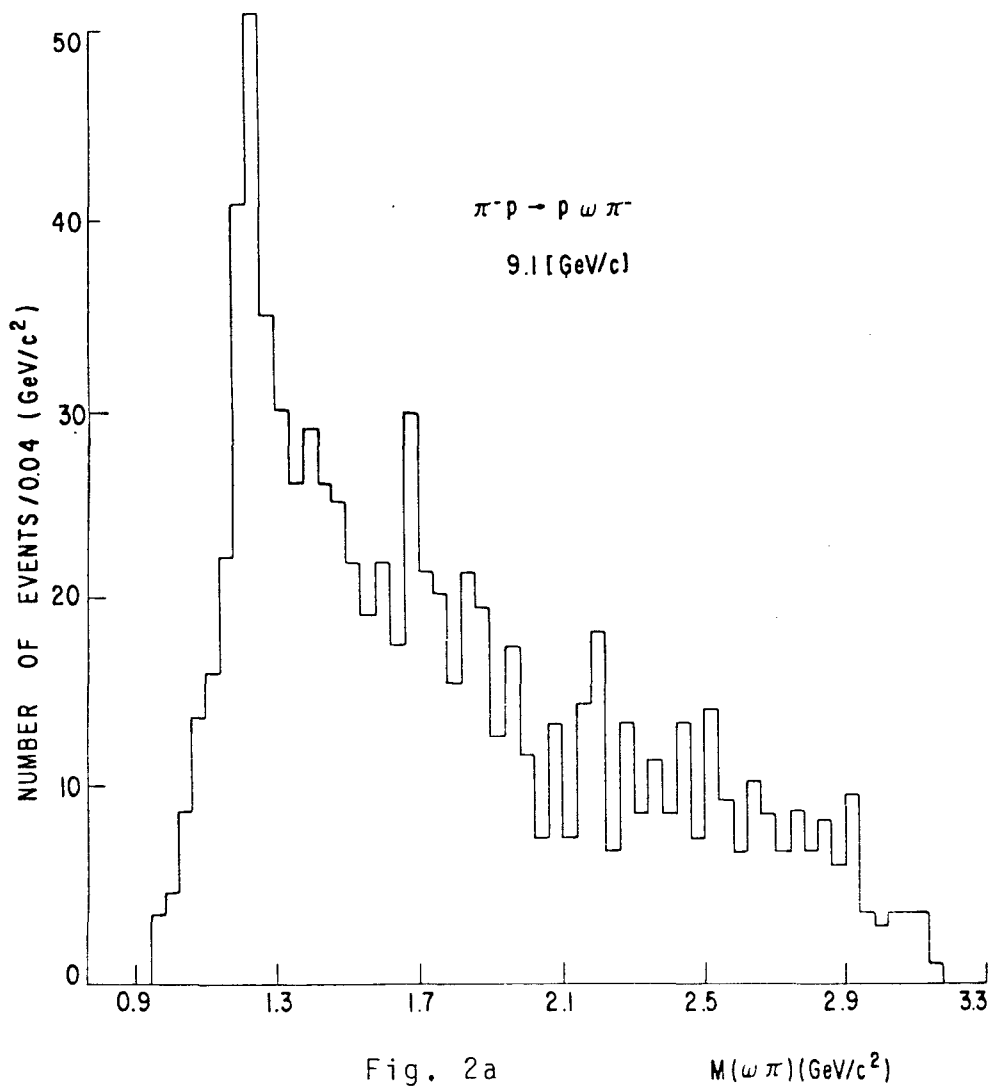


Fig. 2a

$M(\omega\pi)(\text{GeV}/c^2)$

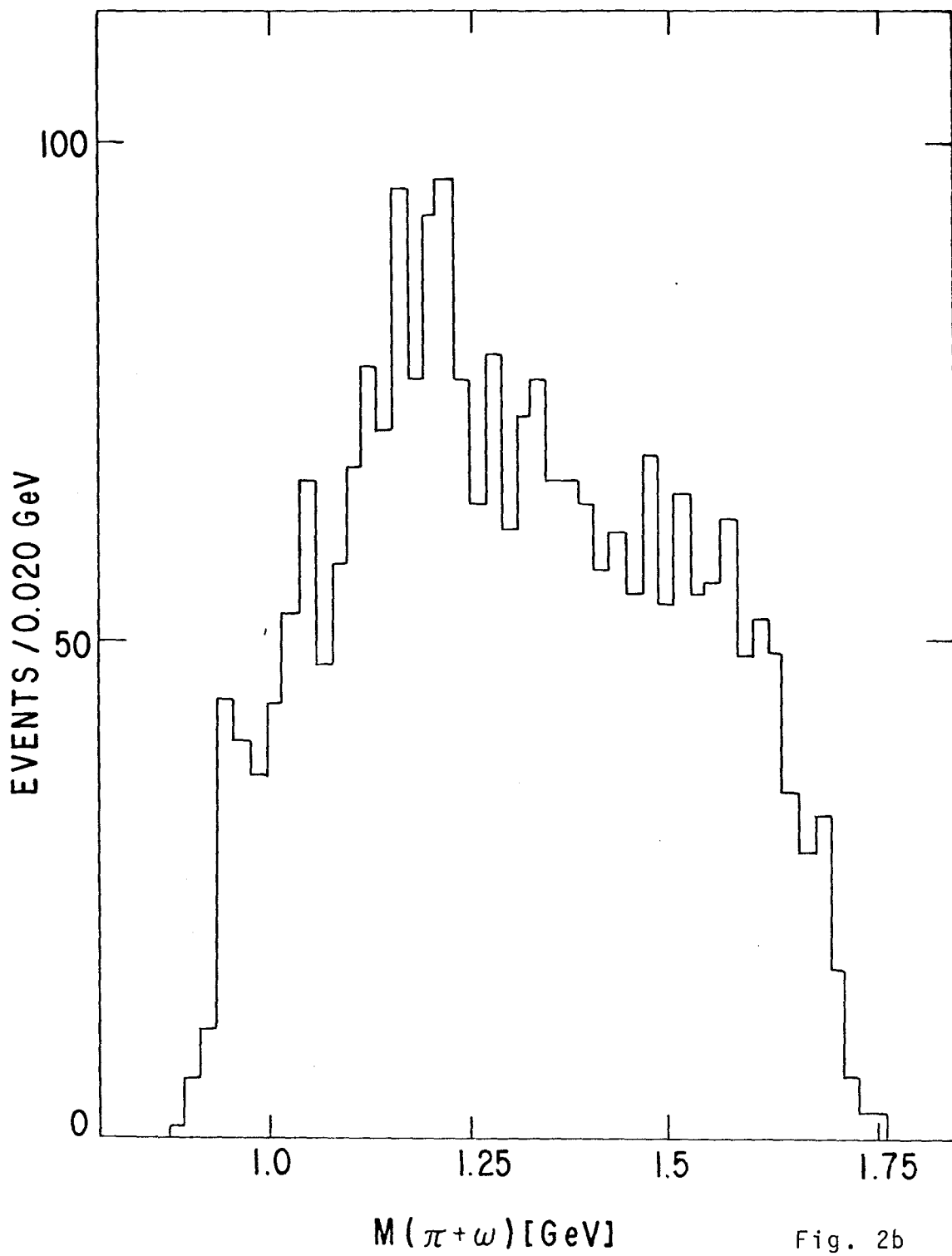


Fig. 2b

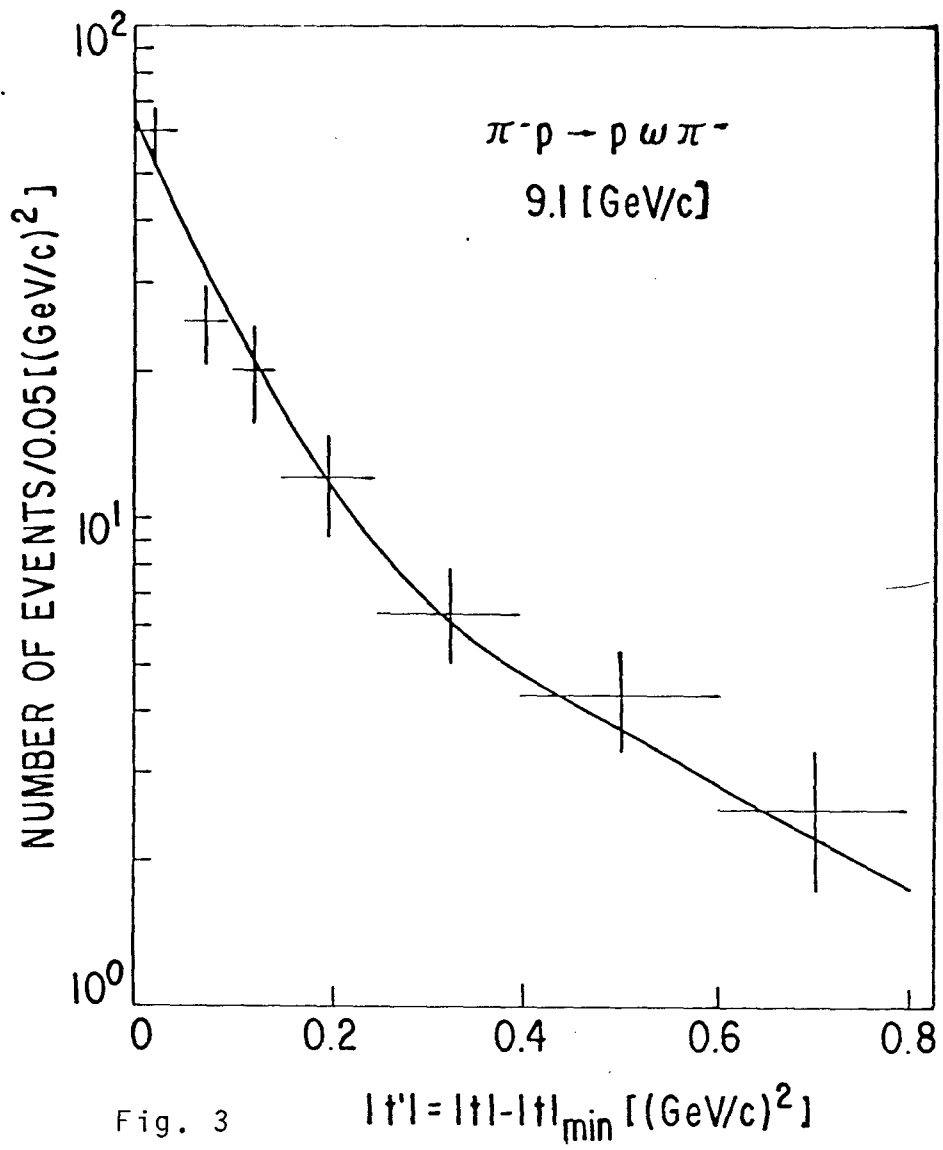


Fig. 3

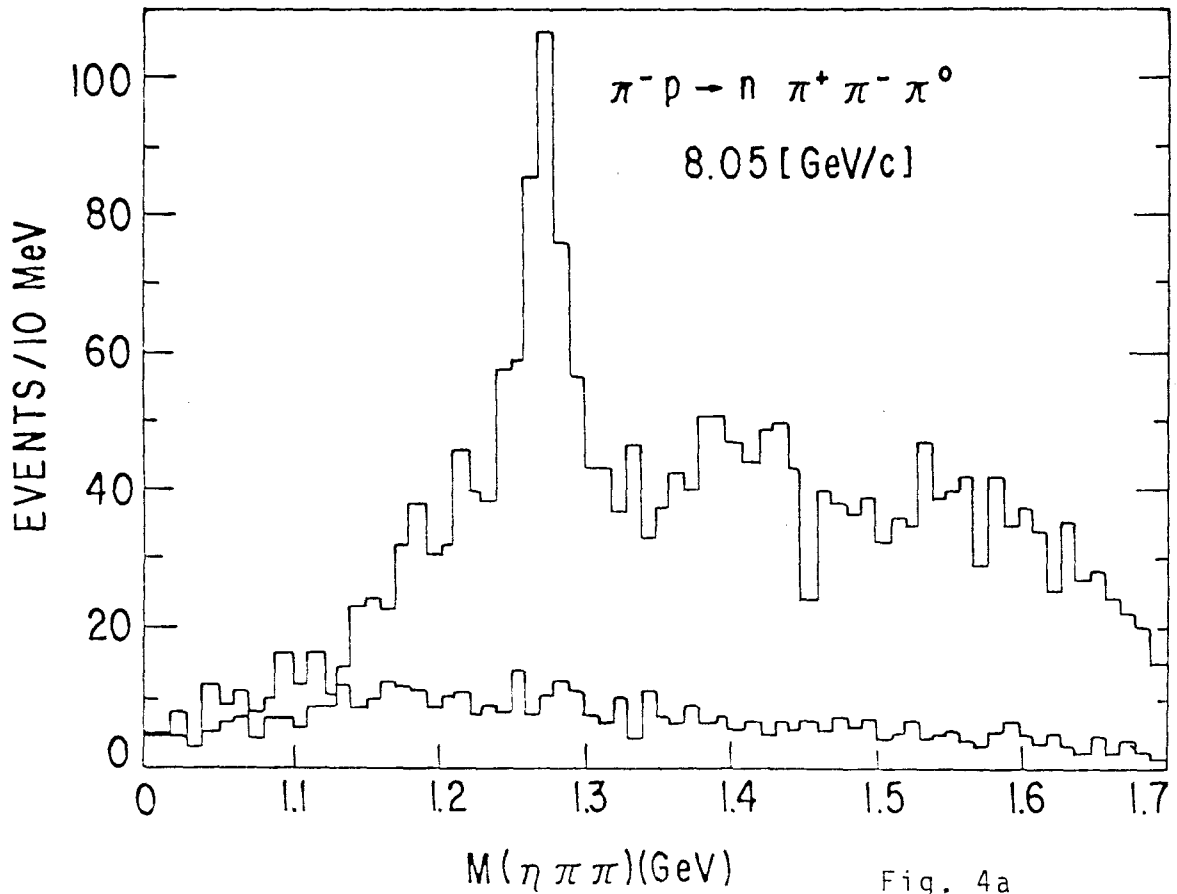


Fig. 4a

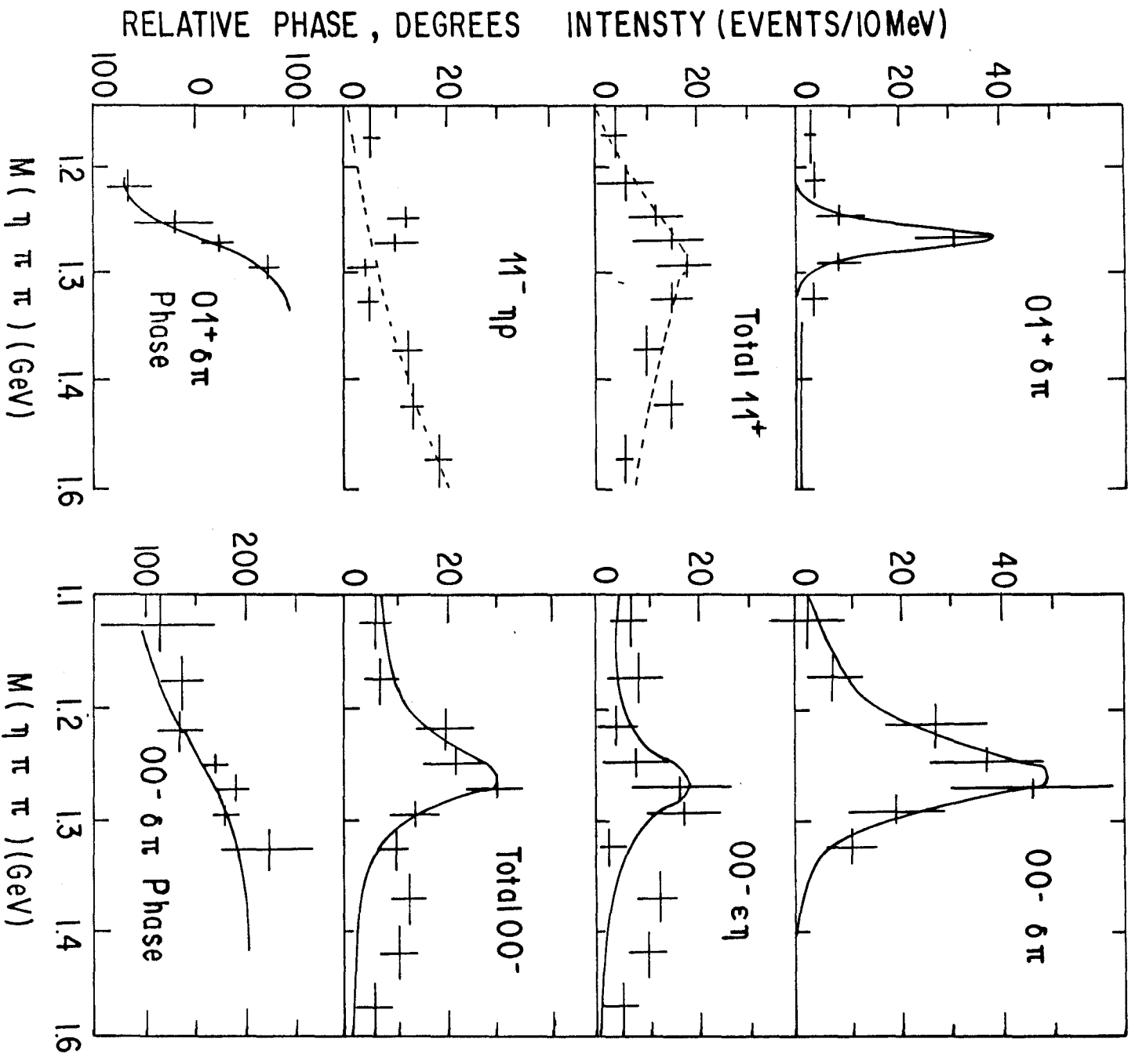


Fig. 4b

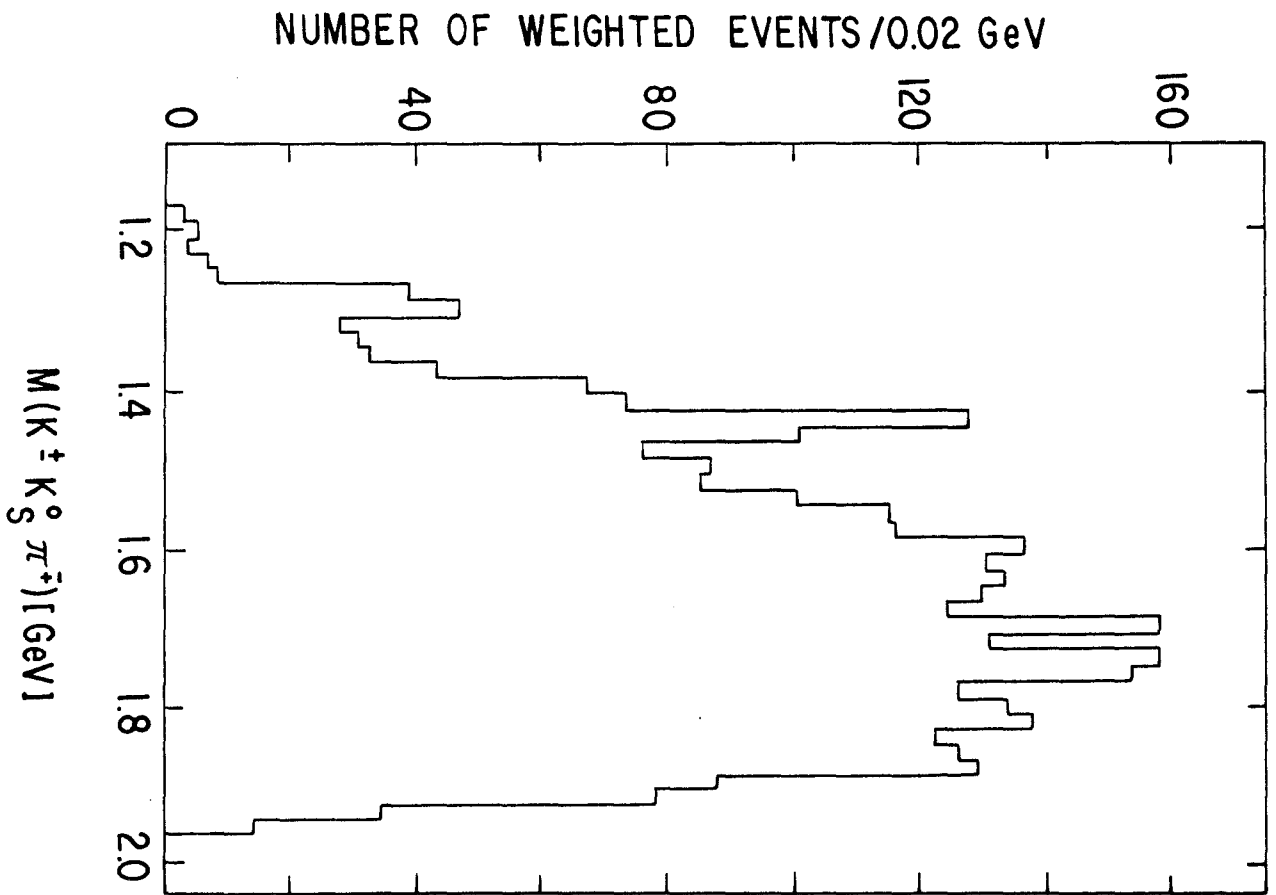


Fig. 5

$K^-p \rightarrow (K \Pi \Pi)^- p$ at 10, 14 and 16 GeV/c

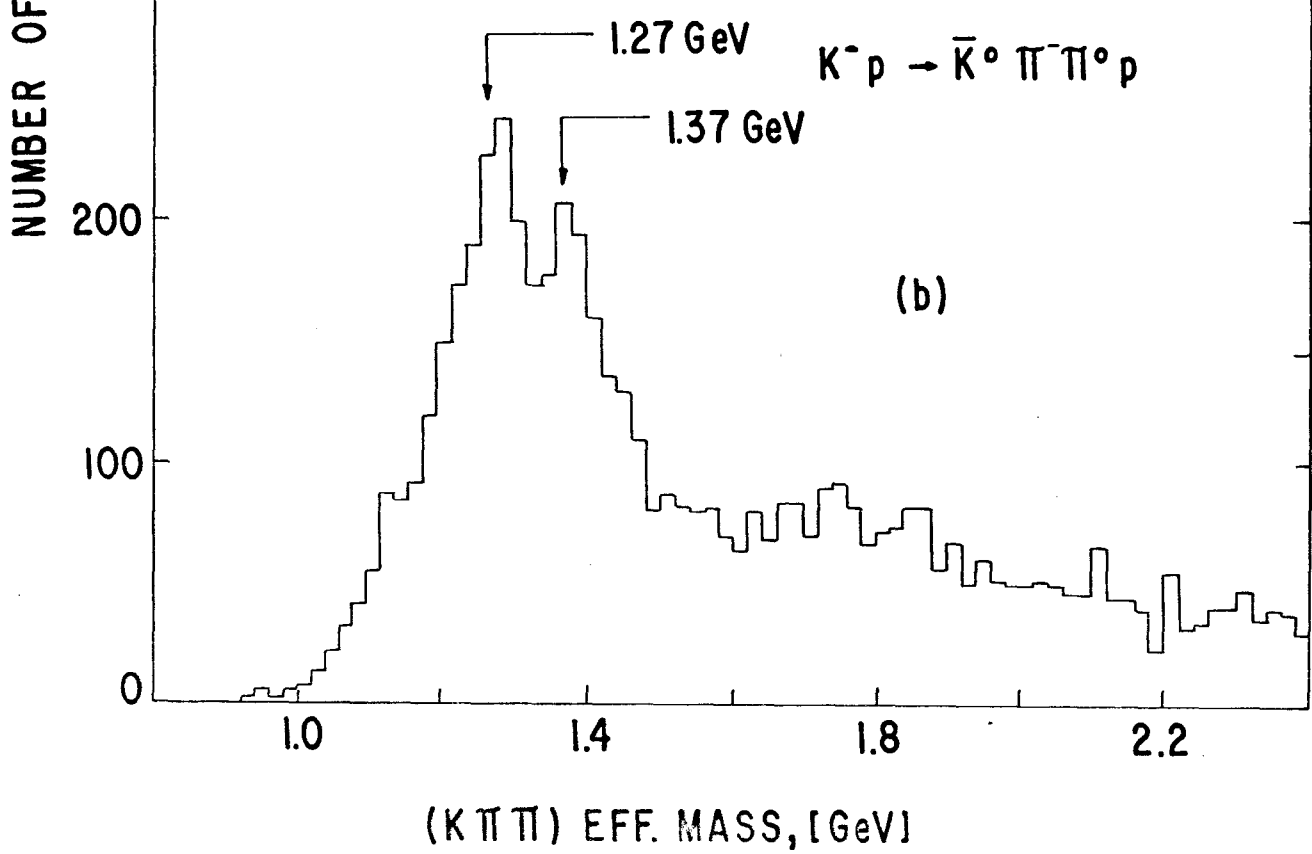
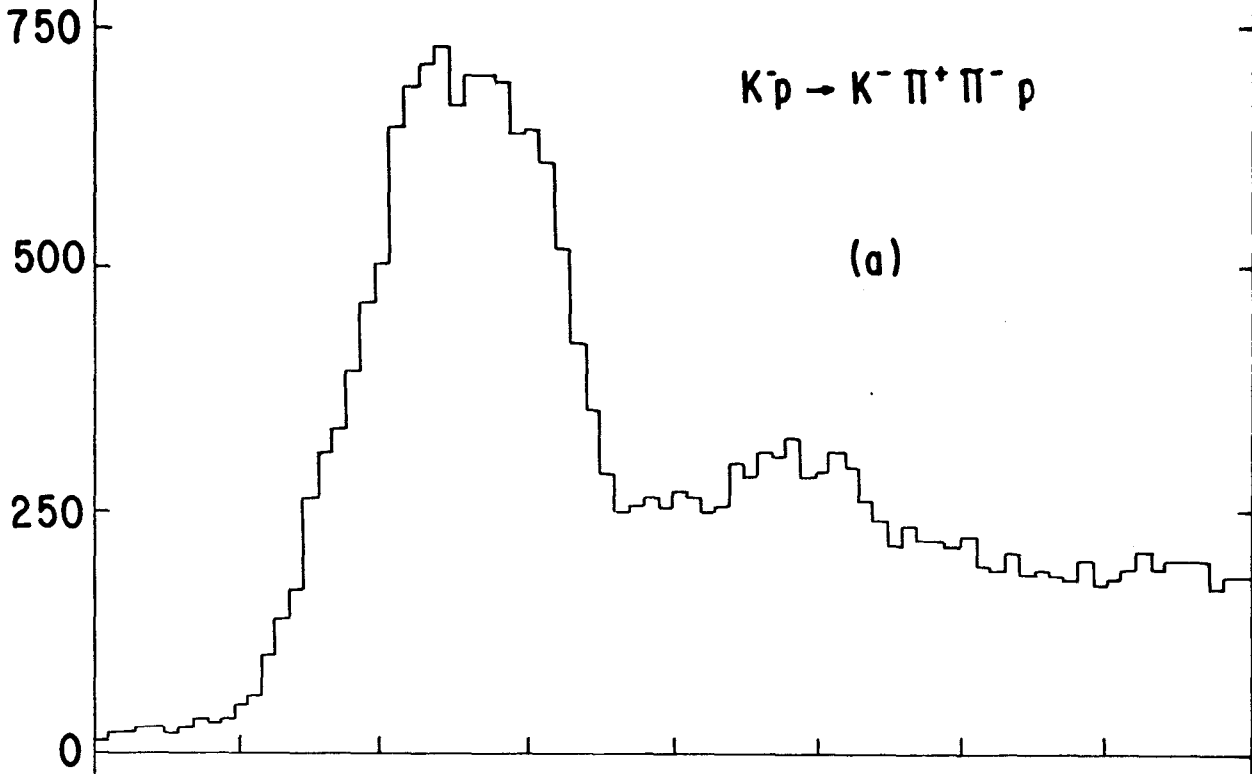


Fig. 6a e 6b

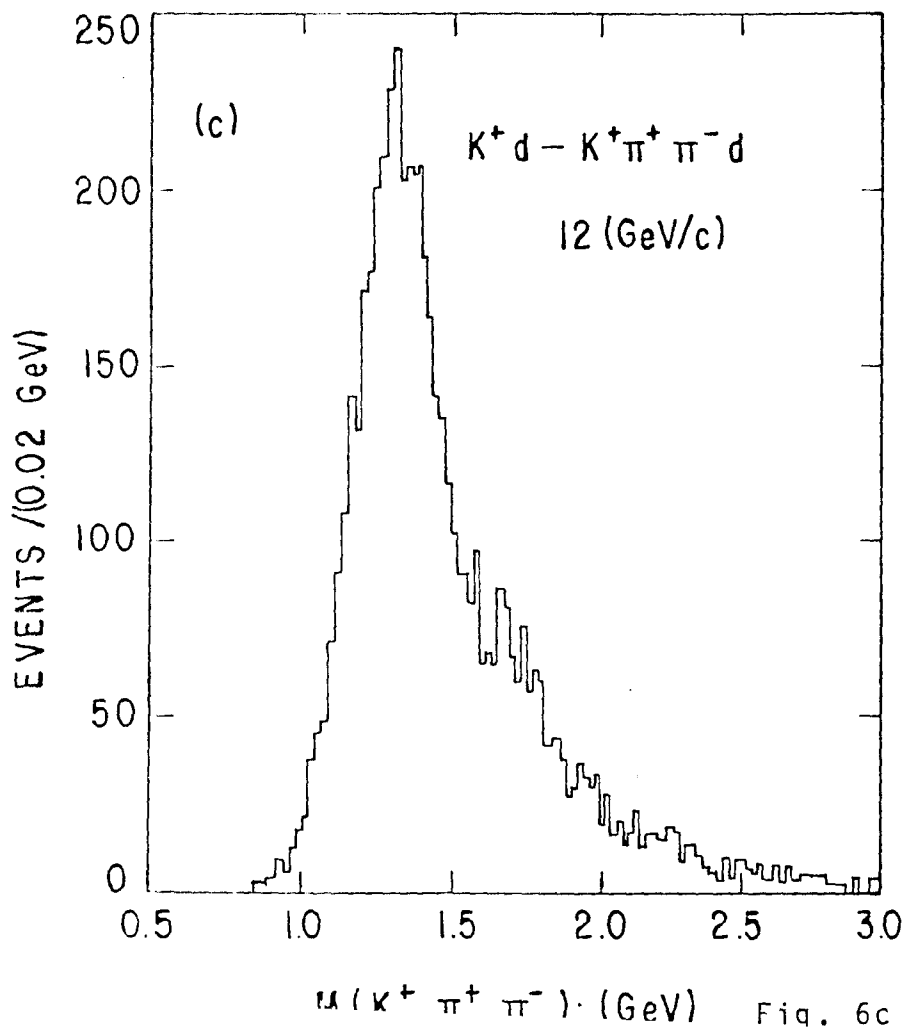


Fig. 6c

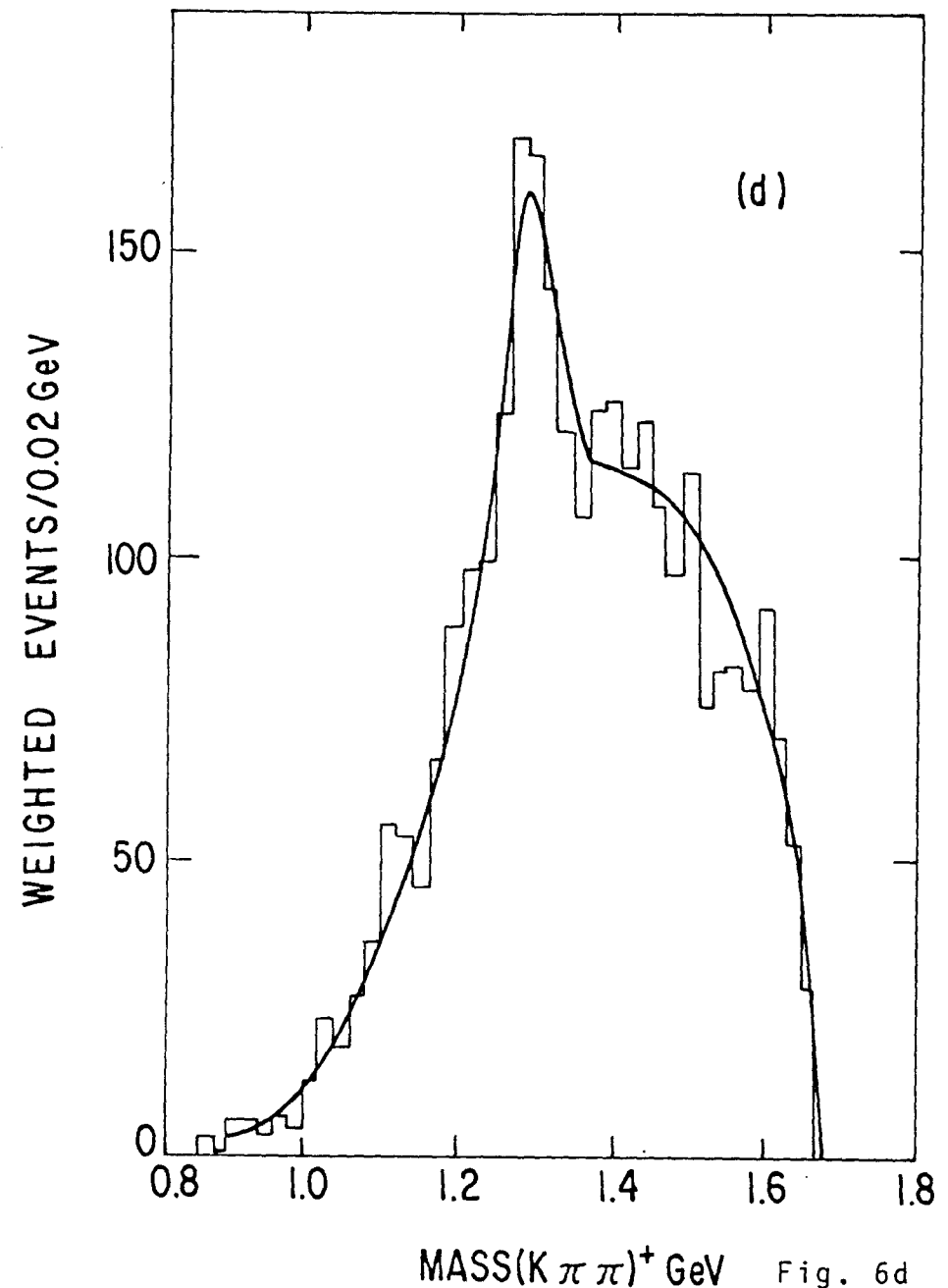


Fig. 6d

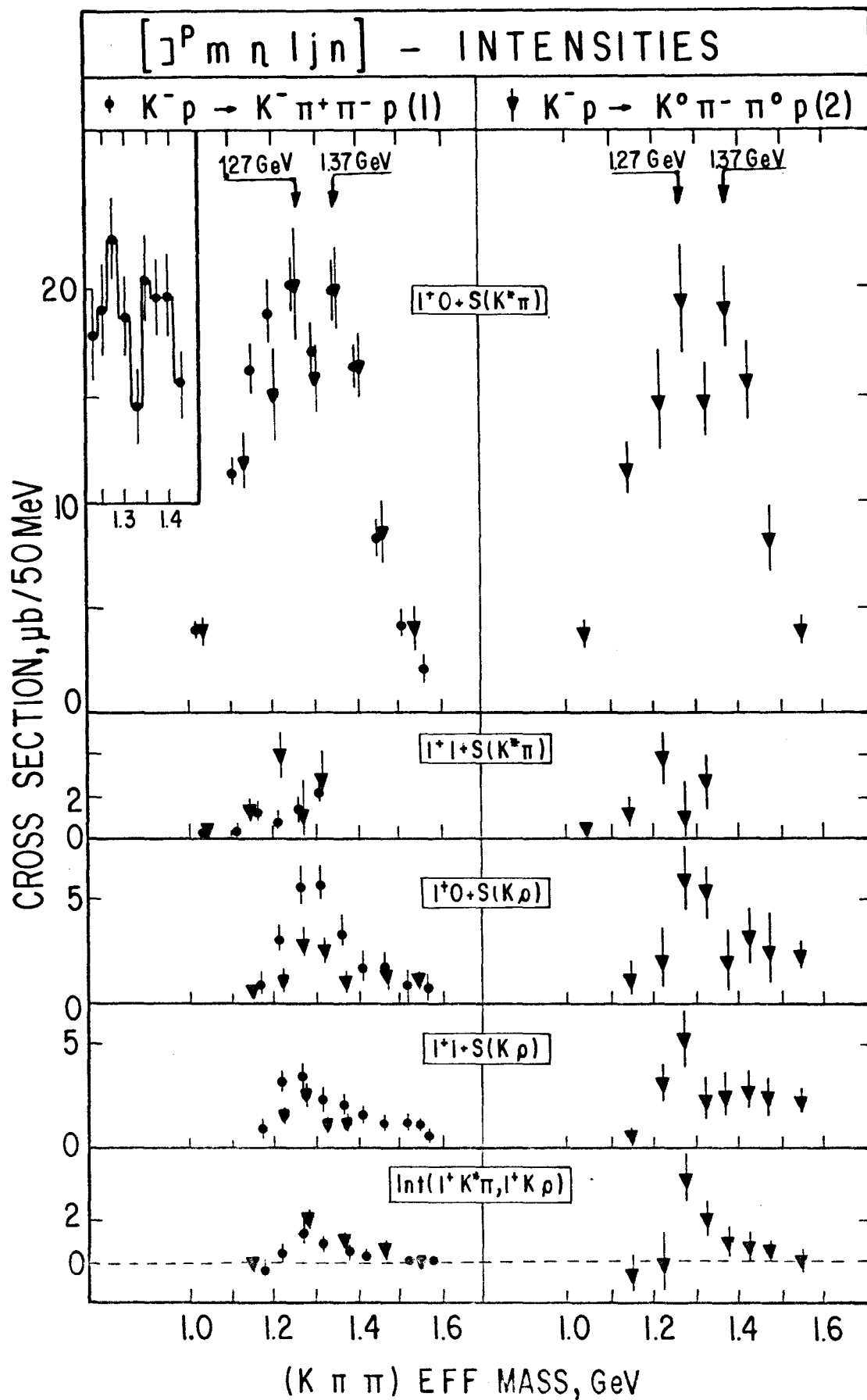


Fig. 7

$K^+ p \rightarrow K^+ \pi^+ \pi^- p$ 13 GeV
 $-t' < 0.3 \text{ GeV}^2$

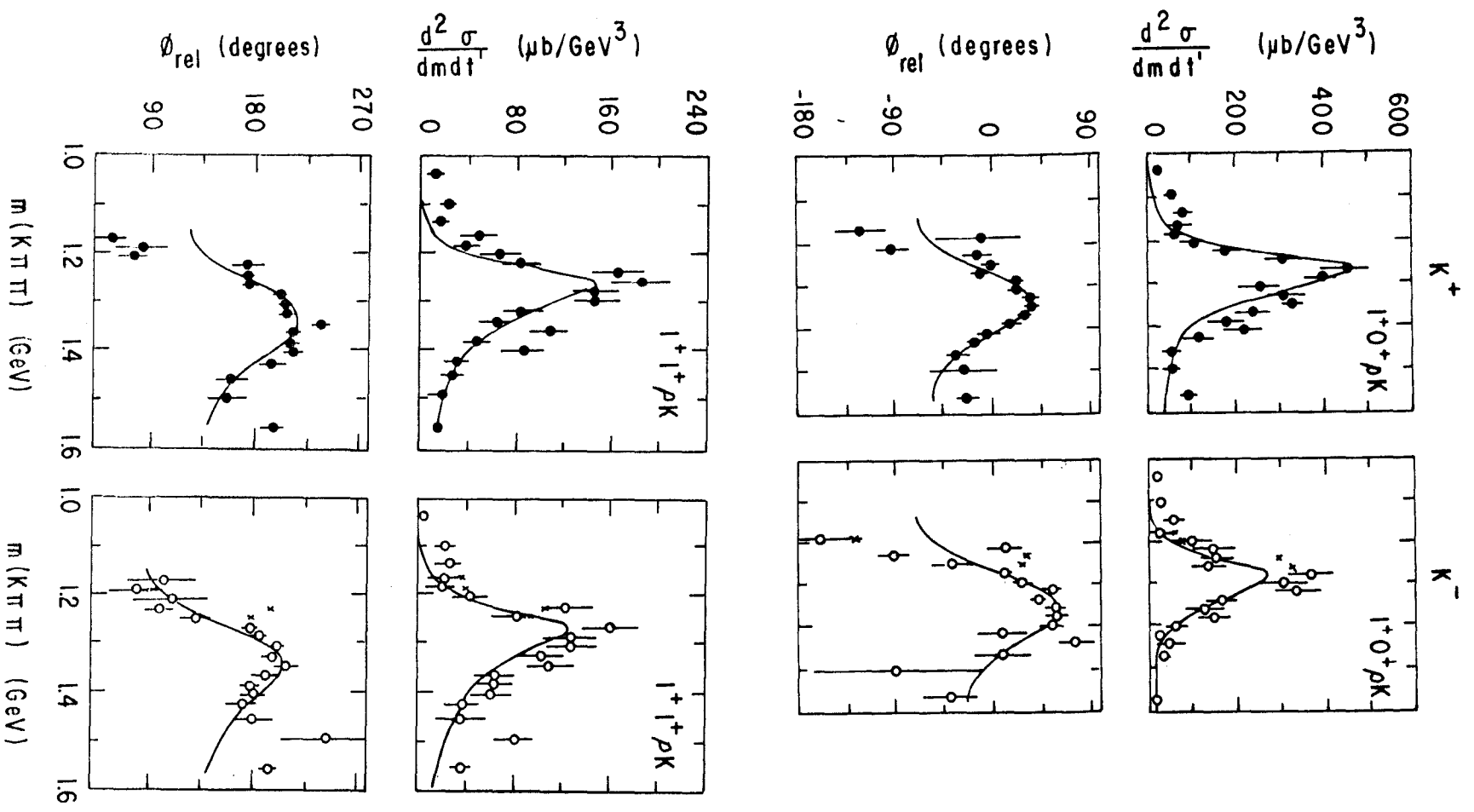


Fig. 8a

$K^+ p \rightarrow K^+ \pi^+ \pi^- p$ 13 GeV
 $-t' < 0.3 \text{ GeV}^2$

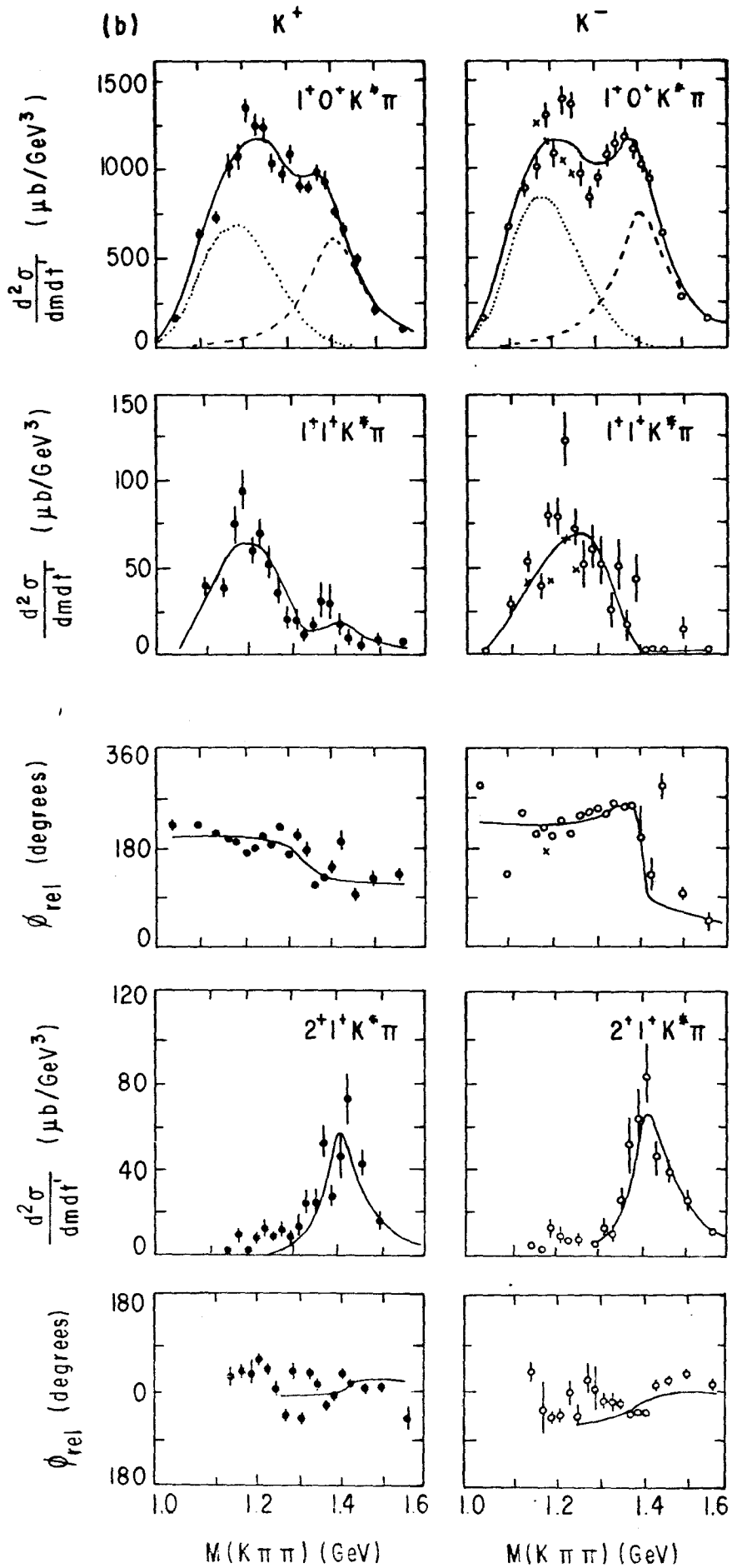


Fig. 8b

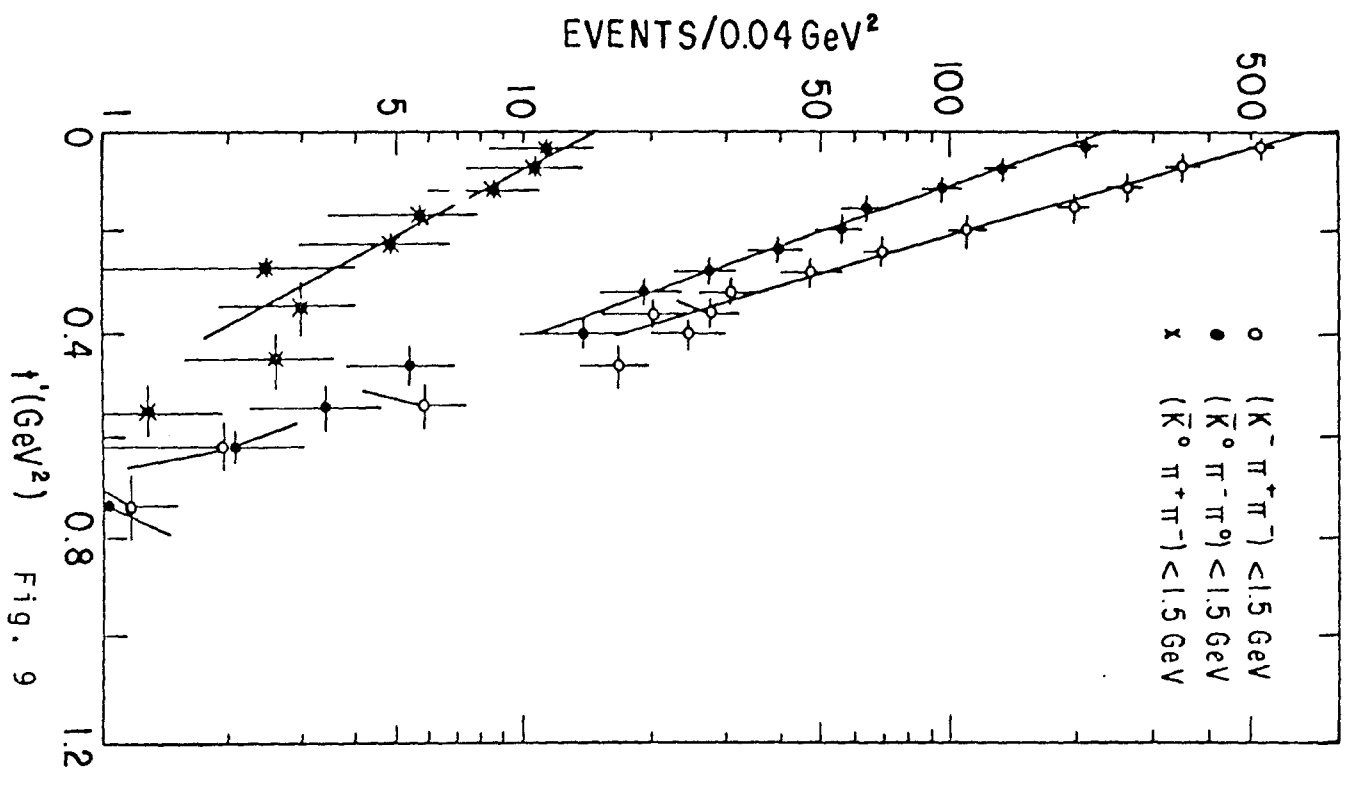


Fig. 9

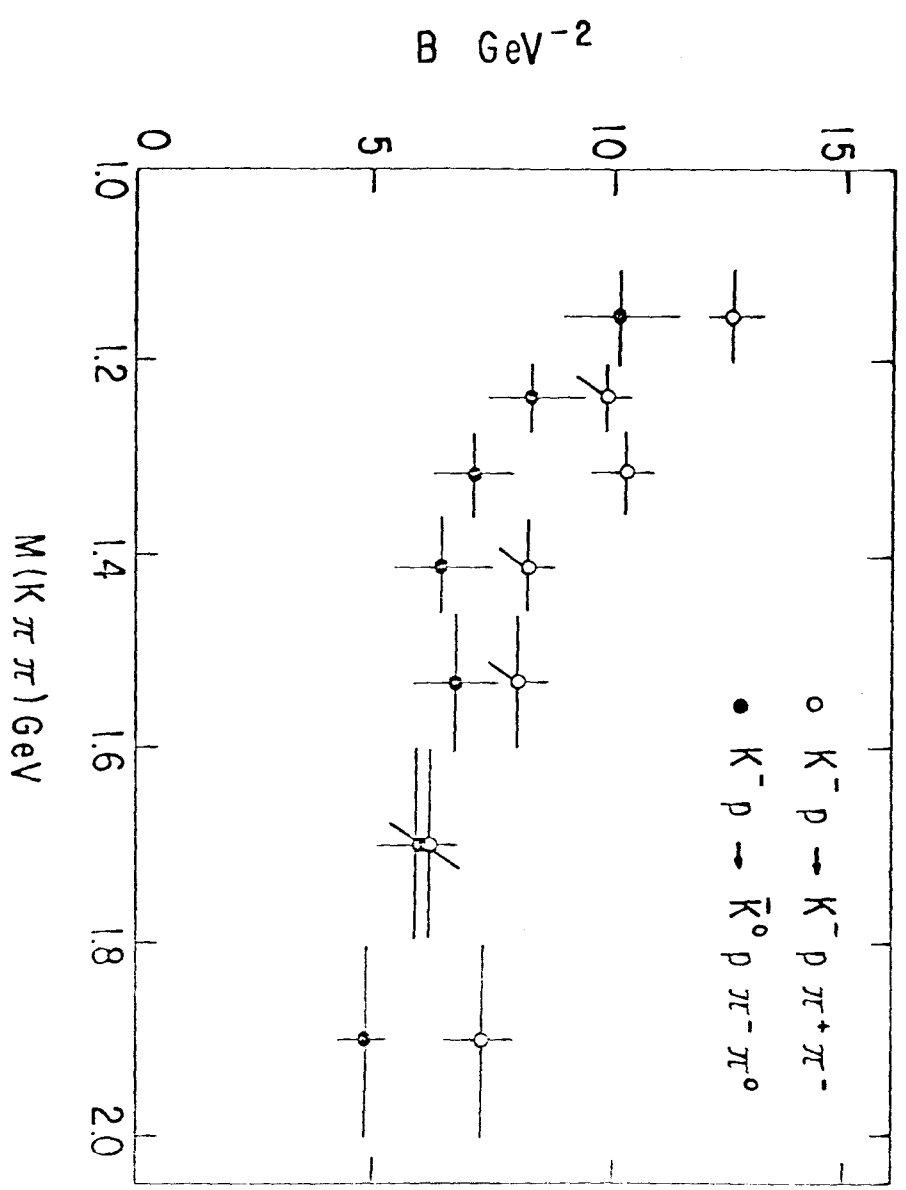


Fig. 10

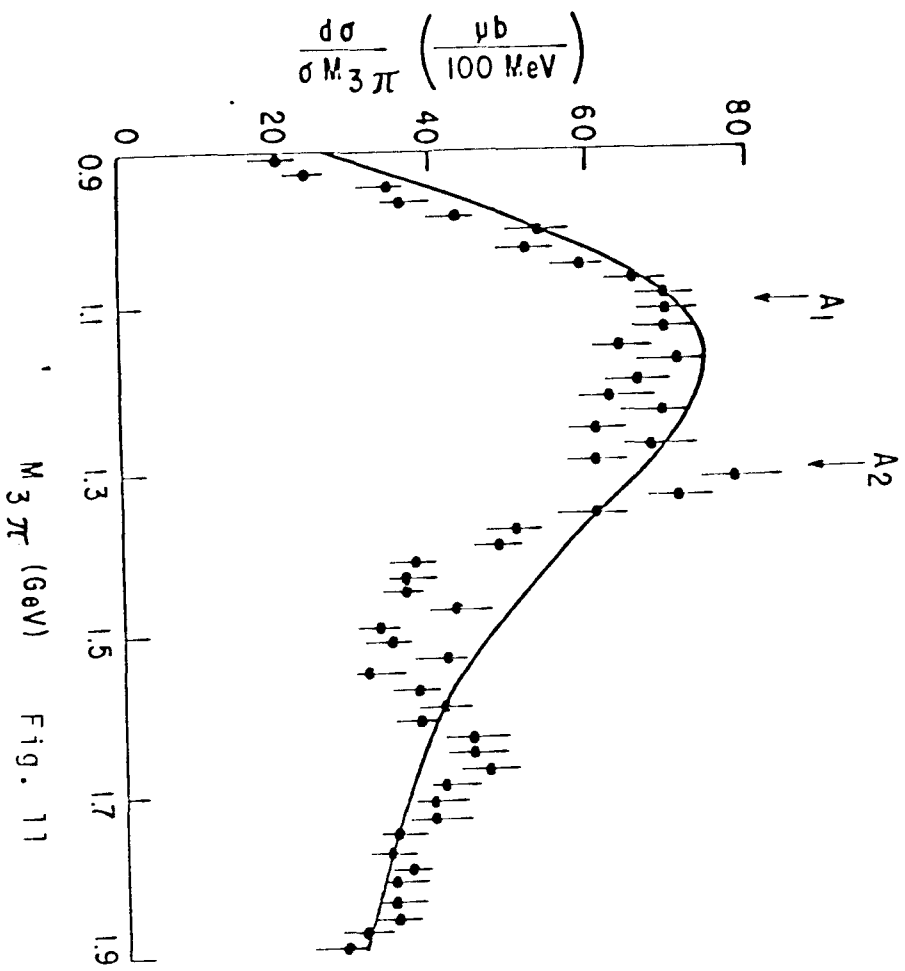


Fig. 11

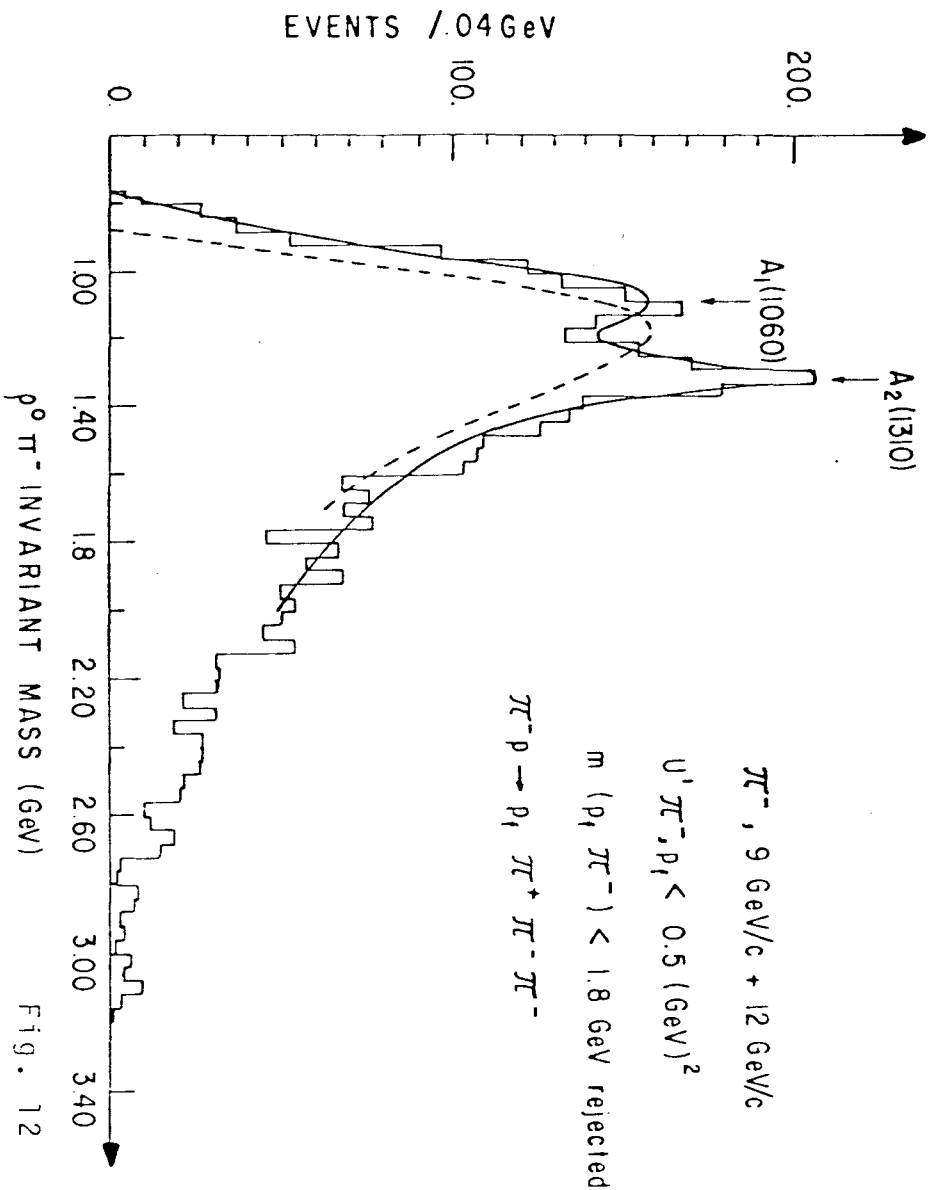


Fig. 12

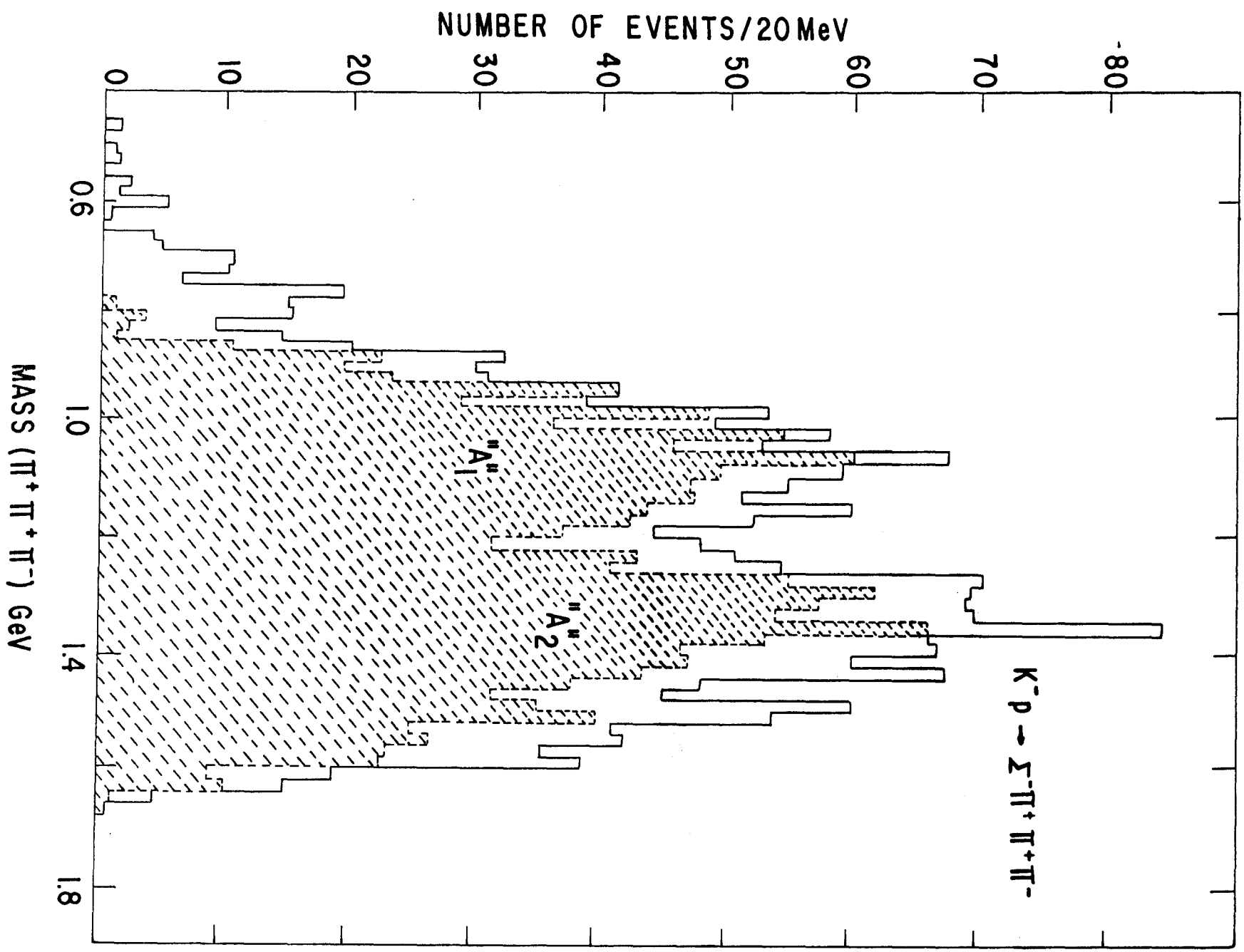


Fig. 13

NUMBER OF EVENTS

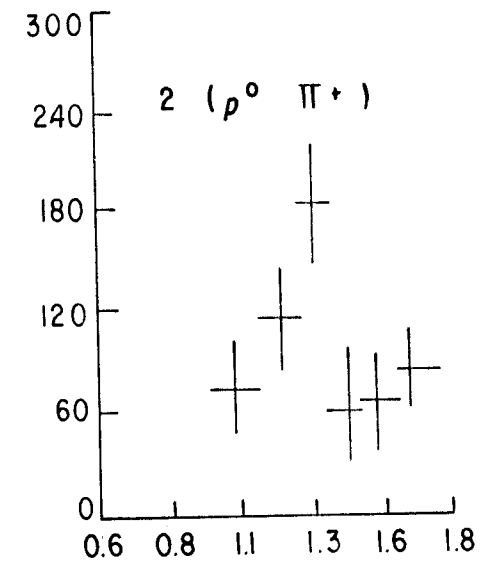
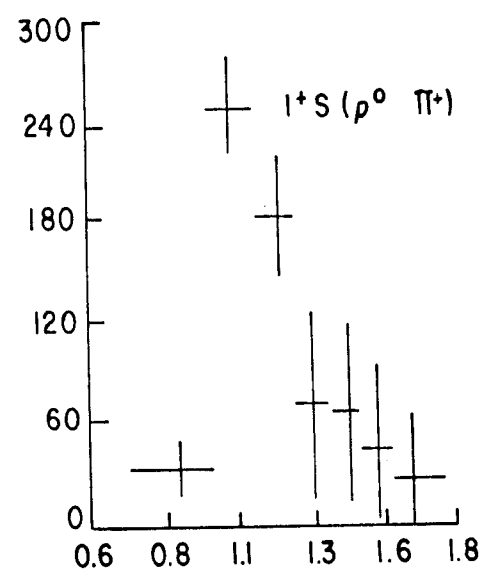
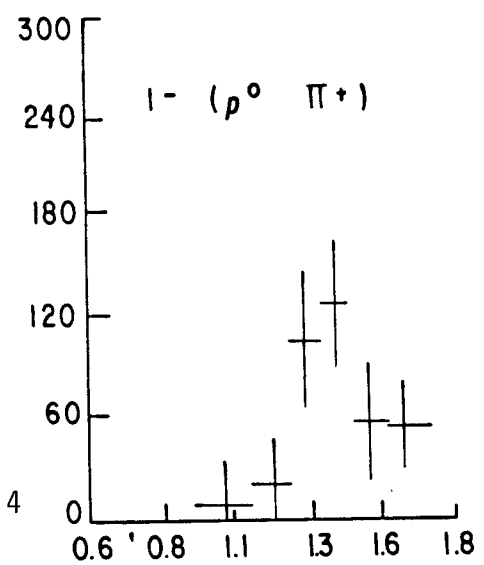
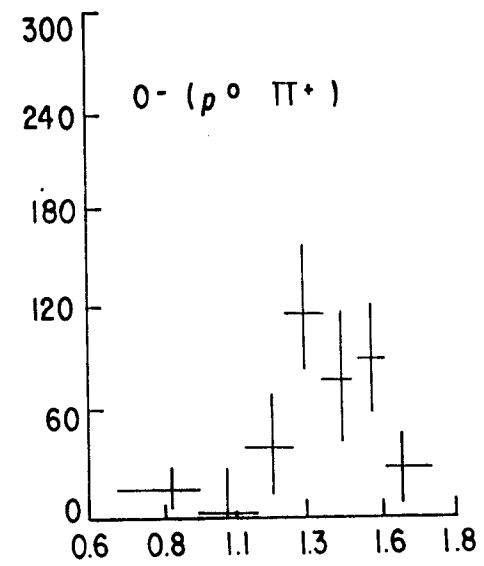
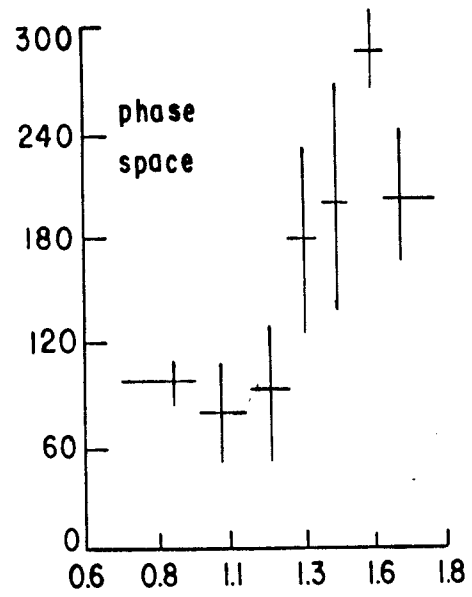


Fig. 14

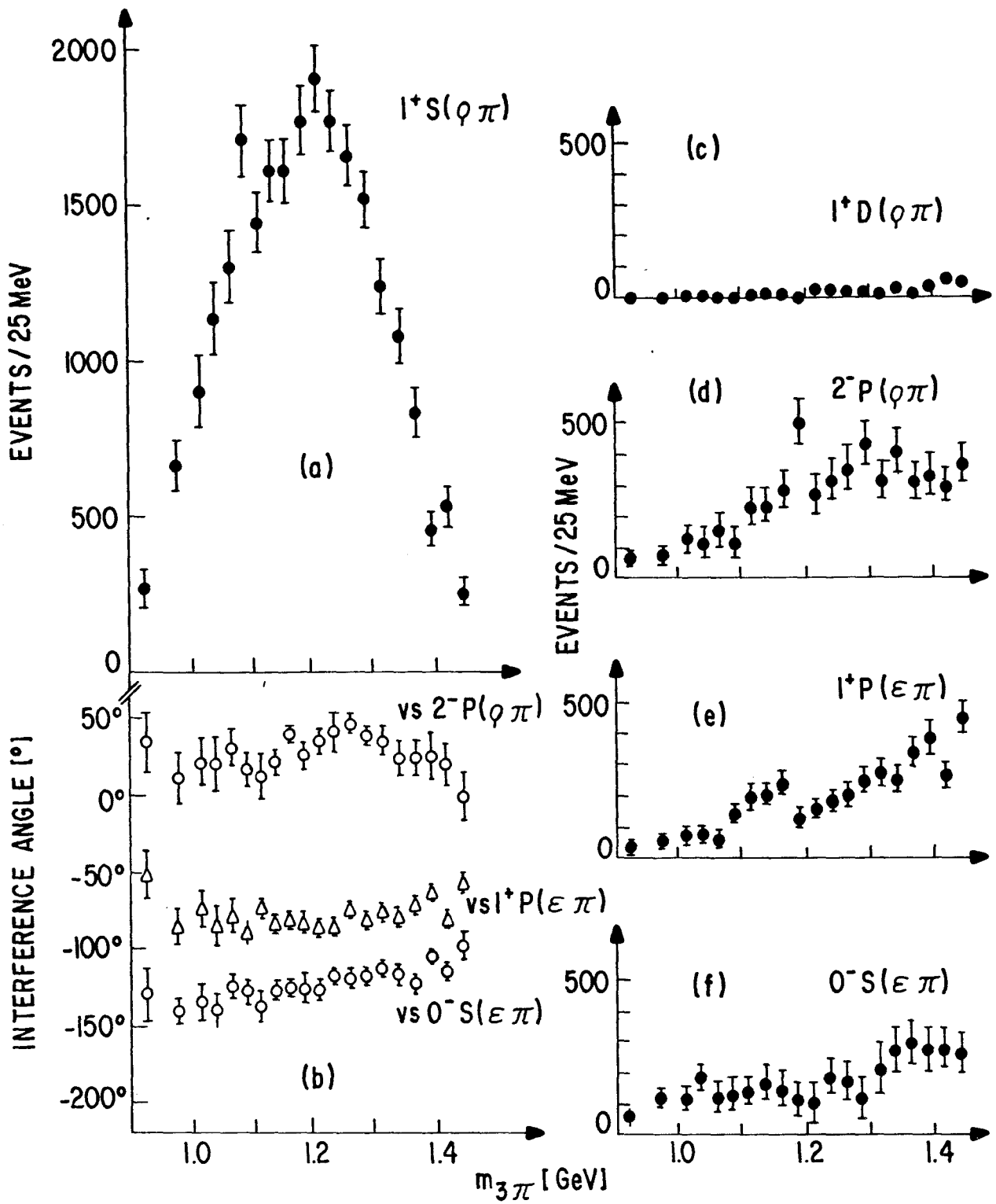


Fig. 15

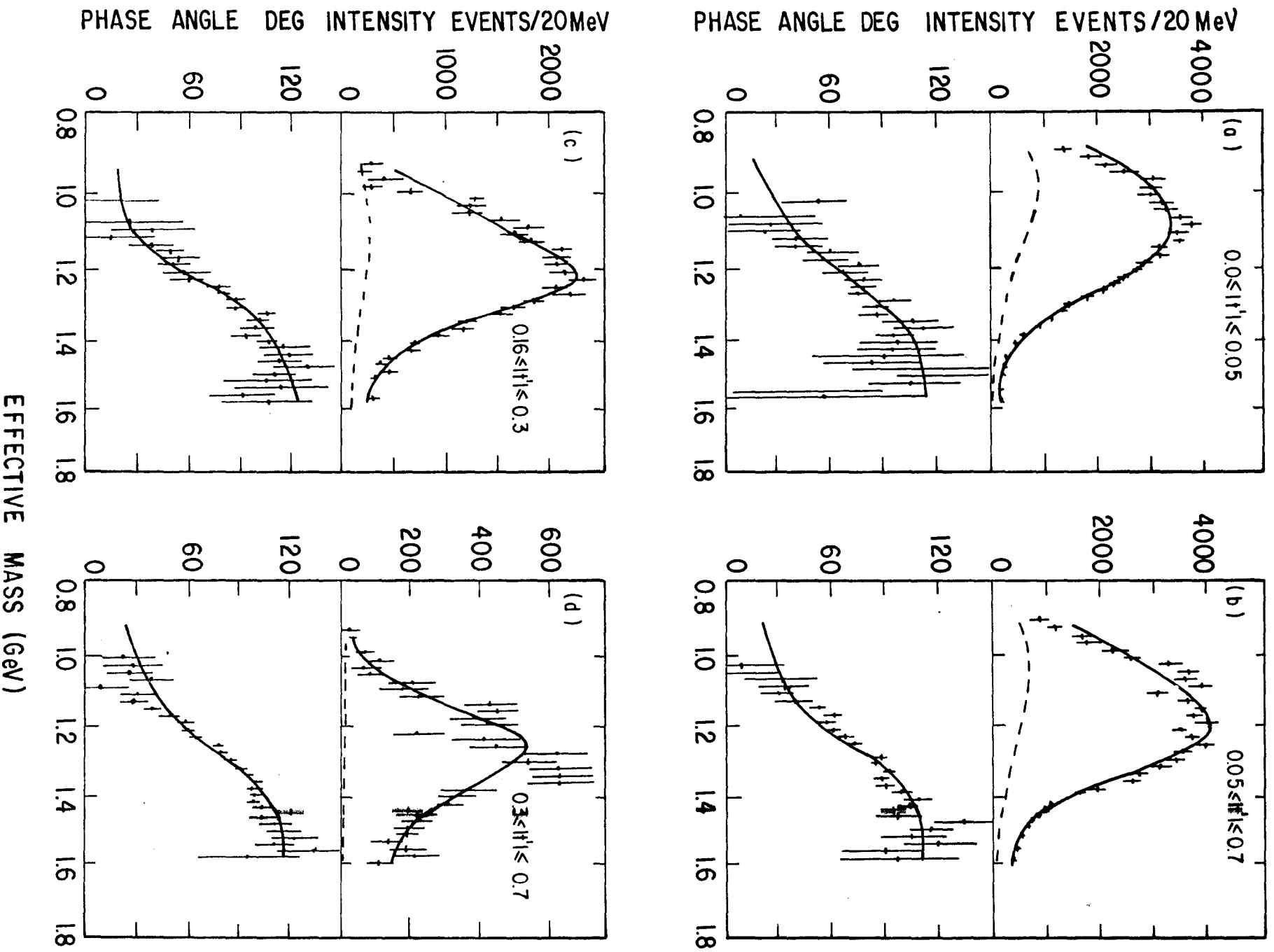


Fig. 16

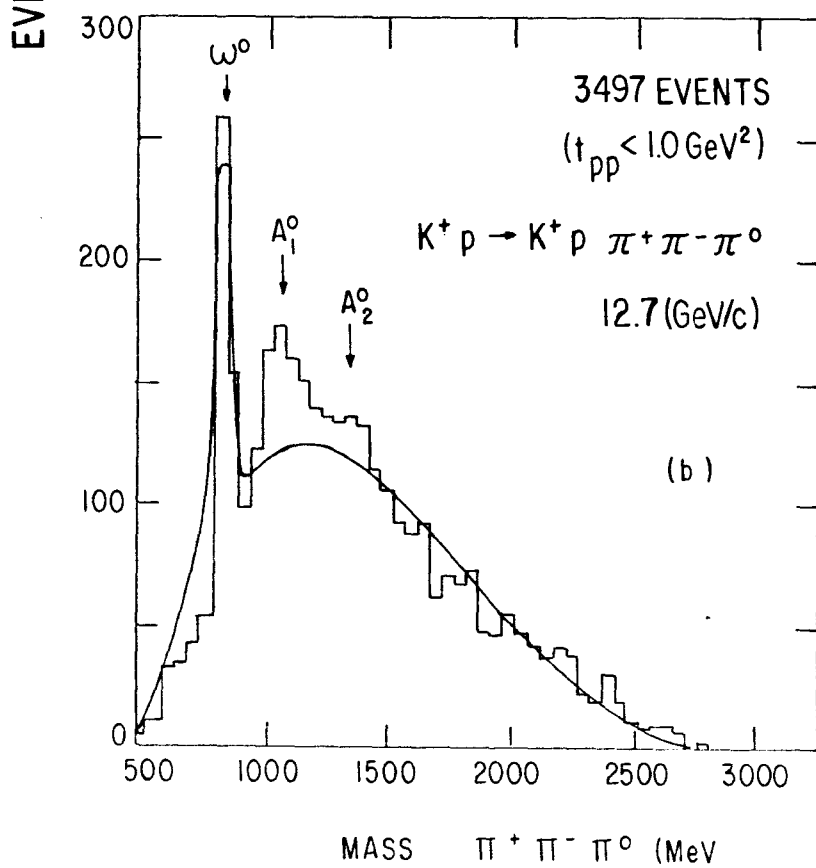
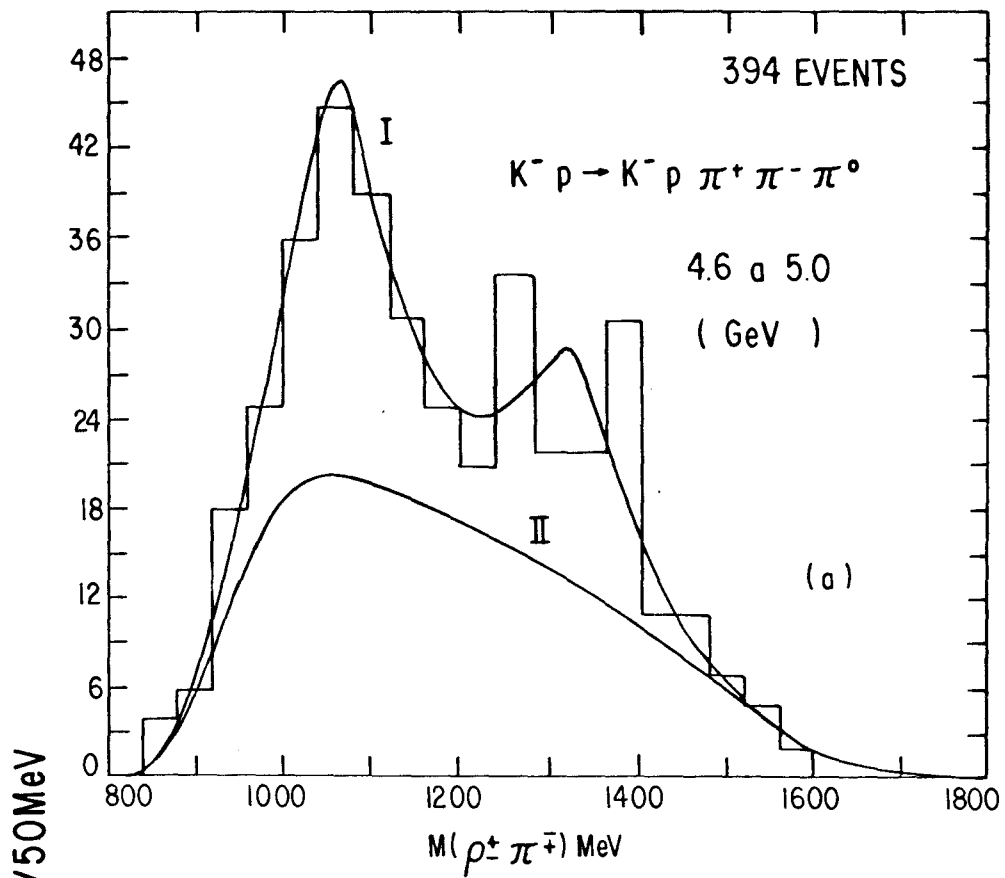


Fig. 17

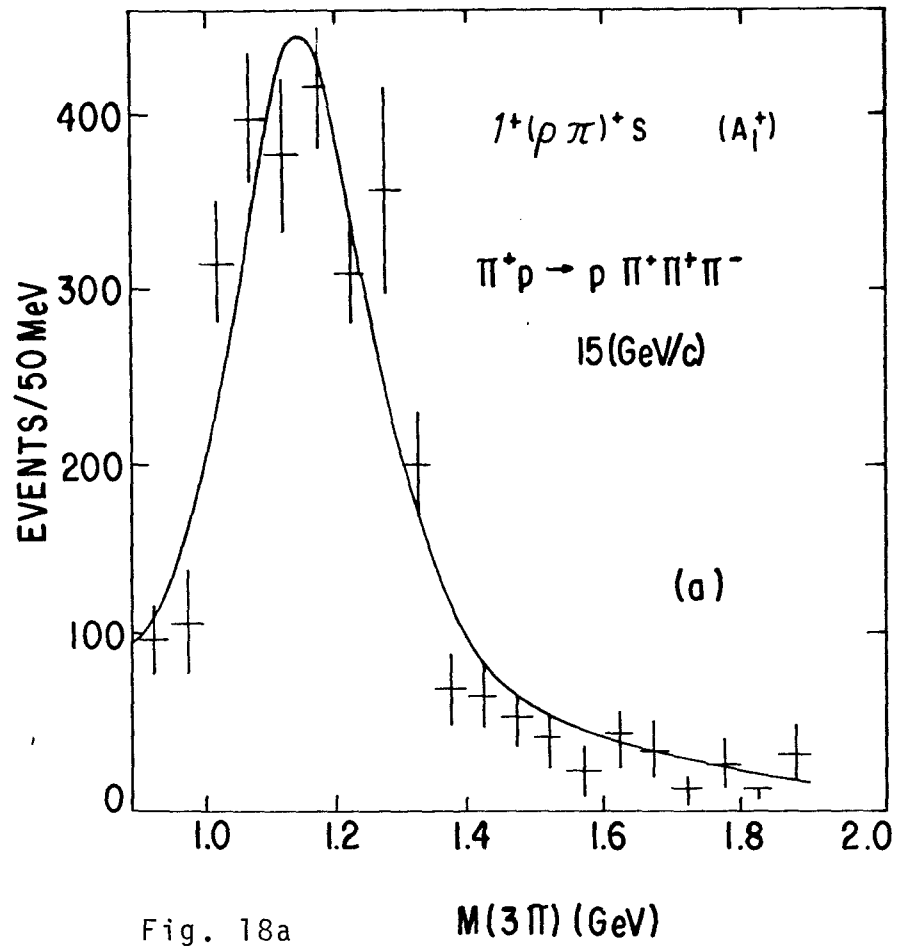


Fig. 18a

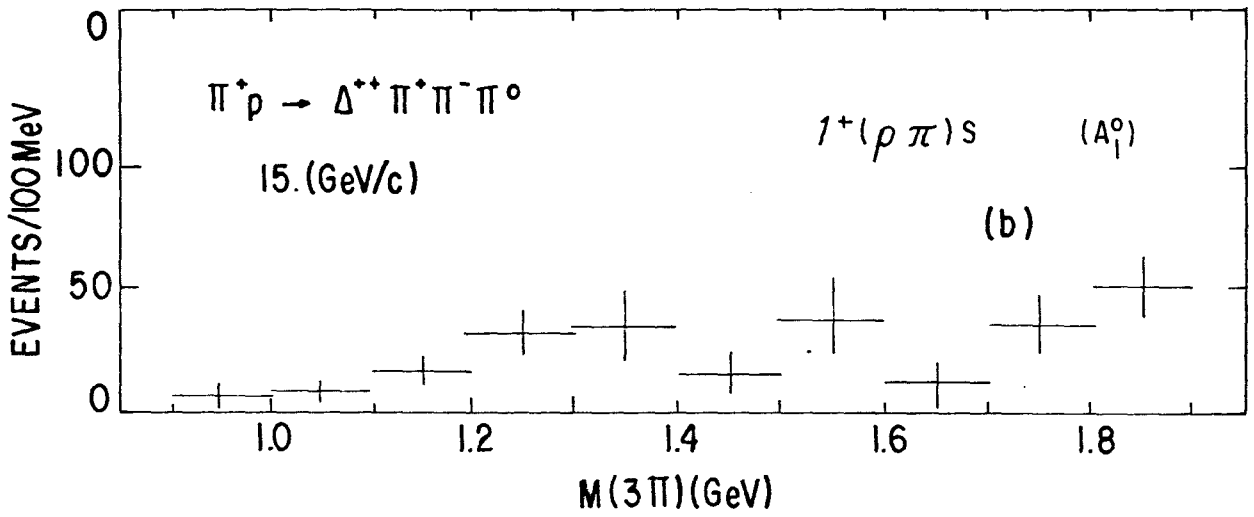


Fig. 18b

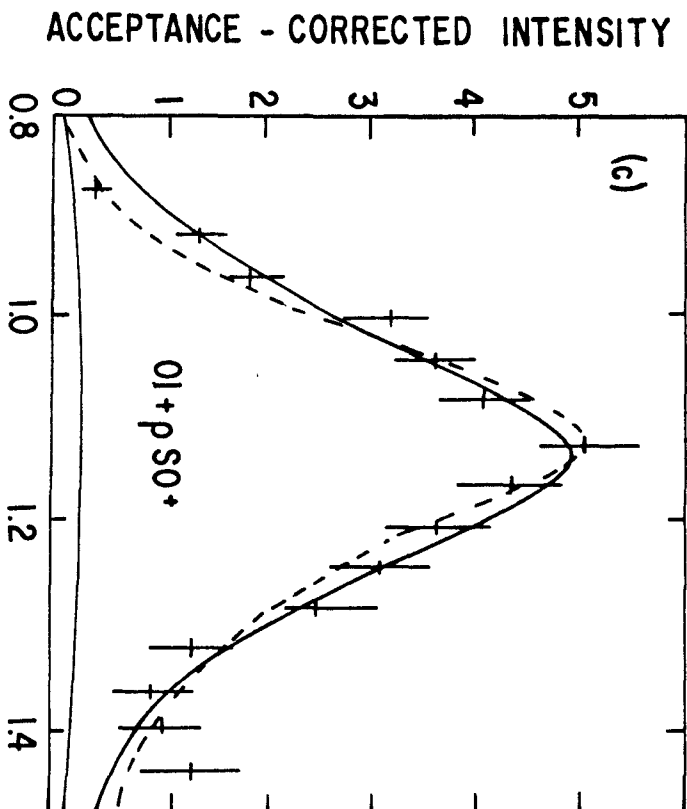
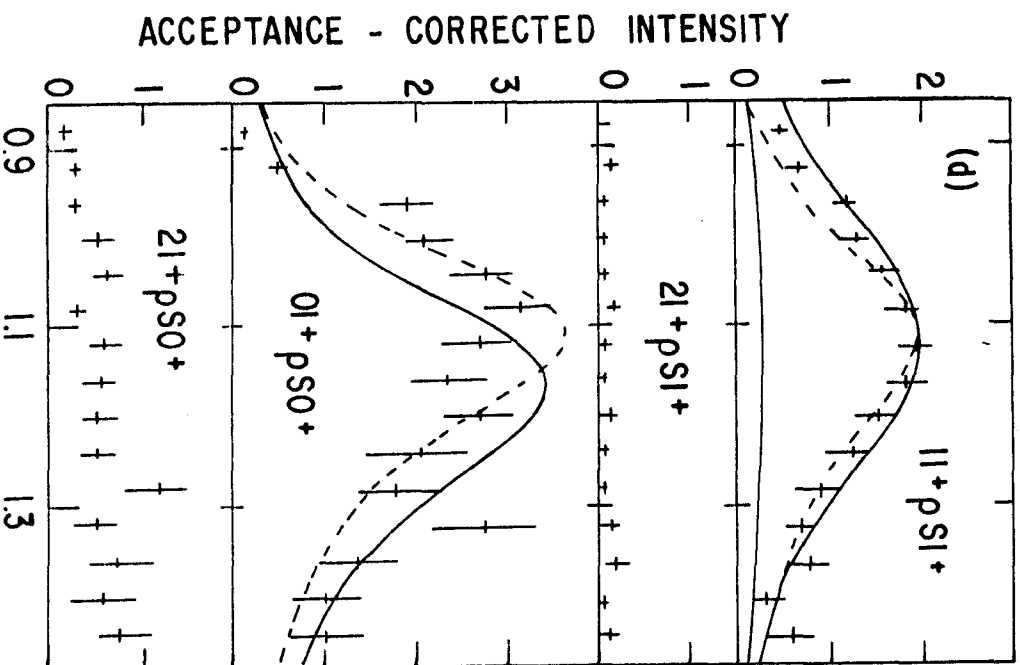
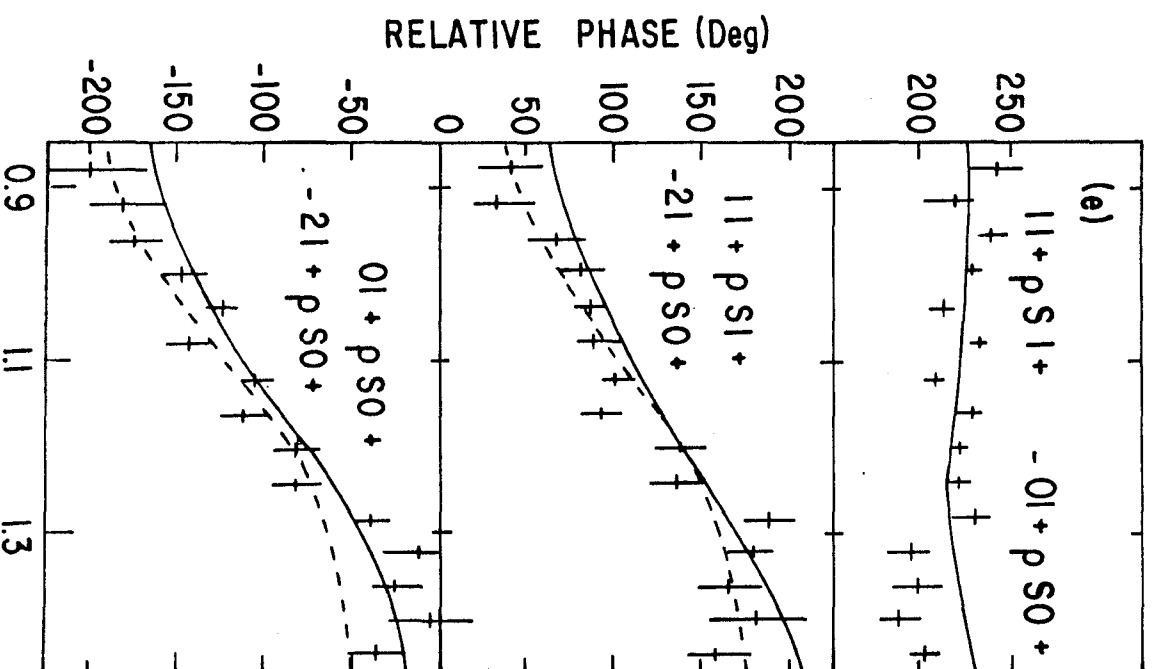


Fig. 18c $M(\pi^+\pi^-\pi^0)$ GeV



$M(\pi^+\pi^-\pi^0)$, GeV Fig. 18d



$M(\pi^+\pi^-\pi^0)$ GeV Fig. 18e

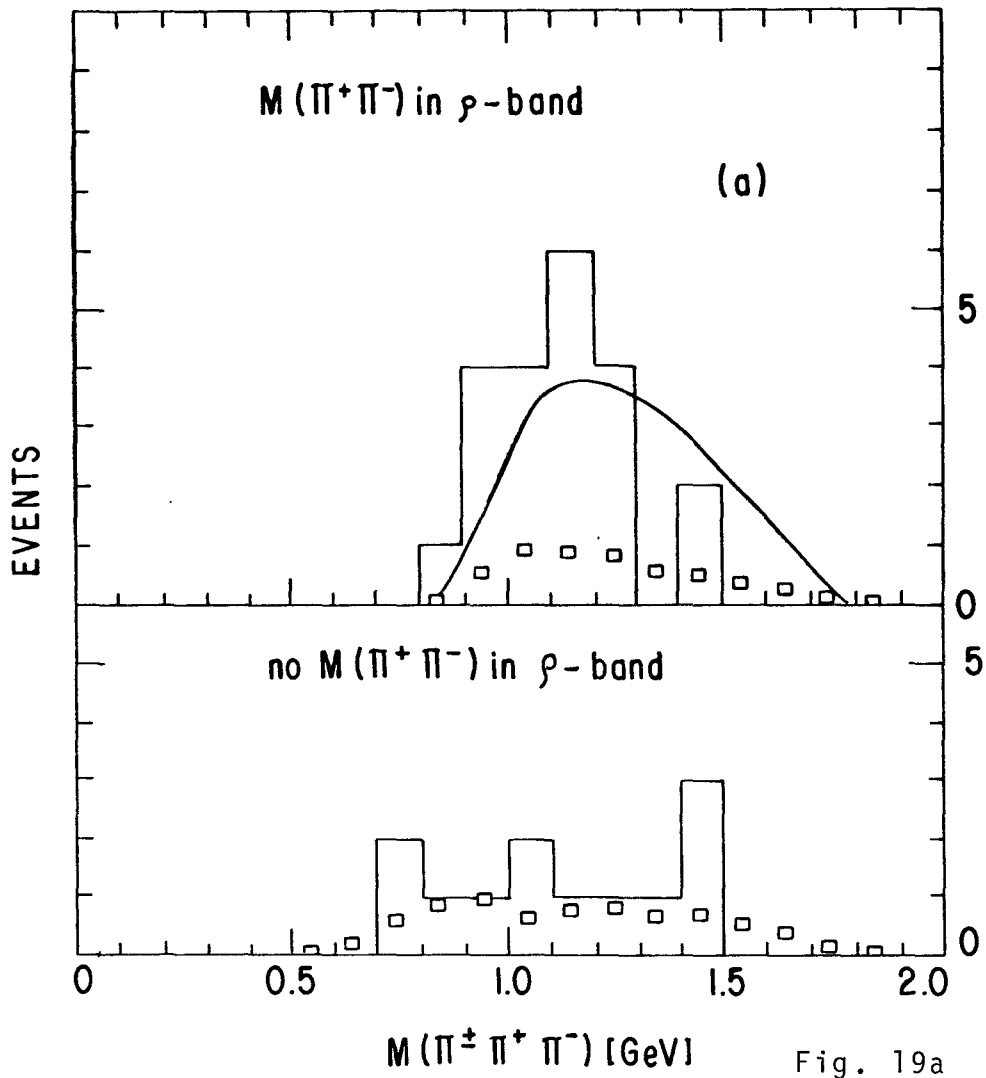


Fig. 19a

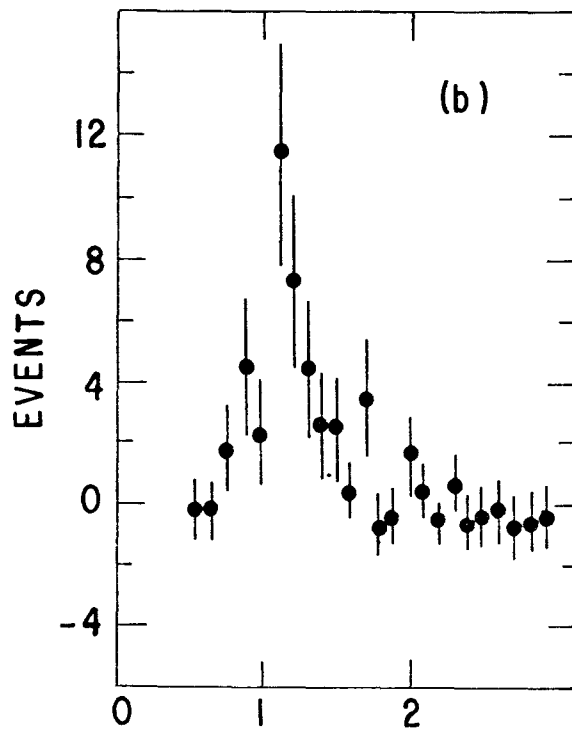


Fig. 19b

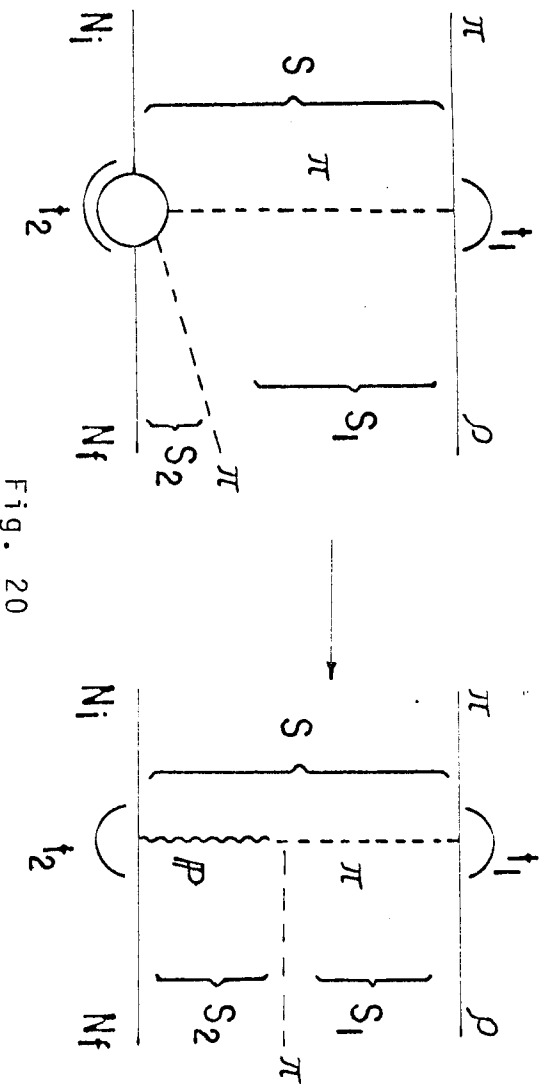


Fig. 20

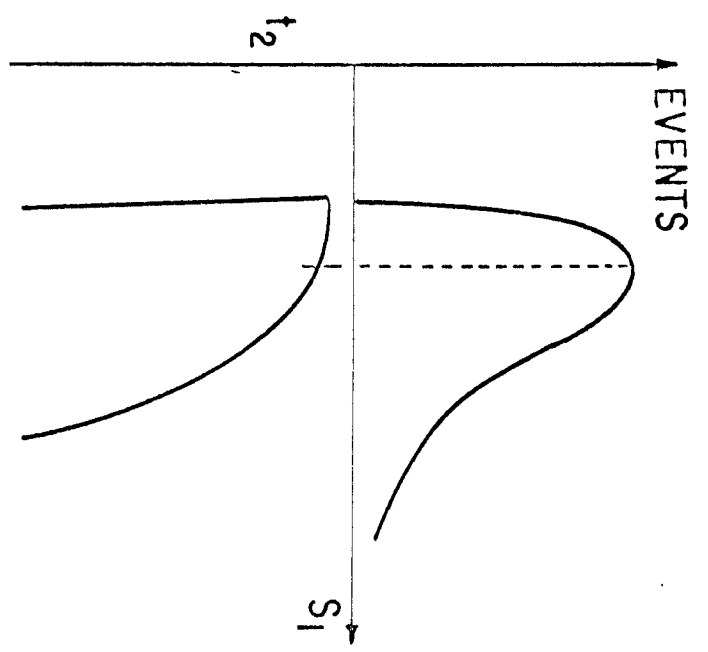
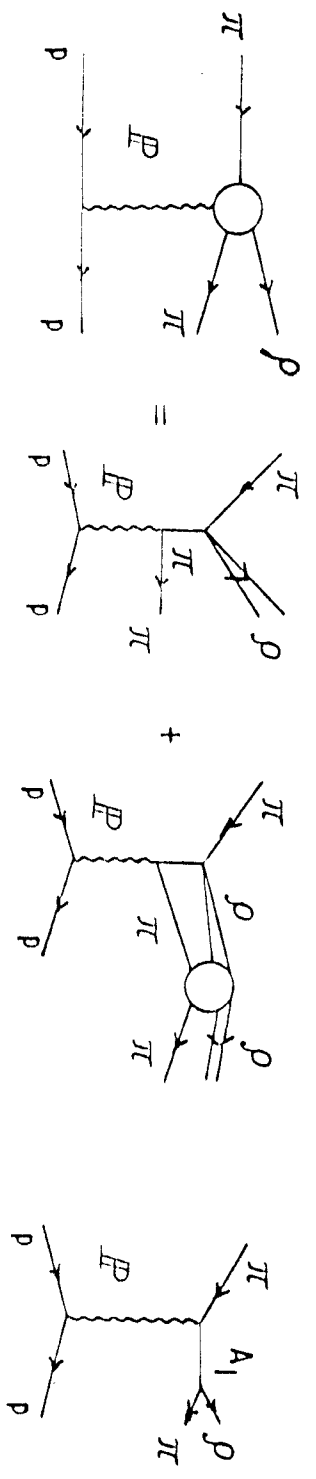


Fig. 21



(a)

(b)

(c)

Fig. 22

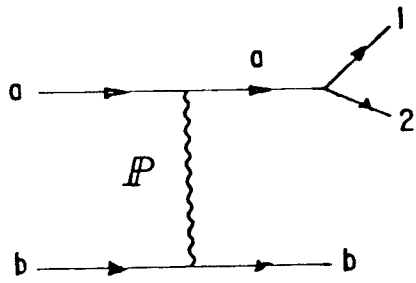


Fig. 23

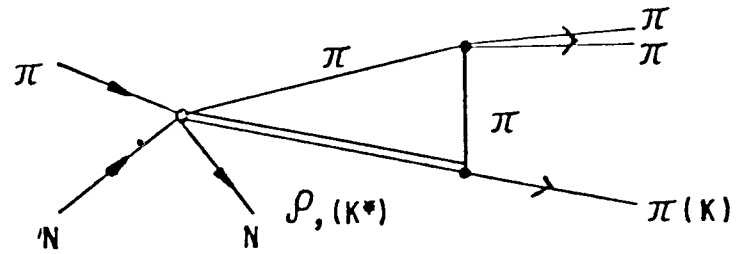


Fig. 24

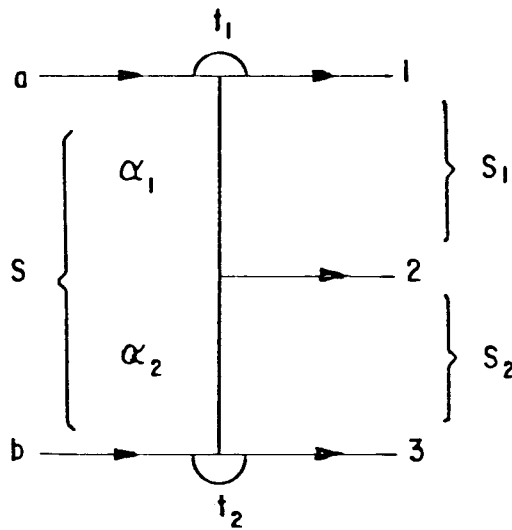


Fig. 25

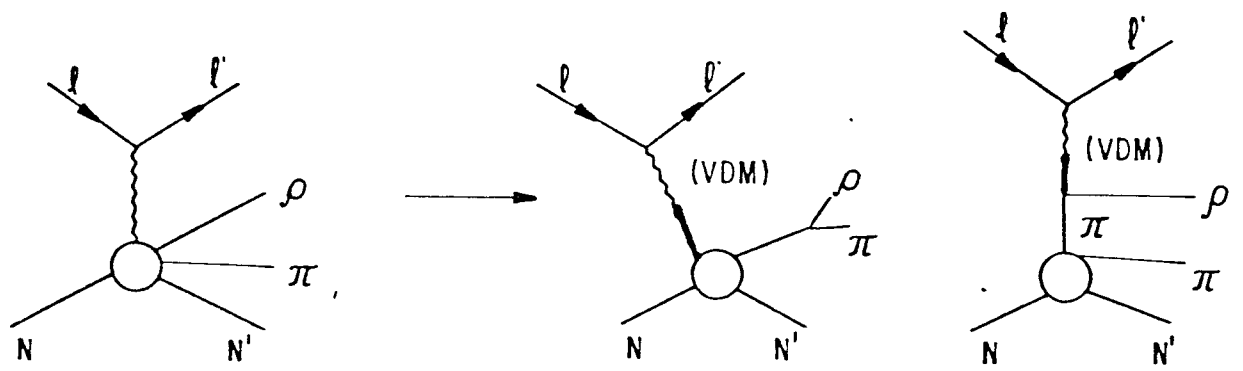


Fig. 26

(a)

(b)

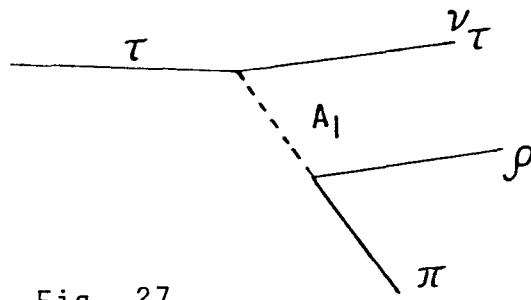


Fig. 27

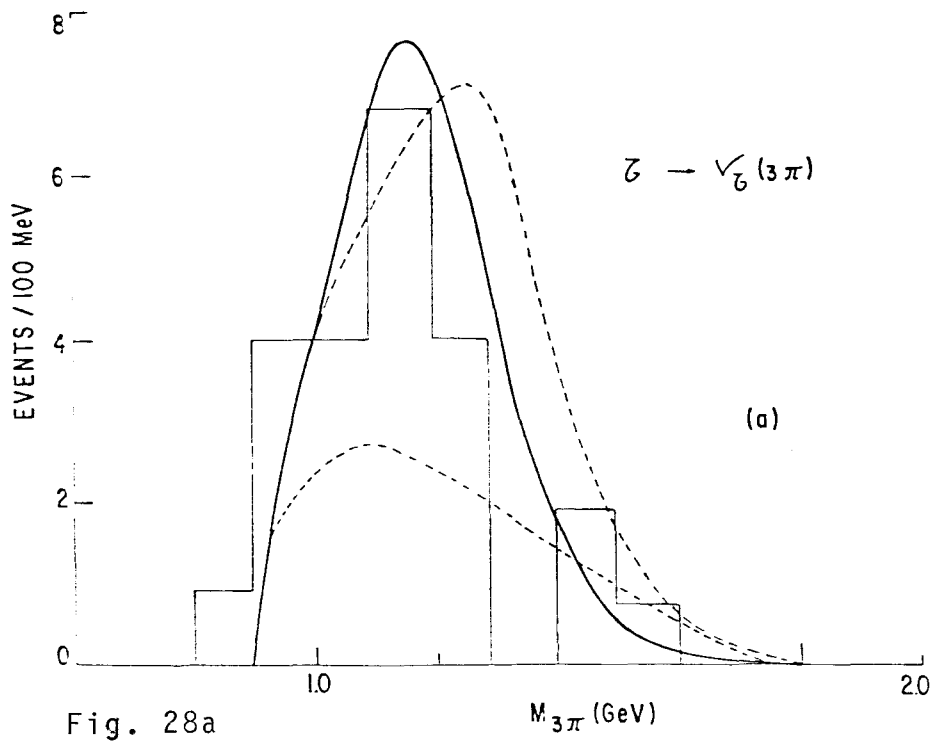


Fig. 28a

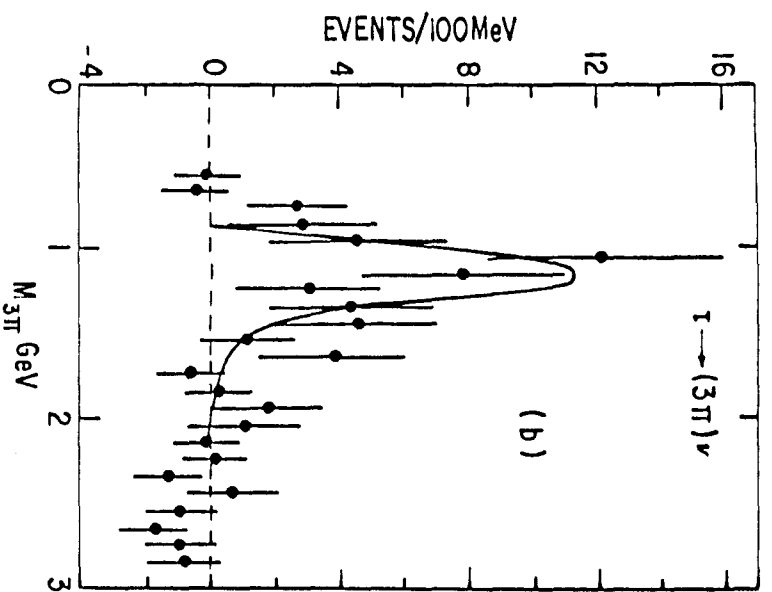


Fig. 28b

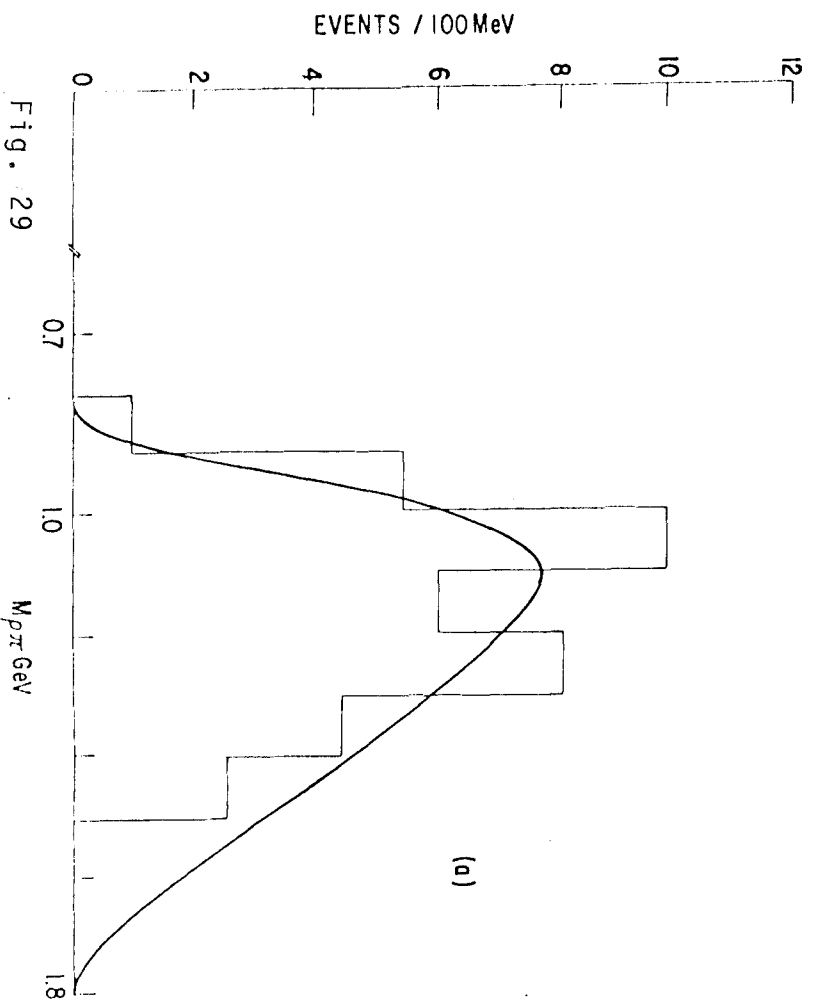


Fig. 29