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A GAUGE THEORY FOR COMPOSITE FIELDS

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Abstract

A gauge theory for composite matter fields is proposed. The cases $SU(N) \otimes SU(N)$ and $SU(2) \otimes U(1)$ are studied. QCD can also be reobtained for composite quarks.

Key-word: Composite fields.

1. INTRODUCTION

Elementary particle physics analysis the structures based on the building block method. The present theoretical understanding of low energy phenomena seems to indicate that gauge field theory is a viable framework for the description of all fundamental forces of nature. The presence of these aspects generates importance for developing a gauge theory for composite fields. It would be an inverse situation to the unification scenario. The effort here is to develop gauge theories for bound states interaction.

Consider a composite matter field. The basic fields would have independent gauge transformations. They are

$$\psi' = U_1\psi \tag{1}$$

$$\phi' = U_2\phi \tag{2}$$

where U_1 and U_2 are matrices involving rotations of two different groups or for a certain group in two different representations. The composite field will be given by

$$\chi = \psi \otimes \phi \tag{3}$$

Substituing in (1) and (2)

$$\chi' = \psi' \otimes \phi' \equiv U\chi$$

where
$$U = U_1 \otimes U_2 \tag{4}$$

For instance, take a rotation just on the field ψ. It gives

$$U = U_1 \otimes 1\!\!1 \tag{5}$$

Similarly to the Yang-Mills case [1] a covariant derivative must be introduced in order to preserve gauge invariance. Local transformations (1) and (2) will generate a composite covariant derivative. Associating different gauge fields $A_\mu (B_\mu)$ to different matter fields $\psi(\phi)$ yields

$$D_\mu(A;B) = D_\mu(A) \otimes 1\!\!1_\phi + 1\!\!1_\psi \otimes D_\mu(B) \tag{6}$$

2. SU(N) ⊗ SU(N)

Consider a general situation where fields transform independently. The local invariance requirement yields

$$D_\mu(A)\, \psi = [\partial_\mu + ig\, t_1^a A_{\mu a}]\, \psi \tag{7}$$

$$D_\nu(B)\, \phi = [\partial_\nu + ig\, t_2^a B_{\mu a}]\, \phi \tag{8}$$

and

$$[D_\mu(A)\psi]' = U_1 [D_\mu(A)\psi] \tag{9}$$

$$[D_\mu(B)\phi]' = U_2 [D_\mu(B)\phi] \tag{10}$$

where t_1^a and t_2^a are SU(N) generators. The compensating gauge fields transform like

$$A_\mu^{a'} t_{1a} = U_1 A_\mu^a t_{1a} U_1^{-1} + \frac{i}{g} (\partial_\mu U_1) U_1^{-1} \quad (11)$$

$$B_\mu^{a'} t_{2a} = U_2 B^a t_{2a} U_2^{-1} + \frac{i}{g} (\partial_\mu U_2) U_2^{-1} \quad (12)$$

The composite covariant derivative defined in (6) is

$$D_\mu(A;B)\chi = \partial_\mu(\psi \otimes \phi) + ig\, A_\mu^a t_{1a} \psi \otimes \phi + ig\, \psi \otimes B_\mu^a t_{2a} \phi \quad (13)$$

with

$$[D_\mu(A;B)\, \chi]' = U[D_\mu(A;B)\, \chi] \quad (14)$$

Rewriting the fields,

$$C_\mu^a = \frac{1}{2} (A_\mu^a - B_\mu^a) \quad (15)$$

$$G_\mu^a = \frac{1}{2} (A_\mu^a + B_\mu^a) \quad (16)$$

and the associated matrices

$$T^a = t_1^a \otimes \mathbb{1}_\phi + \mathbb{1}_\psi \otimes t_2^a \quad (17)$$

$$t^a = t_1^a \otimes \mathbb{1}_\phi - \mathbb{1}_\psi \otimes t_2^a \quad (18)$$

gives,

$$D_\mu(A;B)\,\chi \;=\; D_\mu(G;C)\,\chi$$

where

$$D_\mu(G;C) \;=\; \partial_\mu + ig\,G^{\mu a}T_a + ig\,C^{\mu a}t_a \qquad (19)$$

From (14),

$$D_\mu(G;C)' \;=\; U\,D_\mu(G;C)\,U^{-1} \qquad (20)$$

it yields,

$$G^{a\,\prime}_\mu T_a + C^{a\,\prime}_\mu t_a \;=\; U\,G^a_\mu T_a U^{-1} + U\,C^a_\mu t_a U^{-1} + \frac{i}{g}\,(\partial_\mu U)\,U^{-1} \qquad (21)$$

and

$$\{[D_\mu,D_\nu]\chi\}' \;=\; U\{[D_\mu,D_\nu]\chi\} \qquad (22)$$

Define the following strength tensor

$$[D_\mu(G;C),D_\nu(G;C)] \;=\; ig\,F_{\mu\nu}(G;C) \qquad (23)$$

and calculating the commutation relation

$$[T^a,T^b] \;=\; i\,f^{abc}\,T_c$$

$$[t^a,t^b] \;=\; i\,f^{abc}\,t_c$$

$$[T^a,t^b] \;=\; i\,f^{abc}\,t_c \qquad (24)$$

it yields,

$$\frac{1}{ig} F_{\mu\nu}(G;C) = (\partial_\mu G^a_\nu - \partial_\nu G^a_\mu) T_a + (\partial_\mu C^a_\nu - \partial_\nu C^a_\mu) t_a +$$

$$-g(G_\mu \wedge G_\nu)^a T_a - g(C_\mu \wedge C_\nu)^a T_a - g(G_\mu \wedge C_\nu)^a t_a + g(G_\nu \wedge C_\mu)^a t_a$$

where $$(X_\mu \wedge X_\nu)^a \equiv C^a{}_{bc} X^b_\mu Y^c_\nu \tag{25}$$

(1) and (2) transformations make a basis for a virtual group defined in (4). An appropriate covariant derivative is expressed for this group in (6). Transformations in the considered basis yield a covariant expression (20). This virtual group appears as a method to generate composite matter fields. (25) is covariant under its transformations. The Yang Mill character disappear if the basic group are abelian. Observe that the matrices T^a and t^b do not commute. Calculating the traces

$$\mathrm{tr}(T^a t^b) = \frac{N^2-1}{2} \delta^{ab}$$

$$\mathrm{tr}(T^a t_b) = 0$$

$$\mathrm{tr}(t^a t^b) = \frac{N^2-1}{2} \delta^{ab} \tag{26}$$

Thus

$$\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} = \frac{N^2-1}{2} \Big[(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu)^2 + (\partial_\mu C^a_\nu - \partial_\nu C^a_\mu)^2 +$$

$$- g(\partial_\mu G_\nu - \partial_\nu G_\mu)^a \, [G_\mu \wedge G_\nu) - (C_\mu \wedge C_\nu)]_a +$$

$$- g(\partial_\mu C_\nu - \partial_\nu C_\mu)^a \, [G_\mu \wedge C_\nu) + (G_\nu \wedge C_\mu)]_a +$$

$$+ g^2 [(G_\mu \wedge G_\nu)^2 + (C_\mu \wedge C_\nu)^2 + (G_\mu \wedge C_\nu)^2 + (G_\nu \wedge C_\mu)^2 +$$

$$+ 2(G_\mu \wedge C_\nu)(C_\mu \wedge C_\nu) - (G_\mu \wedge C_\nu)(G_\nu \wedge C_\mu)]\Big] \qquad (27)$$

(27) does not generate mixing propagators as in [2]. QCD reobtained for the case where C^a_μ field is zero. This means to consider just one gauge field with the transformations (11) and (12). The correspoding propagators and vertices are in Fig. 1. In Appendix A it is shown the case SU(2) ⊠ SU(2) as an example.

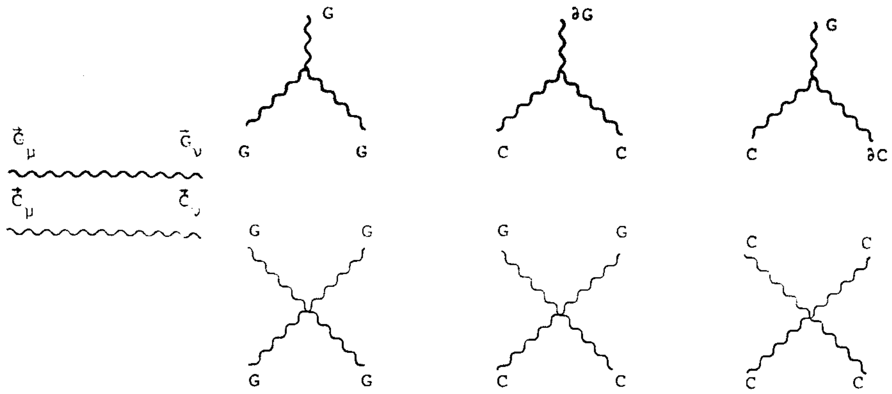

Fig. 1- Feynman graphs for SU(N) ⊠ SU(N).

The matter term in the Lagrangian will be given by

$$i\,\bar{\chi}\, D_\mu(G;C)\,\chi \tag{28}$$

The gauge invariance method generates propagation and interaction for composite matter fields as in Fig. 2.

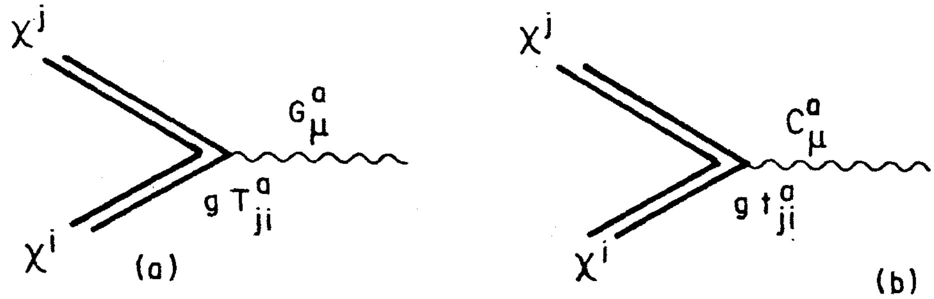

Fig. 2

Composite fields interaction

The gauge method allows there fields to propagate and interact through (31). The difference to the usual case is in the form of the vertex matrix.

3 SU(2) ⊠ U(1)

Here the groups involved have different numbers of generators. Consider that the field ψ belongs to the SU(2) fundamental representation and ϕ transforms under U(1) the rotations

$$U_1 = e^{ig\,\vec{\alpha}(x)\,.\,\frac{\vec{\tau}}{2}} \tag{29}$$

$$U_2 = e^{i\alpha(x)} \tag{30}$$

it yields the covariant derivatives

$$D_\mu(A)\psi = [\partial_\mu + ig\vec{A}_\mu . \frac{\vec{\tau}}{2}]\psi \tag{31}$$

$$D_\mu(B)\phi = [\partial_\mu + ig\vec{B}_\mu . 1]\phi \tag{32}$$

Defining

$$G_\mu = \frac{1}{2}(A^3_\mu + B_\mu) \tag{33}$$

$$C_\mu = \frac{1}{2}(A^3_\mu - B_\mu) \tag{34}$$

The underlying gauge invariance principle will require for the composite field

$$\chi = \begin{pmatrix} \psi_1 & \phi \\ \psi_2 & \phi \end{pmatrix} \tag{35}$$

a composite covariant derivative. Following (6), (13) and (14) we get

$$D_\mu(A^1,A^2,G,C) = \partial_\mu + ig A^1_\mu \frac{\tau_1}{2} + ig A^2_\mu \frac{\tau_2}{2} + ig G_\mu \frac{T^3}{2} + ig C_\mu \frac{t^3}{2} \tag{36}$$

where

$$T^3 = \frac{\tau^3_1}{2} \otimes 1 + \mathbb{1} \otimes 1$$

$$t^3 = \frac{\tau^3_1}{2} \otimes 1 - \mathbb{1} \otimes 1$$

yielding

$$T^3 = \frac{1}{2}\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad t^3 = \frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & -3 \end{pmatrix} \tag{37}$$

Similarly to (26), the strength tensor is

$$\begin{aligned} F_{\mu\nu}(A^1,A^2,G,C) &= (\partial_\mu G_\nu - \partial_\nu G_\mu)\frac{T^3}{2} + (\partial_\mu C_\nu - \partial_\nu C_\mu)\frac{t^3}{2} + \\ &+ [(\partial_\mu A^1_\nu - \partial_\nu A^1_\mu) + g(G_\mu + G_\mu)A^2_\nu - g(G_\nu + C_\nu)A^2_\mu]\frac{\tau^1_1}{2} + \\ &+ [(\partial_\mu A^2_\nu - \partial_\nu A^2_\mu) - g(G_\mu + C_\mu)A^1_\nu + g(G_\nu + C_\nu)A^1_\mu]\frac{\tau^2_1}{2} + \\ &+ g[A^1_\mu A^1_\nu - A^1_\mu A^2_\nu]\frac{\tau^3_1}{2} \end{aligned} \tag{38}$$

Thus

$$\begin{aligned} \text{tr}\, F_{\mu\nu}F^{\mu\nu} &= (\partial_\mu A^1_\nu - \partial_\nu A^1_\mu)^2 + (\partial_\mu A^2_\nu - \partial_\nu A^2_\mu)^2 + \\ &+ 2(\partial_\mu G_\nu - \partial_\nu G_\mu)^2 + 2(\partial_\mu C_\nu - \partial_\nu C_\mu)^2 + \\ &- 4g[(\partial_\mu G_\nu - \partial_\nu G_\mu) - (\partial_\mu C_\nu - \partial_\nu C_\mu)]A^{\mu^1}A^{\nu^2} \\ &+ 2g[(\partial_\mu A^1_\nu - \partial_\nu A^1_\mu)A^{\nu^2} - (\partial_\mu A^2_\nu - \partial_\nu A^2_\mu)A^{\nu^1}](G^\mu + C^\mu) \\ &+ 2g^2[(G_\mu + C_\mu)(A^1_\nu + A^2_\nu)^2 - (G_\nu + C_\nu)(A^{\mu^1}A^{\mu^2} + A^{\mu^2}A^{\nu^2})](G^\mu + C^\mu) \\ &- 2g^2[(A^1_\mu)^2(A^2_\nu)^2 - A^1_\mu A^{\mu 2}A^1_\nu A^{\nu 2}] \end{aligned} \tag{39}$$

The corresponding propagators and vertice are in Fig. 3.

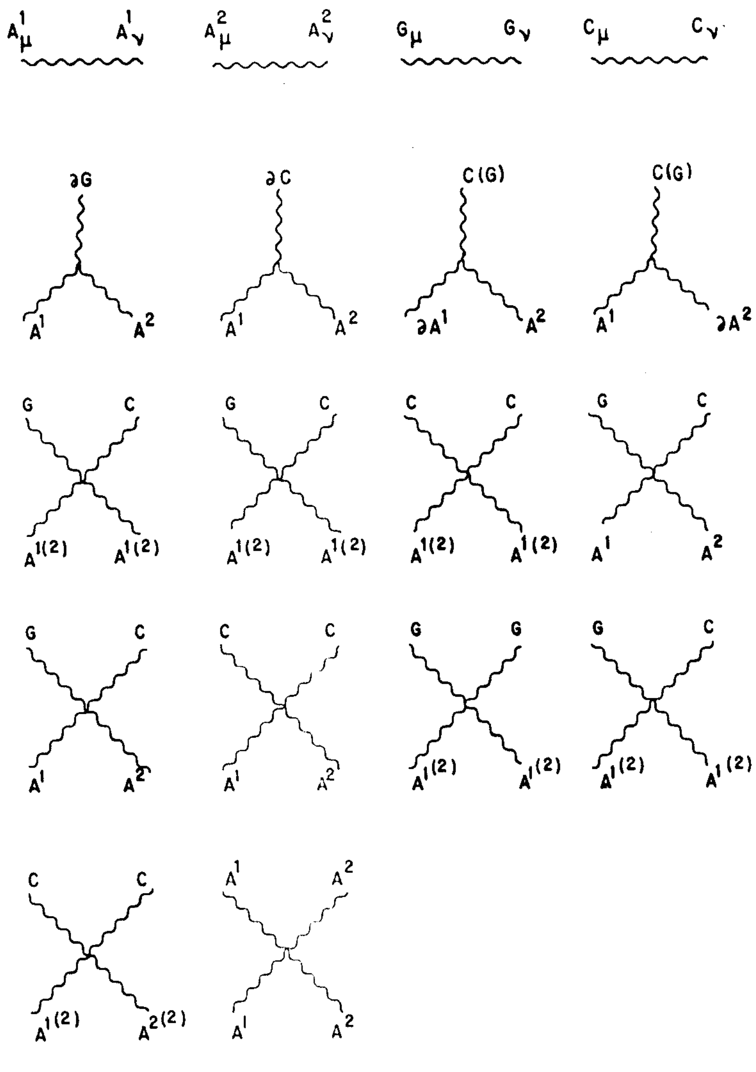

Fig. 3 - Propagators and vertices for SU(2) ⊠ U(1) case.

CONCLUSION

Gauge theories have proved propitious for the description of the dynamics of the building blocks of matter. This happens with the standard model and QCD. Thus the bound state concept of baryons and mesons ought to be a non-perturbative solution of the Euler Lagrange equations in QCD. However for the moment this is only a theoretical speculation. A theory for strong interactions between mesons and baryons does not satisfy in practice the gauge models. A phenomenology based on Regge theory, the S-matrix, dispersion relations, etc. is required. The fields G^a_μ and C^a_μ do not need to be identified as well-defined particles. They rather would be a convenient concept to express the transfer of energy and momentum in a scattering process. Suppose that, based on these fields, it is viable to describe strong interactions. Then, it would be interesting to study the possibility of understanding the pomeron in terms of a parallel model based on (15) and (16).

Our effort here is to generate through gauge invariance a model for composite fields. Each elementary field is assumed to be independent from the other. This means that they will rotate independently under a gauge transformation. The structure of such a bound state will depend on the kind of the gauge invariant Lagrangian associated to the fundamental fields. However this aspect is not the content of this work. Here we assume the presence of a composite field.

The gauge theory for a composite field associates to it a virtual group. This redefinition preserves the number of generators from the basic groups. Consider as example the SU(N) ⊠ SU(N) case. (24) shows the associated virtual group algebra. Observe that the corresponding generators T^a and t^a satisfy the Lie algebra. N varieties of each basic group will construct the composite field. The local physical underlying principle is that the components χ_i of the composite field are indistinguishable. The norm $\bar{\chi}\chi$ is invariant. The local SU(N) ⊠ SU(N) transformations are described by

$$\chi' = U\chi \qquad (40)$$

where U is a space-time dependent matrix.

$$U = e^{ig(t_1^a w_a \oplus t_2^a \lambda_a)}$$

$$U^+ = U^{-1} \quad , \quad \det U = 1 \qquad (41)$$

The structure constant f^{abc} may be calculated from the explicit form of T^j and t^k given in (24) and (26)

$$c^{abc} \sim Tr[T^c \, , [T^a, T^b]] \qquad (42)$$

(i) shows the antisymmetric property of the constant. A similar expression is obtained for t^k.

Consider quarks as composite particles [3]. Observe a model where just one gauge field A_μ^a is associated to the matter fields ϕ^i and ψ^j. From (15) and (16), only the field G_μ^a interacts with the composite field. Let us interpret it as the QCD gluon. In this model composite quarks are given by $\chi_k = f_{kij}\phi^i\psi^j$ where

f_{kij} is a colour structure constant. Thus (30) and (31) indicate that QCD would be regained with composite quarks exchanging just one field. Therefore another option for gluon exchange for perturbative quark-quark scattering is in Fig. 4.

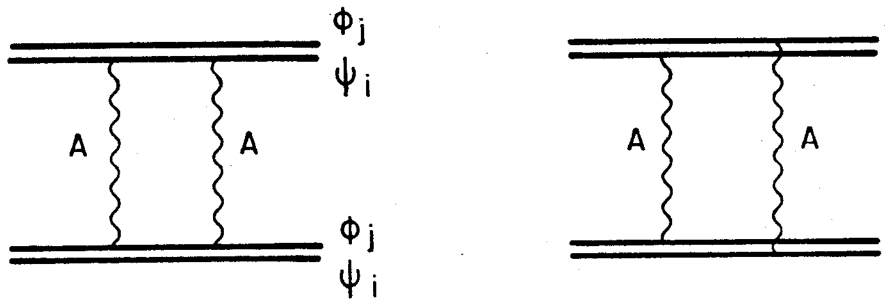

Fig. 4 - Example of a two gluon exchange diagrams for composite quark scattering. From [3] a primitive quark written only in terms of colour is given by $\chi_k = f_{kij}\, \phi^i \psi^j$.

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APPENDIX A - SU(2) ⊠ SU(2)

Consider a field ϕ transforming under a two dimensional representation

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{A1}$$

and also a field ψ

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{A2}$$

The composite field χ is given by

$$\chi = \psi \boxtimes \phi = \begin{pmatrix} \phi_1\psi_1 \\ \phi_1\psi_2 \\ \phi_2\psi_1 \\ \phi_2\psi_2 \end{pmatrix} \tag{A3}$$

From (17),

$$T_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \boxtimes 1\!\!1_\psi + 1\!\!1_\phi \boxtimes \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

giving

$$T_1 = \frac{1}{2}\left[\begin{array}{cc|cc} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ \hline 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right] \tag{A4}$$

From (18),

$$t_1 = \frac{1}{2}\left[\begin{array}{cc|cc} 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ \hline 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{array}\right] \tag{A5}$$

The second matrix component is

$$T_2 = \frac{1}{2}\begin{pmatrix} 0 & -i\,\mathbb{1}_\phi \\ i\,\mathbb{1}_\phi & 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} \tau_2^2 & 0 \\ 0 & \tau_2^2 \end{pmatrix}$$

giving

$$T_2 = \frac{1}{2}\left[\begin{array}{cc|cc} 0 & -i & -i & 0 \\ i & 0 & 0 & -i \\ \hline i & 0 & 0 & -i \\ 0 & i & i & 0 \end{array}\right] \tag{A6}$$

$$t_2 = \frac{1}{2}\left[\begin{array}{cc|cc} 0 & i & -i & 0 \\ -i & 0 & 0 & -i \\ \hline i & 0 & 0 & i \\ 0 & i & -i & 0 \end{array}\right] \tag{A7}$$

Similarly,

$$T_3 = \frac{1}{2}\left[\begin{array}{cc|cc} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{array}\right] \tag{A8}$$

$$t_3 = \frac{1}{2}\left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ \hline 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \tag{A9}$$

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