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CHARGE DISTRIBUTION IN MULTIPLE π-MESON PRODUCTION

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ABSTRACT

A table of charge distribution in multiple π-meson production due to fire ball decay is given for the purpose of practical use. Assuming the charge independence, the distribution function $P_n(r)$ (n = total multiplicity, $r=\pi^+$ multiplicity) is calculated for different I_o = 0,1, and 2 isospin quantum numbers of a fire ball up to n = 10. For I_o = 0, a conventional formula to calculate $P_n(r)$ is given.

INTRODUCTION

The charge distribution of π-mesons in decay products of a fire ball is an important quantity for the analysis of cosmic ray experiment(1). Since the emulsion chamber experiment only detects π^0 mesons via $\pi^0 \to 2\gamma$ decay, it is necessary for the analysis for the fire ball parameters (mass, Lorentz factor etc

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to introduce some theoretical distribution of charged π-ons in the decay product.

It is customly assumed that the interaction is charge independent, and some simple probabilistic argument is applied[23]. However, for low multiplicity events, such a probabilistic calculation is dubious, (see later discussion) and it does not give a correct fluctuation of charge distribution which is also an important quantity. Some theoretical studies on the charge distribution of π-meson system have been done on the basis of group theory[4,5]. In fact they are very powerful in calculating charge correlations. However when we apply the result to, for example, Monte Carlo calculations, we need still simple formulas.

In this note, we carry out an explicit calculation of charge distribution function in multiple π-meson production assuming the charge independence of the interaction. The results are given in the form of tables, which will be useful for practical uses, such as Monte Carlo calculations. Particularly for isospin equal to zero, we construct a simple formula to calculate the distribution function in a sufficient accuracy.

METHOD

The basic assumption here is the charge independence of the interaction which governs the decay of a fire ball.

We also assume that a fire ball has a definite isospin state, namely, a charge state of fire ball is specified by isospin quantum numbers I_0 and M_0, where I_0 is the total isospin, M_0 the Z-component.

Let us consider a decay of a fire ball into n π-mesons. Since a π-meson has isospin I=1, the problem is to calculate the coefficients of the expansion:

$$|I_o,M_o> = \sum_{m_i} C^{I_o,M_o}_{m_1,\ldots m_n} |1,m_1>|1,m_2> \ldots |1,m_n> \qquad (1)$$

where $|I,M>$ denotes the eigenstate of isospin.

Let r be the number of π^+ mesons in the decay product. The probability of finding r π^+ mesons in the decay product is then given by

$$P_n(r) = \sum'_{m_1,\ldots m_n} |C^{I_o,M_o}_{m_1,m_2,\ldots m_n}|^2 \qquad (2)$$

where $\sum'_{m_1,\ldots m_n}$ means the sum over all m_s' with the condition:

$$\sum_{m_1,\ldots m_n} \delta_{m_i,1} = r \qquad (3)$$

The numbers of π^0 and π^- mesons are related to r by

$$n_{\pi^0} = n - 2r + M_o$$
$$n_{\pi^-} = r - M_o \qquad (4)$$

Generally the expansion (1) is not uniquely determined. For n=2, Eq. (1) is nothing but a usual Clebsh-Gordan expansion:

$$|I_o,M_o> = \sum_{m_1m_2} (11m_1m_2|I_oM_o)|1m_1>|1m_2> \qquad (5)$$

For n=3, there are three linearly independent states with the same I_o and M_o formed by 3-mesons, eg.

$$|I_o,M_o\ (I_{12})> =$$

$$\sum_{m_1m_2m_3} (11m_1m_2|I_{12}M_{12})(I_{12}1\ M_{12}m_3|I_oM_o)|1m_1>|1m_2>|1m_3> \qquad (6)$$

where I_{12} can take anyone of the values 0, 1 and 2, provided that

$$|I_{12} - 1| \leq I_o \leq I_{12} + 1$$

The quantum number I_{12} is the isospin of the intermediate state formed by the first and second π mesons.

Other ways of coupling (for example coupling of the second and third mesons to form the intermediate isospin I_{23} and the coupling to the first one to form I_o) are related to the states (6) through the Racah coefficients and are not linearly independent.

In general, the charge distribution function (2) is specified by a set of internal isospin quantum numbers $(I_1, I_2, \ldots I_n)$ as

$$P_n^{|I_1, I_2, \ldots I_n|} = \sum_{m_1, m_2, \ldots m_n} \left| \prod_{i=1}^{n-1} (I_{io} M_i m_i | I_{i+1} M_{i+1}) \right|^2 \qquad (7)$$

where

$$M_i = \sum_{j=1}^{i} m_j, \qquad M_n = M_o$$

$$I_i + 1 \geq I_{i+1} \geq |I_i - 1|$$

with

$$I_1 = 1 \qquad \text{and} \qquad I_n = I_o .$$

It is shown by Pais that the intermediate states $(I_1, I_2, \ldots I_n)$ are further classified by a set of numbers, (N_1, N_2, N_3) for π-meson cloud[4].

In the case of fire ball decay, it may be plausible to assume that all possible configurations can take place with an equal probability. Then the final charge distribution function is given by

$$P_n(r) = \overline{P_n(r)}^{\,|I_1, I_2, \ldots I_n|} \qquad (8)$$

where $\overline{|I_1, I_2, \ldots I_n|}$ indicates the average over all possible configurations $|I_1, I_2, \ldots I_n|$. This procedure corresponds to taking an average over all possible configurations of Pais, (N_1, N_2, N_3) with appropriate weight $\rho(N_1, N_2, N_3)$[4].

Although the calculation of Eq. (7) is straight forward, the numbers of possible values of I and M increase too rapidly with n, so that it is difficult to calculate $P_n(r)$ for large n, even with the aid of large electronic computer.

Tables I-III are the results of explicit calculation of $P_n(r)$ for low n values.

TABLE I

$I_o = 0, \quad M_o = 0$

n \ r	0	1	2	3	4	5
2	$\frac{1}{3}$	$\frac{2}{3}$				
3	0	1				
4	$\frac{1}{15}$	$\frac{8}{15}$	$\frac{6}{15}$			
5	0	$\frac{1}{3}$	$\frac{2}{3}$			
6	$\frac{1}{105}$	$\frac{18}{105}$	$\frac{66}{105}$	$\frac{20}{105}$		
7	0	$\frac{1}{12}$	$\frac{6}{12}$	$\frac{5}{12}$		
8	$\frac{1}{819}$	$\frac{32}{819}$	$\frac{276}{819}$	$\frac{440}{819}$	$\frac{70}{819}$	
9	0	$\frac{1}{58}$	$\frac{6}{29}$	$\frac{31}{58}$	$\frac{7}{29}$	
10	$\frac{1}{6633}$	$\frac{50}{6633}$	$\frac{780}{6633}$	$\frac{2960}{6633}$	$\frac{2590}{6633}$	$\frac{252}{6633}$

TABLE II

$I_o = 1$

n \ r	$M_o = 1$: 0	1	2	3	4	$M_o = 0$: 0	1	2	3	4	$M_o = -1$: 0	1	2	3	4
2	0	1				1	0				1	0			
3	0	$\frac{13}{30}$	$\frac{17}{30}$			$\frac{2}{15}$	$\frac{13}{15}$				$\frac{13}{30}$	$\frac{17}{30}$			
4	0	$\frac{1}{5}$	$\frac{4}{5}$			0	$\frac{3}{5}$	$\frac{2}{5}$			$\frac{1}{5}$	$\frac{4}{5}$			
5	0	$\frac{3}{35}$	$\frac{22}{35}$	$\frac{2}{7}$		$\frac{1}{35}$	$\frac{12}{35}$	$\frac{22}{35}$			$\frac{3}{35}$	$\frac{22}{35}$	$\frac{10}{35}$		
6	0	$\frac{1}{28}$	$\frac{3}{7}$	$\frac{15}{28}$		0	$\frac{5}{28}$	$\frac{9}{14}$	$\frac{5}{28}$		$\frac{1}{28}$	$\frac{3}{7}$	$\frac{15}{28}$		
7	0	$\frac{4}{273}$	$\frac{23}{91}$	$\frac{55}{91}$	$\frac{35}{273}$	$\frac{1}{273}$	$\frac{24}{273}$	$\frac{138}{273}$	$\frac{110}{273}$		$\frac{4}{273}$	$\frac{23}{91}$	$\frac{55}{91}$	$\frac{35}{273}$	
8	0	$\frac{1}{174}$	$\frac{4}{29}$	$\frac{31}{58}$	$\frac{28}{87}$	0	$\frac{7}{174}$	$\frac{10}{29}$	$\frac{31}{58}$	$\frac{7}{87}$	$\frac{1}{174}$	$\frac{4}{29}$	$\frac{31}{58}$	$\frac{28}{87}$	

TABLE III

$I_o = 2$

	$M_o = 2$						$M_o = 1$					$M_o = 0$					$M_o = -1$				$M_o = -2$			
n \ r	0	1	2	3	4	5	0	1	2	3	4	0	1	2	3	4	0	1	2	3	0	1	2	3
2	0	0	1				0	1				$\frac{2}{3}$	$\frac{1}{3}$				1				1			
3	0	0	1				0	$\frac{1}{2}$	$\frac{1}{2}$			0	1				$\frac{1}{2}$	$\frac{1}{2}$			1			
4	0	0	$\frac{11}{21}$	$\frac{10}{21}$			0	$\frac{5}{21}$	$\frac{16}{21}$			$\frac{2}{21}$	$\frac{13}{21}$	$\frac{2}{7}$			$\frac{5}{21}$	$\frac{16}{21}$			$\frac{11}{21}$	$\frac{10}{21}$		
5	0	0	$\frac{2}{7}$	$\frac{5}{7}$			0	$\frac{2}{21}$	$\frac{2}{3}$	$\frac{5}{21}$		0	$\frac{8}{21}$	$\frac{13}{21}$			$\frac{2}{21}$	$\frac{2}{3}$	$\frac{5}{21}$		$\frac{6}{21}$	$\frac{5}{7}$		
6	0	0	$\frac{23}{168}$	$\frac{55}{84}$	$\frac{35}{168}$		0	$\frac{1}{24}$	$\frac{19}{42}$	$\frac{85}{168}$		$\frac{1}{84}$	$\frac{11}{56}$	$\frac{9}{14}$	$\frac{25}{168}$		$\frac{7}{168}$	$\frac{19}{42}$	$\frac{85}{168}$		$\frac{23}{168}$	$\frac{55}{84}$	$\frac{5}{24}$	
7	0	0	$\frac{4}{63}$	$\frac{31}{63}$	$\frac{4}{9}$		0	$\frac{1}{126}$	$\frac{17}{126}$	$\frac{19}{63}$	$\frac{5}{9}$	0	$\frac{2}{21}$	$\frac{11}{21}$	$\frac{8}{21}$		$\frac{1}{63}$	$\frac{17}{63}$	$\frac{38}{63}$	$\frac{1}{9}$	$\frac{4}{63}$	$\frac{31}{63}$	$\frac{4}{9}$	
8	0	0	$\frac{13}{462}$	$\frac{74}{231}$	$\frac{37}{66}$	$\frac{1}{11}$	0	$\frac{1}{460}$	$\frac{17}{115}$	$\frac{251}{460}$	$\frac{7}{23}$	$\frac{1}{693}$	$\frac{61}{1386}$	$\frac{83}{231}$	$\frac{727}{1386}$	$\frac{7}{99}$	$\frac{1}{460}$	$\frac{17}{115}$	$\frac{251}{460}$	$\frac{7}{23}$	$\frac{13}{462}$	$\frac{74}{231}$	$\frac{37}{66}$	$\frac{1}{11}$

TABLE IV

IV-a

n \ r	0	1	2	3	4	5
2	0.333	0.667				
3	0	1				
4	0.0667	0.533	0.400			
5	0	0.333	0.667			
6	0.00952	0.171	0.629	0.191		
7	0	0.0833	0.500	0.417		
8	0.00120	0.0391	0.337	0.537	0.0855	
9	0	0.0172	0.207	0.534	0.241	
10	0.000151	0.00754	0.118	0.446	0.391	0.0380
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
21						

IV-b

n \ r	0	1	2	3	4	5	6	7	8	9	10
2	0.333	0.667									
3	0.0	1.00									
4	0.0667	0.533	0.400								
5	0.0	0.333	0.667								
6	0.00939	0.169	0.634	0.188							
7	0.0	0.0875	0.492	0.421							
8	0.00120	0.0385	0.337	0.539	0.0842						
9	0.0	0.0184	0.219	0.508	0.255						
10	0.000149	0.00745	0.117	0.447	0.391	0.0375					
11	0.0	0.00327	0.0719	0.329	0.447	0.149					
12	0.181×10^{-4}	0.130×10^{-2}	0.322×10^{-1}	0.223	0.470	0.258	0.167×10^{-1}				
13	0.0	0.526×10^{-3}	0.190×10^{-1}	0.152	0.388	0.358	0.835×10^{-1}				
14	0.216×10^{-5}	0.212×10^{-3}	0.758×10^{-2}	0.827×10^{-1}	0.318	0.425	0.159	0.743×10^{-2}			
15	0.0	0.793×10^{-4}	0.429×10^{-2}	0.545×10^{-1}	0.234	0.397	0.265	0.453×10^{-1}			
16	0.257×10^{-6}	0.328×10^{-4}	0.160×10^{-2}	0.253×10^{-1}	0.154	0.377	0.345	0.939×10^{-1}	0.330×10^{-2}		
17	0.0	0.114×10^{-4}	0.870×10^{-3}	0.163×10^{-1}	0.108	0.298	0.368	0.185	0.239×10^{-1}		
18	0.302×10^{-7}	0.489×10^{-5}	0.312×10^{-3}	0.673×10^{-2}	0.595×10^{-1}	0.231	0.388	0.259	0.535×10^{-1}	0.147×10^{-2}	
19	0.0	0.159×10^{-5}	0.162×10^{-3}	0.423×10^{-2}	0.408×10^{-1}	0.170	0.335	0.315	0.123	0.124×10^{-1}	
20	0.353×10^{-8}	0.707×10^{-6}	0.571×10^{-4}	0.161×10^{-2}	0.195×10^{-1}	0.110	0.294	0.362	0.184	0.297×10^{-1}	0.653×10^{-3}
21	0.0	0.215×10^{-6}	0.284×10^{-4}	0.989×10^{-3}	0.131×10^{-1}	0.783×10^{-1}	0.229	0.341	0.253	0.785×10^{-1}	0.631×10^{-2}

DISCUSSION

In most of fire ball decay analysis, it is usually assumed that a fire ball has zero isospin. An immediate consequence of the charge independence is that the average number of π^+ mesons is:

$$\langle n_{\pi^+}\rangle \equiv \sum_r r\, P_n(r) = \frac{1}{3}\, n$$

which also leads to:

$$\langle n_{\pi^-}\rangle = \langle n_{\pi^0}\rangle = \frac{1}{3}\, n$$

The most primitive way of obtaining, probabilistically, such a distribution $P_n(r)$, might seem to generate three numbers n_{π^+}, n_{π^-}, n_{π^0} with the condition $n_{\pi^+} + n_{\pi^-} + n_{\pi^0} = n$, associating for each of them, an apriori probability of 1/3, and to select up cases for which $n_{\pi^+} = n_{\pi^-}$. Such a procedure gives rise to the distribution function $P_n(r)$:

$$P_n(r) \propto \frac{n!}{(r!)^2\,(n-2r)!} \qquad (9)$$

However this is not appropriate, since the averages of n_{π^+} or n_{π^-} or n_{π^0} are not equal to (1/3)n, but we have:

$$\langle n_{\pi^+}\rangle = \langle n_{\pi^-}\rangle < \frac{n}{3} < \langle n_{\pi^0}\rangle \qquad (10)$$

There is another unfavourable factor for the simple probabilistic argument when the multiplicity is low. For odd multiplicity we should always have:

$$P_n(0) = 0 \qquad (n \text{ odd}) \qquad (11)$$

which comes from the symmetry of isospin (G-parity), but the probabilistic calculation does not satisfy this condition.

In Monte Carlo calculations of fire-ball decay, we need $P_n(r)$ up to about $n \sim 20$, for which the exact calculation is almost impracticable due to the rapid increase of the number of intermediate states.

Thus we constructed in the case of $I = 0$ an analytic expression for $P_n(r)$ which reproduces all the numbers (of the Table I) within an error of 5%.

The formula is:

$$\text{for } n = \text{even} \begin{cases} P_n(0) = \dfrac{1}{N_n} \\[2ex] P_n(r) = \dfrac{1}{N_n} \dfrac{n!}{2(1-\frac{r}{n})\ (n-2r)!\ (r!)^2} \end{cases} \qquad (12a)$$

where N is the normalization factor, and for n = odd:

$$P_n(r) = \left(1-q_{n-1}(x_1)\right)P_{n-1}(r-1)+q_{n-1}(x)P_{n-1}(r) \qquad (12b)$$

with $\quad x = \dfrac{2r}{n-1}\quad , \quad x_1 = \dfrac{2r-2}{n-1}\quad ,$

$$q_n(x) = x\left((a_n x + b_n)(x-1)+1\right)$$

with $\quad a_n = -0.714 b_n + 0.259$

where b_n's are chosen so that the average of r gives (1/3)n. The values are:

n	b_n
2	1.083
4	1.210
6	2.203
8	2.811
10	3.164
12	3.414
14	3.612
16	3.775
18	3.912
20	4.028

Table IV gives, in IVa, the same numbers as in Table I, and in IVb, for comparison, values of $P_n(r)$ calculated by our formula (12).

CONCLUDING REMARKS

The exact calculation of charge distribution may be useful for analysing the fire-ball decay in the framework of charge-independent hypothesis. For the decay of isospin 0 state formulae(12-a)and (12-b) can be used without causing a serious error.

Analogous formulae might be constructed for non zero isospin case based on Tables II and III.

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BIBLIOGRAPHY

(1) - Japanese and Brazilian Emulsion Chamber Groups, Prog. Theor. Phys. Suppl. nº 47 (1971), 1.

(2) - Y. Sato, T. Yanagita, N. Ogita, Prog. Theor. Phys. Suppl. nº 47 (1971), 214.

(3) - Y. Fujimoto, S. Hasegawa, K. Kasahara, N. Ogita, A.Osawa, T. Shibata, Prog. Theor. Phys. Suppl. nº 47 (1971), 246.

(4) - A. Pais, Ann. of Phys. 9, (1960), 548.

(5) - A. Di Giacomo, N. Cim., vol. 28 A, n. 3 (1975), 383.