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Effective String Dynamics in Large N QCD

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ABSTRACT

The semiclassical 1/N expansion in the strong coupling regime for spinor quarks was developed and the form of effective action was obtained. An extremum of the effective action that arises in the calculation of the hadronic correlation functions in the large N limit corresponds to a topologically non-trivial configuration of the gauge field. This configuration forms a chromoeletric Nambu string with additional spinor terms that contain in particular the Polyakov spinor factor. In the case when real quarks forming hadrons are replaced by scalar particles the above correlators yield the standard dual resonance amplitudes.

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1 Introduction

This research investigation has as scientific goals the development of methods which stay beyond the frame of perturbation theory and allow one to draw up the physical consequences of non-Abelian $SU(N)$ gauge theory when $N \gg 1$ (at the strong coupling regime), clearing the color confinement problems. Concretely, the questions are:

1. Using of the $1/N$ expansion as a semiclassical expansion over parameter $1/N$, which plays, in the large N limit, the same role as does Planck's constant $\hbar$ in ordinary quantum mechanics. The aim is the extraction of the effective string dynamics from QCD;

2. The non-perturbative calculation of the hadronic field correlators, taking into account quark's spin and flavors;

3. Investigation of the consequences which follow from large N QCD as a topological theory and the application of these results to the hadron physics in the frame of QCD.

1.1 Background and Scientific Significance

The structure and interaction of hadrons are defined by the dynamics of their constituent quarks and gluons, which are the more fundamental fragments of matter. The non-Abelian character of the color interactions in QCD leads to growing up of the effective coupling constant at large distances, where the hadrons are formed. This circumstance gives rise to the necessity of development of non-perturbative methods, which allow to fill the gap connected with long standing problems of color confinement and the mechanism of generation of the string-like configurations of the gauge field, which are responsible for confinement. One of such methods is a semiclassical $1/N$ approximation. The effective string dynamics can arise actually in the frame of this approach.

1.2 Preliminary Studies

In recent years I have been working on the topological trend in the semiclassical $1/N$ approximation and effective string dynamics in $SU(N)$ gauge field theory. I have found that, within the scalar quark approximation, the open string binding quarks in a hadron is obtained from non-perturbative calculations of the hadronic field correlators when $N \gg 1$, at the strong coupling regime. This approach differs from others attempting to decide this problem (lattice calculation, loop equations, etc.) and stays nearer to Witten's conception of the master field. A topologically non-trivial configuration of the gauge field, which forms a chromoeletric string, provides the steady extremum of the effective two-dimensional action arising in the calculation of the correlators in the framework of the semiclassical $1/N$ expansion for the 4-dimensional gauge theory. This string is a Nambu string with extra constraints, which restrict the space of quantum states. One of this constraints is a consequence of topological quantization of the chromoeletric flux on the string world sheet. This flux quantization follows from the condition that saddle point solutions of the effective field equations must be single valued functions. This leads to the quantization of the string action.

The partition function of the string is expressed only in terms of the Euler characteristic of the string world sheet. This means that the $SU(N)$ gauge theory reduces to a topological field theory in our approximation. The Weyl anomaly, which is proportional to the Liouville action, is eliminated by the second constraint of the constant scalar curvature R at arbitrary dimension of target space. This string is a non-critical one (i.e. out of dimension $D = 26$).

The procedure for renormalization of the string tension was determined.

The Gell-Mann-Low function for the running coupling constant in the non-perturbative phase of $SU(N)$ gauge theory is identical to the first term in the expansion of the β-function at the strong coupling approximation in the field theory on a lattice. The calculation of the chromoeletric string correlators in the scalar quark approximation yields the Koba-Nielsen formula for the dual resonance amplitude. The above differences are displayed mainly in the calculation of the partition function but, as it turns out, disappear from the expressions for the correlators [1].

1.3 Research Design

The research design is the generalization of the above results to the case of realistic quarks possessing Lorentz spin and flavor degrees of freedom. For realization of this design it is necessary to start using spinning particle propagators, which are defined as a path integral over additional Grassmanian variables describing the quark spin in the presence of the external gauge field. Afterwards, it is necessary to determine the form of the effective spinor action corresponding to non-perturbative phase of QCD at leading order in the parameter $1/N$. The variation of this action will lead to the equations of motion and corresponding boundary conditions for the gauge and quark fields.

Then, it is necessary to find the steady (topologically non-trivial) solution of these equations, which describes a string-like configuration of gauge field binding quarks in the hadron, to substitute this solution into the effective action and to sum over the string world sheets. It will allow to calculate the partition function and correlators of the chromoeletric string binding spinor quarks.

In the next stage one will have to investigate the analytical properties of the scattering amplitudes for hadrons and elucidate the hadron mass spectrum.

In order to reach these aims one suggests to use analytical methods. Then, at the final stage for obtaining of quantitative predictions for interaction cross-sections, invariant mass distributions, etc., and the comparison of these results to the experimental data, it will be necessary to use extensive computer calculations.

2 Connected Part of the Hadronic Field Correlators

To extract the effective string dynamics from QCD, let us consider the $1/N$ non perturbative evaluation of the hadronic field correlator

$$K(1,...n) = \langle M(x_1) M(x_2) ... M(x_n) \rangle \tag{1}$$

in the strong coupling regime

$$N \gg Ne^2 \gg 1, \tag{2}$$

where e is the gauge coupling constant.

$M(x_i)$ is a field operator of the constituent meson (pseudo scalar)

$$M(x_i) = \bar{\psi}^c_{\beta,m}(x_i)(T_{a_i})_{mn}(\gamma_5)_{\beta\alpha}\psi^c_{\alpha,n}. \tag{3}$$

Here $\psi(x)$ is a quark field, T_a is a flavor symmetry group matrix, α, β are the Lorentz spin group indices and c is an index of the color gauge $SU(N)$ group ($c = 1, ..., N$).

The operator (3) is a color singlet. The correlator (1) is given by the following path integral

$$K(1,...,n) = Z^{-1}\int d\mu[A]\,\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{iS_{YM}[A,\psi]}\{M(x_1)...M(x_n)\}, \tag{4}$$

where Z is a normalization constant and

$$S_{YM}[A,\psi] = \int d^4x\left[-\frac{1}{4}G^a_{\mu\nu}(x)G^{a,\mu\nu}(x) + \bar{\psi}^c_\beta(x)\left(i\widehat{\mathcal{D}} - m_0\right)^{cc'}_{\beta\alpha}\psi^{c'}_\alpha(x)\right],$$

$$\widehat{\mathcal{D}}^{cc'}_{\beta\alpha} = (\gamma^\mu)_{\beta\alpha}\mathcal{D}^{cc'}_\mu, \qquad \mathcal{D}^{cc'}_\mu = \left(\partial_\mu - ie\frac{\lambda^a}{2}A^a_\mu\right)^{cc'}. \tag{5}$$

Let us take the integration over $\bar{\psi}$ and ψ. After this, the connected part of (4) is

$$K(1,...,n) = Z^{-1}\sum_{\substack{\text{perm}\\(x_i,x_k)}}(-1)^P\int d\mu[A]\left[\det S_F(A)\right]^{-1}e^{iS_{YM}[A]}\times$$

$$\times\,\mathrm{Tr}\left\{\prod_{i=1}^{n}\left[iT_{a_i}\gamma_5 S_F(x_i,x_{i+1};A)\right]\right\}, \tag{6}$$

where

$$\mathrm{Tr}\left\{\prod_{i=1}^{n}\left[\gamma_5 S_F(x_i,x_{i+1};A)\right]\right\} = \left[(\gamma_5)_{\beta\alpha}S_{F,\alpha\gamma}(x_1,x_2;A)\right]\times$$

$$\times\left[(\gamma_5)_{\gamma\delta}S_{F,\delta\rho}(x_2,x_3;A)\right]...\left[(\gamma_5)_{\sigma\omega}S_{F,\omega\beta}(x_n,x_1;A)\right], \tag{7}$$

the symbol P stands for the parity of the permutation of the fermion fields, and the quark propagator is given by

$$iS^{cc'}_{F,\alpha\beta}(x',x;A) = \left\langle 0\left|T\left(\psi^c_\alpha(x')\bar{\psi}^{c'}_\beta(x)\right)\right|0\right\rangle_A =$$

$$= \left\langle x',\alpha,c\left|i\widehat{S}_F(A)\right|x,\beta,c'\right\rangle, \tag{8}$$

with

$$\widehat{S}_F(A) = \left(i\widehat{\mathcal{D}}(A) - m_0\right)^{-1}, \tag{9}$$

having diagonal structure in the flavor indices (if there is not involved an electroweak field).

Consequently, we have the Chan-Paton factor in (6), $\mathrm{Tr}\{T_{a_1}T_{a_2}...T_{a_n}\}$, where $a_i = 1, ..., 8$ for the $SU(3)$ flavor group.

Formula (7) can be written as a path integral over trajectories in superspace by using a representation for the Fermi particle propagator [2] [3]. In the coherent basis (up to integral terms) we have the following expression:

$$\gamma_5 S_F(x_1, x_2; A) =$$

$$= \int\limits_{x_1=x(\gamma_1)}^{x_2=x(\gamma_2)} \mathcal{D}x_\mu(\gamma)\,\mathcal{D}\psi_\mu(\gamma)\,\mathcal{D}\psi_5(\gamma)\,\mathcal{D}\lambda(\gamma)\,\mathcal{D}\chi(\gamma)\,\mathcal{D}\bar{\xi}_c(\gamma)\,\mathcal{D}\xi_c(\gamma)\,e^{iS}, \tag{10}$$

where (see [3], [4], [5])

$$S[x_\mu, \psi_\mu, \psi_5, \lambda, \chi, \xi_c] = \int\limits_{\gamma_1}^{\gamma_2} d\gamma L = \int\limits_{\gamma_1}^{\gamma_2} d\gamma\,(L_0 + L_m + L_{int}),$$

$$L_0 = -\frac{\lambda}{2}\dot{x}^\mu(\dot{x}_\mu - i\chi\psi_\mu) - \frac{i}{2}\psi^\mu\dot{\psi}_\mu, \tag{11}$$

$$L_m = -\frac{\lambda}{2}m_0^2 + \frac{i}{2}\left(\psi_5\dot{\psi}_5 + m_0\chi\psi_5\right), \tag{12}$$

$$L_{int} = i\bar{\xi}_c\dot{\xi}_c + e\bar{\xi}_c\left(\frac{\lambda^a}{2}\right)_{cc'}\left(A^a_\mu\dot{x}^\mu - \frac{i}{2}\lambda\psi^\mu G^a_{\mu\nu}\psi^\nu\right)\xi_{c'}, \tag{13}$$

and $\dot{x}_\mu = dx_\mu/d\gamma$, etc.

The variables χ, ψ_μ, ψ_5, ξ_c are anticommuting, λ and x_μ are commuting, $\lambda(\gamma)$ is the one-dimensional metric on the trajectory of the particle with spin.

The action S is real, gauge invariant and it is reparameterization invariant if a change of the parameter $\gamma \to \gamma'$ is accompanied by the variable transformations

$$\lambda \to \lambda' = \lambda\frac{d\gamma}{d\gamma'} = \frac{\lambda}{\dot{\gamma}'},$$

$$\chi \to \chi' = \frac{\chi}{\dot{\gamma}'}, \tag{14}$$

with x_μ, ψ_μ, ψ_5, ξ_c unchanged.

In addition, each of the three pieces of L (11), (12), (13) is invariant up to a total γ-derivative under the following infinitesimal supersymmetry transformation generated by an anticommuting variable $\alpha(\gamma)$:

$$\delta x^\nu = i\alpha\psi^\nu,$$

$$\delta\psi^\nu = -\frac{\alpha\left(\dot{x}^\nu - \frac{i}{2}\chi\psi^\nu\right)}{\lambda},$$

$$\delta\lambda = -i\alpha\chi,$$

$$\delta\chi = 2\dot{\alpha},$$

$$\delta\psi_5 = m_0\alpha,$$

$$\delta\xi_c = e\alpha\psi^\mu A^a_\mu \left(\frac{\lambda^a}{2}\right)_{cc'} \xi_{c'}. \tag{15}$$

This invariance is essential for eliminating the non-physical degree of freedom ψ_0.

After discarding internal quark loops, which are $O(1/N)$ (that is, letting $\det S_F \to 1$) we have the following expression for each part in the sum over permutations (6):

$$K(1,...,n) = i^n Z^{-1} \mathrm{Tr}\{T_{a_1}...T_{a_n}\} \times$$
$$\times \int \mathcal{D}A_\mu(x)\,\mathcal{D}x_\mu(\gamma)\,\mathcal{D}\psi_\mu(\gamma)\,\mathcal{D}\psi_5(\gamma)\,\mathcal{D}\lambda(\gamma)\,\mathcal{D}\chi(\gamma)\,\mathcal{D}\bar{\xi}_c(\gamma)\,\mathcal{D}\xi_c(\gamma)\left\{e^{iS}\times\right.,$$
$$\left.\times\, e^{-\bar{\xi}_d(0)\xi_d(0)}\xi_c(1)\,\bar{\xi}_c(0)\right\}, \tag{16}$$

where

$$S\left[A^a_\mu, x_\mu, \psi_\mu, \psi_5, \lambda, \chi, \xi_c\right] = -\frac{1}{4}\int d^4x G^a_{\mu\nu}(x)\, G^{a,\mu\nu}(x) -$$
$$-\oint_\Gamma d\gamma \left[\frac{1}{2}\left(\frac{\dot{x}^2}{\lambda} + \lambda m_0^2\right) - i\bar{\xi}\left(\frac{d}{d\gamma} - ie\hat{A}_\mu\right)\xi\right] +$$
$$+\frac{1}{2}\oint_\Gamma d\gamma\left\{i\frac{\chi\dot{x}_\mu\psi^\mu}{\lambda} - i\psi^\nu\dot{\psi}_\nu + i\left(\psi_5\dot{\psi}_5 + m_0\chi\psi_5\right) -\right.$$
$$\left.-\, ie\lambda\psi^\mu\psi^\nu G^a_{\mu\nu}\bar{\xi}_c\left(\frac{\lambda^a}{2}\right)_{cc'}\xi_{c'}\right\}. \tag{17}$$

In (17) γ is the parameterization of the closed contour Γ $(0 \le \gamma \le 1)$ defined by the closed curve $x_\mu(\gamma)$, $x_\mu(0) = x_\mu(1)$. The x_μ are the coordinates of the quark trajectory, forming a closed contour Γ, passing through the fixed points x_i $(i = 1,...,n)$, $x_i \equiv x(\gamma_i)$, as is shown in Fig.1.

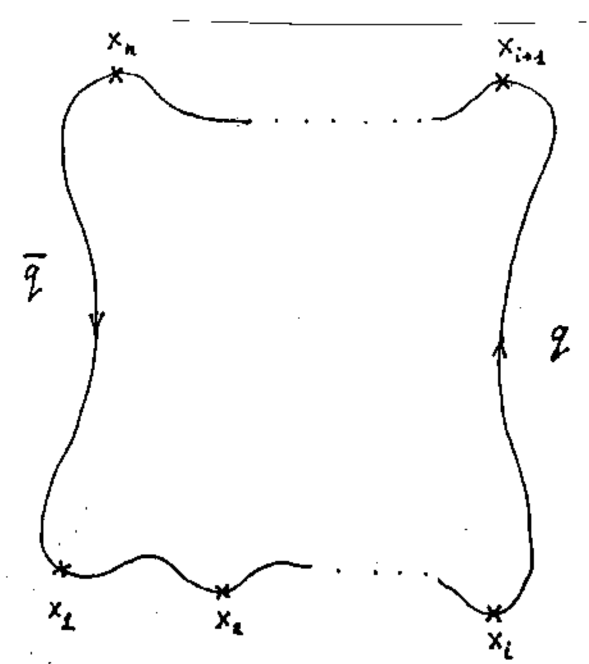

Figure 1: contour Γ passing through $x_i = x(\gamma_i)$

3 The Semiclassical $1/N$ Approximation

The semiclassical $1/N$ expansion is used for the calculation of the correlators (16). According to F. Berezin and L. Yaffe [6] the parameter $1/N$ plays in the large N limit the same role as does the Planck constant $\hbar$ in ordinary semiclassical approximation in quantum mechanics. To leading order in $1/N$ it is necessary to take into account only the contributions from planar (in color space) gluon diagrams. This statement means that is necessary to sum in the path integral (16) only a certain subclass of the gauge fields whose contribution is the most important in the above approximation. We will use topological arguments to look for the above subclass of this field configurations.

For the semiclassical $1/N$ approximation to be applicable it is necessary to have a stable extremal. It is well known that, in non-linear theories the stable field configurations are given by the topologically non-trivial solutions of the classical equations of motion (instantons, monopoles, σ-model solutions, etc.).

On our task, the variation of action (17) $\delta S/\delta\bar{\xi}_c = 0$ leads to the equation of motion for the color spinor fields $\xi_c(\gamma)$,

$$\frac{d\xi_c}{d\gamma} = ie\left(\frac{\lambda^a}{2}\right)_{cd}\xi_d\left(A^a_\mu\dot{x}^\mu - \frac{i}{2}\lambda\psi^\mu G^a_{\mu\nu}\psi^\nu\right). \tag{18}$$

The formal solution of eq. (18) is

$$\hat{\xi}_c(\gamma) = \left[P\exp\left\{ie\int\limits_0^\gamma d\tau\left[A_\mu\dot{x}^\mu - \frac{i}{2}\lambda\psi^\mu\psi^\nu G_{\mu\nu}\right]\right\}\right]_{cd}\xi_d(0), \tag{19}$$

where $\xi_d(0)$ is the initial condition.

The non-abelian value $P\exp\{...A_\mu...\}$ in (19) has the correct behavior under gauge transformations

$$A_\mu(x) \to A^\Omega_\mu(x) = \Omega(x)A_\mu(x)\Omega^{-1}(x) - \frac{i}{e}\left(\partial_\mu\Omega(x)\right)\Omega^{-1}(x),$$

$$G_{\mu\nu}(x) \to \Omega(x)G_{\mu\nu}(x)\Omega^{-1}(x),$$
$$P\exp\{...A_\mu...\} \to P\exp\left\{...A^\Omega_\mu...\right\} = \Omega(x(\gamma))P\exp\{...A_\mu...\}\Omega^{-1}(x(0)).$$

This exponent is an element of $SU(N)$ the color group. It is defined on the contour Γ and maps Γ into the group $SU(N)$. This mapping is trivial for $N \geq 2$ since the homotopical group $\pi_1[SU(N)] = 0$. The single exception is provided by the case of quasi-abelian gauge field in (19). In that case we have a mapping of Γ into the subgroup $U(1)$. The first homotopical group of the mapping $\pi_1[U(1)] = \mathbf{Z}$, where $\mathbf{Z}$ is the group of integers. Moreover, since $\hat{\xi}(\gamma)$ in (19) is a solution of the differential equation (18) we demand that it is single valued on the closed contour Γ. Hence, if we neglect temporarily the (second) term with magnetic moment of the quark in formula (19), we will have the quantization of chromoeletric flux of the quasi-abelian gauge field on the arbitrary surface Σ with boundary $\partial\Sigma = \Gamma$. A specific example of this type was realized in my paper [7]. This quantization of the flux can stabilize the field configuration only if it is the whole flux of field, but not of some random part of it. Since the contour Γ forms a one-dimensional boundary, this requirement can be satisfied only by quasi-two-dimensional fields of the following type

$$A_\mu^a(x(z))\big|_\Sigma \equiv \frac{\partial x_\mu}{\partial z_i} A^{a,i}(z)\bigg|_\Sigma, \tag{20}$$

"living" on surfaces Σ with the boundary $\partial\Sigma = \Gamma$. Here the equation $x_\mu = x_\mu(z^i)$ defines the imbedding Σ into external flat target space ($i = 1, 2$; $a = 1, ..., N^2 - 1$; $\mu = 1, ..., 4$). Thus the topologically non-trivial configurations$\left\{A, \hat{\xi}[A]\right\}$ are realized in the subclass of quasi-two-dimensional fields A only. Consequently, at the semiclassical $1/N$ calculation it is necessary to take into account the contribution of such fields only.

The selection of this subclass of the fields (20) is carried out by inserting under the path integration sign $\int \mathcal{D}A_\mu^a(x) ...$ in the correlator (16) the corresponding projection operator

$$\Pi = \int \mathcal{D}\left[A_\mu^a(x(z))\right]_\Sigma \delta\left\{A_\mu^a(x(z))\big|_\Sigma - A_\mu^a(x)\right\} =$$

$$= \int \mathcal{D}\left[x_\mu(z)\right]_\Sigma \mathcal{D}\left[A^{a,i}(z)\right]_\Sigma J_\Sigma(A(z), x(z))\, \delta\left\{\left[\frac{\partial x_\mu}{\partial z^i} A^{a,i}(z)\right]_\Sigma - A_\mu^a(x)\right\}. \tag{21}$$

Here $J_\Sigma(A, x)$ is the Jacobian of the transformation to the new variables $A^{a,i}(z)$, $x_\mu(z)$. Integration over the initial fields $A_\mu^a(x)$ with the help of the δ-functions, contained in Π, reduces the calculation of the vacuum expectation value in (16) to the integration over two-dimensional field $A^{a,i}(z)|_\Sigma$ for fixed surface Σ and subsequent summation over the surfaces Σ. As a result of the transition to the new fields the two-dimensional Y-M action arises on each surface,

$$S_\Sigma^{eff}[A, \xi, x, \psi, \psi_5, \lambda, \chi] = -\frac{1}{4}\int d^2z \sqrt{-h}\, h^{il} h^{kn} G_{ik}^a G_{ln}^a -$$

$$-\oint_{\partial\Sigma} d\gamma \frac{1}{2}\left(\frac{\dot{x}^2}{\lambda} + \lambda m_0^2\right) + i\oint_{\partial\Sigma} d\gamma \bar{\xi}_c \left(\frac{d}{d\gamma} - i\epsilon A_i \dot{z}^i\right)_{cd} \xi_d +$$

$$+\frac{1}{2}\oint_{\partial\Sigma} d\gamma \left\{ i\frac{\chi}{\lambda}\dot{z}^i\psi_i - i\psi^\mu(\gamma)\dot{\psi}_\mu(\gamma) + i\left(\psi_5\dot{\psi}_5 + m_0\chi\psi_5\right) - \right.$$

$$\left. -i\epsilon\lambda\psi^i\psi^k G_{ik}^a(z(\gamma))\, \bar{\xi}_c \left(\frac{\lambda^a}{2}\right)_{cc'} \xi_{c'} \right\}, \tag{22}$$

where

$$G_{ik}^a = \partial_i A_k^a - \partial_k A_i^a + \epsilon f^{abc} A_i^b A_k^c,$$

$$\psi^i = \psi^\mu e_\mu^i, \quad e_\mu^i = h^{ik} e_{k,\mu} = h^{ik}\frac{\partial x_\mu}{\partial z^k}, \tag{23}$$

with

$$h = \det h_{ik}, \quad h_{ik} = \left(\partial x_\mu / \partial z^i\right)\left(\partial x^\mu / \partial z^k\right) \tag{24}$$

being the induced metric on the world sheet Σ, $\epsilon = e/\delta$, a two-dimensional charge and δ, a ultraviolet regulator with dimension of length (δ has the meaning of a little (but non-vanishing) thickness of the surface Σ).

The spin field ψ^μ can be expanded into tangential and transverse parts,

$$\psi^\mu = \psi_\parallel^\mu + \psi_\perp^\mu, \quad \psi_\parallel^\mu = \sum_{k=1}^{2} e_k^\mu \psi^k, \quad \psi_\perp^\mu = \sum_{k=1}^{2} \left(\psi^\nu n_\nu{}^k\right) n_k{}^\mu, \tag{25}$$

where $n_k{}^\mu e_\mu^i = 0$, $i, k = 1, 2$.

After this, the term $\psi^\mu \dot{\psi}_\mu$ in the action (22) is represented as

$$\psi^\mu \dot{\psi}_\mu = \psi_\parallel^\mu \dot{\psi}_{\parallel\mu} + \psi_\perp^\mu \dot{\psi}_{\perp\mu} + 2\psi_\perp^\mu \dot{\psi}_{\parallel\mu}.$$

The field $\psi_\perp$ has not any interactions and one can take Gaussian integration over it

$$\int \mathcal{D}\psi_\perp \exp\left\{\frac{1}{2}\oint_\Gamma d\gamma \left[\psi_\perp^\mu \dot{\psi}_{\perp\mu} - 2\dot{\psi}_\parallel^\mu \psi_{\perp\mu}\right]\right\} =$$

$$= \text{const} \cdot \exp\left\{-\frac{1}{2}\oint_\Gamma d\gamma \psi_\parallel^\mu \dot{\psi}_{\parallel\mu}\right\},$$

where const $= \det\left(d/d\gamma\right)$.

It results that the kinetic term of the tangential field $\psi_\parallel^\mu \dot{\psi}_{\parallel\mu}$ is cancelled and $\psi_\parallel$ is a non-dynamical field.

Now we have the following effective action on each surface Σ,

$$S_\Sigma^{eff}\left[A, \xi, x, \psi, \psi_5, \lambda, \chi\right] = -\frac{1}{4}\int_\Sigma d^2z\sqrt{-h}h^{il}h^{kn}G_{ik}^a G_{ln}^a -$$

$$-\frac{1}{2}\oint_{\partial\Sigma} d\gamma \left(h_{ik}\frac{\dot{z}^i \dot{z}^k}{\lambda} + \lambda m_0^2\right) + i\oint_{\partial\Sigma} d\gamma \bar{\xi}_c \left(\frac{d}{d\gamma} - i\epsilon A_i \dot{z}^i\right)_{cd} \xi_d +$$

$$+\frac{1}{2}\oint_{\partial\Sigma} d\gamma \left\{i\frac{\chi}{\lambda}\dot{z}^i\psi_i - i\epsilon\lambda\psi^i\psi^k G_{ik}^a \bar{\xi}_c \left(\frac{\lambda^a}{2}\right)_{cc'} \xi_{c'} + i\left(\psi_5 \dot{\psi}_5 + m_0 \chi \psi_5\right)\right\}, \tag{26}$$

where $h_{ik} = \left(\partial x_\mu / \partial z^i\right)\left(\partial x^\mu / \partial z^k\right)$.

4 The Saddle Point Configuration

The variation $\delta S_\Sigma^{eff} / \delta A_i^a = 0$ leads to the equation of motion for the $A^{cl}(z) \equiv \hat{A}$ on the fixed surface Σ,

$$\partial_i \left[\sqrt{-h}G^{a,ik}\right] - \epsilon f^{abc}\sqrt{-h}G^{b,ik}A_i^c = 0, \tag{27}$$

and to the following boundary condition on $\partial\Sigma = \Gamma$:

$$\sqrt{-h}G^{a,ik}\left(z\left(\gamma\right)\right) e_{is}\dot{z}^s = \epsilon T^a\left(\gamma\right)\dot{z}^k - i\epsilon\lambda f^{abc}A_i^b T^c \psi^i \psi^k, \tag{28}$$

which takes into account the presence of quarks on the boundary $\partial\Sigma$. Here e_{si} is the antisymmetric tensor and $T^a(\gamma)$ is the color spin operator of the quark:

$$T^a(\gamma) = \bar{\xi}_c(\gamma)\left(\frac{\lambda^a}{2}\right)_{cd}\xi_d(\gamma), \tag{29}$$

$$\frac{dT^a}{d\gamma} = \epsilon f^{abc} A_i^c T^b \dot{z}^i + i\frac{\epsilon\lambda}{2} f^{abc} G_{ik}^b \psi^i \psi^k T^c, \tag{30}$$

according to the equation (18) for $\xi(\gamma)$.

The equation of motion (27) is fulfilled by a solution

$$\widehat{G}^{a,ik}(z) = \epsilon e^{ik} I^a(z)\left(\sqrt{-h(z)}\right)^{-1}, \tag{31}$$

if the color vector $I^a(z)$ is covariantly constant

$$\mathcal{D}_i^{ab} I^b(z) = \frac{\partial I^a}{\partial z^i} - \epsilon f^{abc} I^b A_i^c = 0, \tag{32}$$

and if the following boundary condition

$$I^a(z)\big|_{z\in\partial\Sigma} = T^a(\gamma), \tag{33}$$

holds on $\partial\Sigma$.

In this case, the vector $T^a(\gamma)$ will be also covariantly constant because the term with $f^{abc}G_{ik}^b\psi^i\psi^k T^c$ is cancelled and

$$\frac{dT^a}{d\gamma} = \epsilon f^{abc} A_i^c T^b \dot{z}^i, \tag{34}$$

$$T^2 = T^a T^a = C_2(F) = \frac{N^2-1}{2N}. \tag{35}$$

The potential $\hat{A}_i^a$ corresponding to (31) in the particular gauge in which

$$I^a(z) = \text{const.}, \tag{36}$$

has a form

$$\hat{A}_i^a(z) = \frac{I^a a_i(z)}{\epsilon}, \tag{37}$$

where $a_i(z)$ is an abelian two-dimensional field. In this case, the boundary condition (28) is fulfilled also. After substitution of the self consistent saddle point configuration of the fields $\left\{\hat{A},\hat{\xi}\right\}$ we have the following expression for the action

$$S_\Sigma^{eff}\left[\hat{A},\hat{\xi},...\right] = k_0\int_\Sigma d^2z\sqrt{-h} - \frac{1}{2}\oint_{\partial\Sigma} d\gamma\left(h_{ik}\frac{\dot{z}^i\dot{z}^k}{\lambda} + \lambda m_0^2\right) +$$

$$+\frac{1}{2}\oint_{\partial\Sigma} d\gamma\left\{i\frac{\chi}{\lambda}\dot{z}^i\psi_i + i\left(\psi_5\dot{\psi}_5 + m_0\chi\psi_5\right)\right\}, \tag{38}$$

where

$$k_0 = \frac{e^2}{2\delta^2}\left(\frac{N^2-1}{2N}\right), \quad e^2 = \frac{e_0^2}{N}, \quad \frac{e}{\delta} \equiv \epsilon. \tag{39}$$

Now the correlator has a form

$$K(1,...,n) \approx i^n Z^{-1}\mathrm{Tr}\{T_{a_1}...T_{a_n}\} \times$$

$$\times \int \mathcal{D}x_\mu(z)\,\mathcal{D}x_\mu(\gamma)\,\mathcal{D}\psi_{\|}(\gamma)\,\mathcal{D}\psi_5(\gamma)\,\mathcal{D}\lambda(\gamma)\,\mathcal{D}\chi(\gamma)\,\mathcal{D}\bar{\xi}(0)\,\mathcal{D}\xi(0) \times \{\xi_c(1)\,\bar{\xi}_c(0)$$

$$\times\, e^{iS_\Sigma^{eff}[x_\mu,\chi,\lambda,\psi,\psi_5]-\bar{\xi}_d(0)\xi_d(0)}\Big\}. \tag{40}$$

Here x_μ enters in S^{eff} through the induced metric $h_{ik}[x]$,

$$\psi_{\|}^\mu = \psi^i e_i^\mu, \quad \mathcal{D}\psi_{\|}^\mu = \mathcal{D}\psi^i\left(\det e_i^\mu\right)^{-1} = \sqrt{-h}\mathcal{D}\psi^i. \tag{41}$$

Then, let us integrate over ψ_5 and χ. After this K will be as

$$K(1,...,n) \approx i^n Z^{-1}\mathrm{Tr}\{T_{a_1}...T_{a_n}\} \times$$

$$\times \int \mathcal{D}x_\mu(z)\,\mathcal{D}x_\mu(\gamma)\,\mathcal{D}\psi_{\|}(\gamma)\,\mathcal{D}\lambda(\gamma)\,\mathcal{D}\bar{\xi}(0)\,\mathcal{D}\xi(0) \times \{\xi_c(1)\,\bar{\xi}_c(0)$$

$$\times\, e^{iS_\Sigma^{eff}[x,\lambda,\psi]-\bar{\xi}_d(0)\xi_d(0)}\Big\},$$

$$S_\Sigma^{eff}[x,\lambda,\psi] = k_0\int_\Sigma d^2z\sqrt{-h} - \frac{1}{2}\oint_{\partial\Sigma} d\gamma\left(\frac{\dot{x}^2}{\lambda}+\lambda m_0^2\right) +$$

$$+\frac{i}{2m_0^2}\oint_{\partial\Sigma} d\gamma\left(\frac{\dot{z}^k\psi_k}{\lambda}\right)\frac{d}{d\gamma}\left(\frac{\dot{z}^i\psi_i}{\lambda}\right). \tag{42}$$

5 The Solution of Equation (18)

Let us consider the solution of equation (18) by taking into account the saddle point configuration $\hat{A}$ (37). We have

$$\frac{d\xi_c}{d\gamma} = i\epsilon\left(\frac{\lambda^a}{2}\right)_{cd}\xi_d\left(\hat{A}_i\dot{z}^i - i\lambda\psi^k\psi^i\widehat{G}_{ki}^a\right) =$$

$$= i\left(\frac{\lambda^a}{2}\right)_{cd}\xi_d I^a a_i(z(\gamma))\,\dot{z}^i + \epsilon^2\lambda\psi^k\psi^i\left(\frac{\lambda^a}{2}\right)_{cd}\xi_d e_{ki} I^a\sqrt{-h}. \tag{43}$$

The color vector $I^a(z)$ at the boundary $z \in \partial\Sigma$ equals the color quark spin $T^a(\gamma)$ (see equation (33)), that is, becomes the operator in the space of color functions $\xi_c(\gamma)$. The action of such operators is given by the formula [8]

$$Af(\xi) = \int\prod_i d\zeta_i d\bar{\zeta}_i A\left(\bar{\xi},\zeta\right) f\left(\bar{\zeta}\right) e^{\sum_i(\bar{\xi}-\bar{\zeta})_i\zeta_i}, \tag{44}$$

where $A\left(\bar{\xi},\zeta\right)$ is the normal symbol of the operator A,

$$A\left(\bar{\xi},\zeta\right)=\frac{\langle\xi\left|A\right|\zeta\rangle}{\langle\xi|\zeta\rangle}.$$

The application of (44) to the function $f\left(\xi\right)=\xi$ gives

$$\left(\frac{\lambda^{a}}{2}\right)_{cd}T^{a}\xi_{d}=C_{2}\left(F\right)\xi_{c}, \tag{45}$$

that is

$$\frac{d\xi_{c}}{d\gamma}=i\left[a_{i}\dot{z}^{i}-i\epsilon^{2}\lambda\psi^{k}\psi^{i}e_{ki}\sqrt{-h}\right]C_{2}\left(F\right)\xi_{c}. \tag{46}$$

The solution is

$$\hat{\xi}_{c}\left(\gamma\right)=e^{i\phi(\gamma)}\xi_{c}\left(0\right), \tag{47}$$

where the phase $\phi\left(\gamma\right)$ equals

$$\phi\left(\gamma\right)=C_{2}\left(F\right)\int_{0}^{\gamma}d\gamma'\left[\frac{dz^{i}}{d\gamma'}a_{i}-i\lambda\epsilon^{2}\psi^{k}\psi^{i}e_{ki}\sqrt{-h}\right]. \tag{48}$$

The first integral in (48) is

$$\int_{0}^{1}d\gamma\frac{dz^{i}}{d\gamma}a_{i}=\oint_{\partial\Sigma}dz^{i}a_{i}=\frac{1}{2}\int_{\Sigma}d^{2}z\sqrt{-h}F_{ik}\frac{e^{ik}}{\sqrt{-h}}=$$

$$=\int_{\Sigma}d^{2}z\sqrt{-h}\left({}^{*}F\right)=\epsilon^{2}\int_{\Sigma}d^{2}z\sqrt{-h}, \tag{49}$$

according to formulae (31) and (37).

The condition of single valuedness for the phase $\phi\left(\gamma\right)$ gives the following result

$$\phi\left(1\right)=2\pi Q,\quad Q=\pm1,\pm2,.... \tag{50}$$

¿From this we have the first constraint

$$\pi\left|Q\right|=k_{0}\left[\int_{\Sigma}d^{2}z\sqrt{-h}-i\oint_{\partial\Sigma}d\gamma\lambda\psi^{k}\psi^{i}e_{ki}\sqrt{-h}\right]. \tag{51}$$

The second constraint insures the constancy condition of the scalar curvature of the world sheet Σ,

$$R=\text{const.}\sim\epsilon^{2}. \tag{52}$$

This condition is needed for the topological non-triviality ($Q\neq0$) and provides also suppression of the contribution from Gaussian fluctuations of the gauge field by a factor $1/N$ [9].

6 Path Integral for the Correlator

Let us continue the evaluation of the correlator K. Integration over $\xi(0)$, $\bar{\xi}(0)$ in the formula (32) gives a factor N (number of colors) in the expression

$$K(1,...,n) = i^n N Z^{-1} \mathrm{Tr}\{T_{a_1}...T_{a_n}\} \times$$

$$\times \int \mathcal{D}x_\mu(z)\,\mathcal{D}x_\mu(\gamma)\,\mathcal{D}\psi_i(\gamma)\,\mathcal{D}\lambda(\gamma)\exp\{iS + \text{constraints}\}. \tag{53}$$

According to the character of the saddle point configuration of the fields, the contour Γ should be viewed as the boundary $\Gamma = \partial\Sigma$. To take this circumstance into account one should introduce into the integral (53) the δ-function $\delta[x_\mu(\gamma) - x_\mu(z(\gamma))]$ and integrate with its help over $x_\mu(\gamma)$. As a result we obtain

$$K(1,...,n) = i^n N Z^{-1} \mathrm{Tr}\{T_{a_1}...T_{a_n}\} \times$$

$$\times \int \mathcal{D}x_\mu(z)\,\mathcal{D}\psi_i(\gamma)\,\mathcal{D}\lambda(\gamma)\,e^{iS^{eff}}, \tag{54}$$

where the integral over $x_\mu(z)$ is extended also on the second term in the action (42), but with the points x_k on $\partial\Sigma$ held fixed. Consequently, the last expression may be written in the form

$$K(x_1,...,x_k,...,x_n) = \left\langle \oint_\Gamma \prod_{k=1}^{n} d\gamma_k \lambda(\gamma_k)\,\delta(x_k - x(z(\gamma_k))) \right\rangle, \tag{55}$$

Going over to the momentum representation we obtain

$$K(p_1,...,p_k,...,p_n) = \left\langle \oint_\Gamma \prod_{k=1}^{n} d\gamma_k \lambda(\gamma_k)\,\frac{e^{ip_k\cdot x(z(\gamma_k))}}{(2\pi)^{4/2}} \right\rangle, \tag{56}$$

where the averaging operation is defined by an integral of the type (54), but with points no longer held fixed.

In Euclidean space we have

$$K(p_1,...,p_n) \approx \int \mathcal{D}x_\mu(z)\,\mathcal{D}\psi_i(\gamma)\,\mathcal{D}\lambda(\gamma)\exp\{-S + \text{constraints}\} \times$$

$$\times \frac{1}{n!}\prod_{k=1}^{n}\left[\oint d\gamma_k \lambda(\gamma_k)\,\frac{e^{-ip_k\cdot x(z(\gamma_k))}}{(2\pi)^2}\right], \tag{57}$$

where the Euclidean action is

$$S = k_0 \int_\Sigma d^2z\sqrt{h} + \frac{1}{2}\oint_{\partial\Sigma} d\gamma\left(\frac{\dot{x}^2}{\lambda} + \lambda m_0^2\right) +$$

$$+ \frac{i}{2m_0^2}\oint_{\partial\Sigma} d\gamma \left(\frac{\dot{z}^k\psi_k}{\lambda}\right)\frac{d}{d\gamma}\left(\frac{\dot{z}^i\psi_i}{\lambda}\right). \tag{58}$$

The factor $n!$ in the above expression is introduced to take into account that the interacting particles are identical.

In order to evaluate the correlator (57) it is necessary to learn first of all how to calculate the simpler expression for the partition function

$$Z = \int \mathcal{D}x_\mu (z)\, \mathcal{D}\psi_i (\gamma)\, \mathcal{D}\lambda (\gamma) \exp\left\{-S^{eff}\right\}. \tag{59}$$

In Euclidean space, the quantization condition (51) has a form

$$\pi |Q| = k_0 \left[\int\limits_\Sigma d^2 z\sqrt{h} - i \oint\limits_{\partial\Sigma} d\gamma \lambda \psi^k \psi^i e_{ki} \sqrt{h} \right]. \tag{60}$$

As a consequence of this condition, the full partition function (59) breaks up into a sum of contributions from various topological sectors:

$$Z = \sum_Q Z_{|Q|}, \quad Z_{|Q|} = Z_{Q^+} + Z_{Q^-}. \tag{61}$$

The calculation of $Z_{|Q|}$ is made with the help of the relation [1]

$$\int \mathcal{D}x_\mu \exp\left\{ -k_0 \int\limits_\Sigma d^2 z \sqrt{h} \right\} \doteq$$

$$\int \mathcal{D}x_\mu \mathcal{D}g_{ab} \exp\left\{ -\frac{k_0}{2} \int\limits_\Sigma d^2 z \sqrt{g} g^{ab} \partial_a x_\mu \partial_b x_\mu \right\}, \tag{62}$$

which is correct only in the leading order of saddle point approximation. The simbol $\doteq$ indicates *ad difinitio* that the integral over inner metric g_{ab} is equal to the integrand evaluated at the saddle point.

Now

$$Z_{Q^\pm} \doteq \int \mathcal{D}x_\mu (z)\, \mathcal{D}g_{ab} (z)\, \mathcal{D}\lambda (\gamma)\, \mathcal{D}\psi_i (\gamma) \exp\left\{ -\frac{k_0}{2} \int\limits_\Sigma d^2 z \sqrt{g} g^{ab} \partial_a x_\mu \partial_b x_\mu - \right.$$

$$-\frac{1}{2} \oint\limits_{\partial\Sigma} d\gamma \left(\frac{\dot{x}^2}{\lambda} + \lambda m_0^2 \right) - \frac{1}{2m_0^2} \oint\limits_{\partial\Sigma} d\gamma \left(\frac{\dot{z}^k \psi_k}{\lambda} \right) \frac{d}{d\gamma} \left(\frac{\dot{z}^i \psi_i}{\lambda} \right) +$$

$$\left. \alpha f_1^\pm [g] + \beta f_2 [g] \right\}, \tag{63}$$

where $f_1^\pm$ and f_2 are the constraints that take into account the additional conditions (52) and (60) and α, β are Lagrange multipliers.

After introduction into the integral over $\lambda(\gamma)$ of a δ-function [1], which takes into account the condition of agreement between the metric on Σ and the metric on $\partial\Sigma$, $\delta \left[\lambda (\gamma) - \left(g_{ab} \dot{z}^a \dot{z}^b \right)^{1/2} / m_0 \right]$, we obtain

$$Z_{Q^\pm} \doteq \int \mathcal{D}x_\mu (z)\, \mathcal{D}g_{ab} (z)\, \mathcal{D}\psi_{\|\mu} (s) \exp\left\{ -\frac{k_0}{2} \int\limits_\Sigma d^2 z \sqrt{g} g^{ab} \partial_a x_\mu \partial_b x_\mu - \right.$$

$$-\frac{m_0}{2}\oint\limits_{\partial\Sigma} ds\dot{x}^2 - \frac{m_0}{2}\oint\limits_{\partial\Sigma} ds - \frac{1}{2}\oint\limits_{\partial\Sigma} ds\left(\dot{x}_\mu\psi_{\|\mu}\right)\frac{d}{ds}\left(\dot{x}_\nu\psi_{\|\nu}\right) +$$

$$+\alpha^{\pm}\left(\pm\pi Q - k_0\int\limits_{\Sigma} d^2z\sqrt{g} + \frac{ik_0}{m_0}\oint\limits_{\partial\Sigma} ds\psi_{\|\mu}\psi_{\|\nu}\partial_a x_\mu\partial_b x_\nu\frac{e^{ab}}{\sqrt{g}}\right) +$$

$$+\beta\left(\oint\limits_{\partial\Sigma} ds\mathcal{H}_g + \frac{ik_0}{m_0}\oint\limits_{\partial\Sigma} ds\psi_{\|\mu}\psi_{\|\nu}\partial_a x_\mu\partial_b x_\nu\frac{e^{ab}}{\sqrt{g}} - q\right)\Bigg\}, \tag{64}$$

where

$$ds = \left(g_{ab}\dot{z}^a\dot{z}^b\right)^{1/2} d\gamma, \quad \dot{x}_\mu = \frac{dx_\mu}{ds},$$

and $\mathcal{H}_g$ is the geodesic curvature of the boundary $\partial\Sigma$ [1].

The term

$$\exp\left\{-\frac{1}{2}\oint\limits_{\partial\Sigma} ds\left(\dot{x}_\mu\psi_{\|\mu}\right)\frac{d}{ds}\left(\dot{x}_\nu\psi_{\|\nu}\right)\right\}, \tag{65}$$

generates the well known spin factor [10], but there are another terms, which contain the spinor field ψ. The calculation of the integral over ψ in formula (64) demands further investigation.

If the Lorentz spin of the quarks is neglected we have the more simple expression for the action

$$S^{\pm}_{\Sigma} = -\frac{k_0}{2}\int\limits_{\Sigma} d^2z\sqrt{g}g^{ab}\partial_a x_\mu\partial_b x_\mu - \frac{m_0}{2}\oint\limits_{\partial\Sigma} ds\left(\dot{x}^2 - 1\right) +$$

$$+\alpha^{\pm}\left(\pm\pi Q - k_0\int\limits_{\Sigma} d^2z\sqrt{g}\right) + +\beta\left(\oint\limits_{\partial\Sigma} ds\mathcal{H}_g - q\right). \tag{66}$$

In this case, the partition function Z_Q is expressed only through the Euler characteristic χ of the world sheet Σ (see [1]). This means that the $SU(N)$ gauge theory reduces to a topological field theory in our approximation.

Despite the above differences of this approach from the standard string model, the final calculation of the correlators in the scalar quark approximation yields the Koba-Nielsen formula for the dual resonance amplitudes [1].

The further taking into account of the contribution of Grassmanian field ψ_μ in formula (64) can lead to the shift of the mass spectrum and cure the tachyon problem in the string theory.

7 Conclusion

In the strong coupling approximation for spinor quarks a semiclassical $1/N$ expansion has been developed and the form of the effective spinor action has been obtained. The topologically non-trivial configuration of the gauge field that forms the chromoeletric string and binds the spinor quarks in the hadron has been found. The substitution of this field into the action produces the string action with additional spinor terms which contain in particular the Polyakov spinor factor. The other spinor term probably corresponds to Thomas interaction of the spin in a non-inertial frame.

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