Nonlinear electrodynamics and the Pioneer 10/11 spacecraft anomaly

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Abstract. – The occurrence of the phenomenon known as photon acceleration is a natural prediction of nonlinear electrodynamics (NLED). This would appear as an anomalous frequency shift in any modelization of the electromagnetic field that only takes into account the classical Maxwell theory. Thus, it is tempting to address the unresolved anomalous, steady, but time-dependent, blueshift of the Pioneer 10/11 spacecrafts within the framework of NLED. Here we show that astrophysical data on the strength of the magnetic field in both the Galaxy and the local (super)cluster of galaxies support the view on the major Pioneer anomaly as a consequence of the phenomenon of photon acceleration. If confirmed, through further observations or lab experiments, the reality of this phenomenon should prompt to take it into account in any forthcoming research on both cosmological evolution and origin and dynamical effects of primordial magnetic fields, whose seeds are estimated to be very weak.

The Pioneer 10/11 spacecraft anomaly. – Since 1998, Anderson *et al.* have continuously reported an anomalous frequency shift derived from about ten years study of radio-metric data from Pioneer 10: 03/01/1987-22/07/1998, Pioneer 11: 05/01/1987-01/10/1990, and of Ulysses and Galileo spacecrafts [1]. The observed effect mimics a constant acceleration acting on the spacecraft with magnitude $a_P = (8.74 \pm 1.33) \times 10^{-8}$ cm s^{-2} and a steady frequency drift $\frac{d\Delta\nu}{dt} \simeq 6 \times 10^{-9}$ Hz/s which equates to a "clock acceleration": $\frac{d\Delta\nu}{dt} = \frac{a_P}{c}\, \nu$ (‡), where t is the one way signal travel time. An independent analysis for the period 1987 - 1994 confirms the previous observations [2]. In addition, by removing the spin-rate change contribution yields an apparent anomalous acceleration $a_P = (7.84 \pm 0.01)\times\ 10^{-8}$ cm s^{-2}, of the same amount for both Pioneer 10/11 [3, 4]. Besides, it has been noted that the magnitude of a_P compares nicely to cH_0, where H_0 is the Hubble parameter today.

Unlike other spacecrafts as the Voyagers and Cassini which are three-axis stabilized (hence, not well-suited for a precise reconstitution of trajectory because of numerous attitude controls), the Pioneer 10/11, Ulysses and the by-now destroyed Galileo are attitude-stabilized by spinning about an axis (parallel to the axis of the high-gain antenna) which permits precise acceleration estimations to the level of 10^{-8} cm s^{-2} (single measurement accuracy averaged over 5 days). Besides, because of the proximity of Ulysses and Galileo to the Sun, the data from both spacecrafts were strongly correlated to the solar radiation pressure unlike the data from the remote Pioneer 10/11. Let us point out that the motions of the four spacecrafts are modelled by general relativistic equations (see [3], section IV) including the perturbations from heavenly bodies as small as the large main-belt asteroids (the Sun, the Moon and the nine planets are treated as point masses). Proposals for dedicated missions to test the Pioneer anomaly are now under consideration [5]. In search for a possible origin of the anomalous blueshift, a number of gravitational and non-gravitational potential causes have been ruled out by Anderson *et al* [3]. According to the authors, none of these effects may explain a_P and some are 3 orders of magnitude or more too small. The addition of a Yukawa force to the Newtonian does not work easily. An additional acceleration is predicted by taking into account the Solar quadrupole moment [6]. Although this entails a blueshift, it decreases like the inverse of the power four of the heliocentric radius, being of the order of a_P only below 2.1 AU. Meanwhile, the claim that the Modified Newtonian Dynamics (MOND) may explain a_P in the strongly Newtonian limit of MOND [7,8] is not obvious at all. Therefore, the alternative that the Pioneer anomaly does not result from a real change in velocity (see [3], section X) deserves to be investigated.

Indeed, a direct interpretation of the observational data from the spacecrafts implies merely an anomalous time-dependent blueshift of the photons of the communication signals. On the other hand, in using a time dependent potential [10, 11] to explain the Pioneer 10/11 data one may be pointing out to the need of an effective metric for the photons. In fact, what is needed is just a time variation of the 4-momentum of the photon along its path. Thus the atomic energy levels would not be affected, only the motion of the photon being concerned.

In summary, prosaic explanations, non-gravitational forces and modified dynamics or new interaction (long or short range) force terms do not work [6–9]. Gravitational origin of the anomaly is rouled out by the precision of the planetary ephemeris (see Anderson et al. [1], Iorio [12], and others [13]) and the known bounds on dark matter within the orbital radius of Uranus or Neptune [14]. Hence, the Pioneer anomaly seems not to be related to the gravitational [1, 12, 13], but rather to the EM sector (since these two are the only long range interactions known today). Non-metric fields can also be regarded as gravitational fields and there is a lot of space for speculation. The possibility of an interaction of the EM signal with the solar wind leading to a change of the frequency of the EM signal is now rouled out (see Anderson et al. [3]). It is clearly the equation of motion of the photon that is concerned, that is, what happens to the photon during its propagation from the Pioneer 10/11 antennas to the receivers on Earth. Now, classical (Maxwell theory) or quantized (QED) linear electrodynamics does not allow for a change of the frequency of a photon during its propagation in a linear medium without invoking diffusion due to the interaction with the surrounding matter (hence a smear out of the image of the source). Indeed, for such a phenomenon to occur, one needs to consider a general Lagrangian density $L = L(F)$ for which its second derivative w.r.t. F: $d^2L/dF^2 = L_{FF} \neq 0$. Therefore, the Pioneer anomaly, if not an artifact, may be a result of NLED as we show below. Indeed, relation (‡) above translates in covariant notation into $\frac{dx^\nu}{dl}\ \nabla_\nu\ k^\mu = \frac{a_P}{c^2}\ k^\mu$, where l is some affine parameter along a ray defined by $k^\mu = \frac{dx^\mu}{dl}$ (see [17]). The latter equation departs from the classical electrodynamics one $\frac{dx^\nu}{dl}\, \nabla_\nu\, k^\mu = 0$ (see [29], section 87) and suggests the NLED effect dubbed photon acceleration. The concept of photon acceleration, which follows from the description of photon propagation in NLED, was introduced by Ref. [15]. We explain next why the anomaly shows up in some situations and not others. For experimental tests of NLED and further theoretical predictions see [19].

NLED and A Lagrangian for All Scales: From Cosmology to the Solar System. – Indeed, all these requirements are achieved by considering NLED based on a Lagrangian density $L(F)$ that includes terms depending non-linearly on the invariant $F = F_{\mu\nu}\ F^{\mu\nu}, F = 2(B^2c^2 - E^2)$ [15,16,18], instead of the usual Lagrangian density $L = -\frac{1}{4}F$ of the classical electromagnetism in a vacuum. Hereafter we investigate the effects of nonlinearities in the evolution of EM waves, described onwards as the surface of discontinuity of the EM field. Extremizing the Lagrangian with respect to the potentials A_μ yields the following field equation [18]:

$$\nabla_\nu (L_F F^{\mu\nu}) = 0, \tag{1}$$

where ∇_ν defines the covariant derivative, and $L_F = dL/dF$. Besides this, we have the cyclic identity:

$$\nabla_\nu F^{*\mu\nu} = 0 \quad \Leftrightarrow \quad F_{\mu\nu|\alpha} + F_{\alpha\mu|\nu} + F_{\nu\alpha|\mu} = 0\,. \tag{2}$$

Taking the discontinuities of the field equation yields [20]

$$L_F f_\lambda{}^\mu k^\lambda + 2L_{FF} F^{\alpha\beta} f_{\alpha\beta} F^{\mu\lambda} k_\lambda = 0\,. \tag{3}$$

The discontinuity of the Bianchi identity renders:

$$f_{\alpha\beta}k_\gamma + f_{\gamma\alpha}k_\beta + f_{\beta\gamma}k_\alpha = 0. \tag{4}$$

To obtain a scalar relation, we contract Eq.(4) with $k^\gamma F^{\alpha\beta}$, resulting

$$(F^{\alpha\beta} f_{\alpha\beta} g^{\mu\nu} + 2F^{\mu\lambda} f_\lambda{}^\nu) k_\mu k_\nu = 0\,. \tag{5}$$

We have two distinct cases: $F^{\alpha\beta} f_{\alpha\beta} = \chi$, or 0. If it is zero, such a mode propagates along standard null geodesics. When it is χ, we obtain, from Eqs.(3) and (5), the propagation equation for the field discontinuities

$$\left(g^{\mu\nu} - 4\frac{L_{FF}}{L_F} F^{\mu\alpha} F_\alpha{}^\nu \right) k_\mu k_\nu = 0\,. \tag{6}$$

Taking the derivative of this expression, we obtain

$$k^\nu \nabla_\nu k_\alpha = 4 \left(\frac{L_{FF}}{L_F} F^{\mu\beta} F_\beta{}^\nu\ k_\mu k_\nu \right)_{|\alpha}. \tag{7}$$

Eq.(7) shows that the nonlinear Lagrangian introduces a term acting as a force accelerating the photon.

Photon acceleration in NLED. – If NLED is to play a significant role at the macroscopic scale, this should occur at the intermediary scales of clusters of galaxies or the interclusters medium, wherein most observations show that the magnetic fields are almost uniform (and of the same order of magnitude [22, 23]), unlike the dipolar magnetic fields of the Sun and planets. However, galaxies are gravitationally bound systems, whereas the cosmic expansion is acting at the cluster of galaxies scale. Thus, the magnetic field ($\mathbf{B}$) in clusters of galaxies

(IGMF) depends on the cosmic time. So, the **B** that is relevant to this study is that of the local cluster of galaxies [24]. (As for the contribution of the CMB radiation see [30]). Recently, Vallée [25] has speculated that the 2 μG magnetic field he has observed within the local supercluster of galaxies in cells of sizes of about 100 kpc may extend all the way to the Sun. We explore further this idea in the framework of NLED and show that it is capable to provide an explanation of the Pioneer anomaly from first principles.

Relation (6) may be casted in the form

$$g_{\mu\nu}k^{\mu}k^{\nu} = 4\frac{L_{FF}}{L_F}b^2, \tag{8}$$

where $b^{\mu} = F^{\mu\nu}k_{\nu}$ and $b^2 = b^{\mu}b_{\mu}$. As $E = 0$, one can write, after averaging over the angular-dependence [26]: $b^2 = -\frac{1}{2}||\vec{k}||^2 B^2 c^2 = -\frac{1}{4}||\vec{k}||^2 F$, with $||\vec{k}|| = \omega/c = 2\pi\nu/c$. By inserting this relation in (8) yields

$$g_{\mu\nu}k^{\mu}k^{\nu} = -\frac{\omega^2}{c^2}\,F\,\frac{L_{FF}}{L_F}. \tag{9}$$

Taking the x^{α} derivative of Eq.(9) we obtain

$$2g_{\mu\nu}k^{\mu}(k^{\nu})_{|\alpha} + k^{\mu}k^{\nu}(g_{\mu\nu})_{|\alpha} = -(\frac{\omega^2}{c^2}\,F\,\frac{L_{FF}}{L_F})_{|\alpha}. \tag{10}$$

The cosmological expansion will be represented by $g_{\mu\nu} = a^2(\eta)g^{(local)}_{\mu\nu}$, with a the scale factor, η the conformal time, and $g^{(local)}_{\mu\nu}$ the local metric. So, Eq.(10) yields: $2g_{\mu\nu}k^{\mu}(k^{\nu})_{|0} + 2\frac{\dot{a}}{a}\,g_{\mu\nu}k^{\mu}k^{\nu} = -\,(\frac{\omega^2}{c^2}\,F\,\frac{L_{FF}}{L_F})_{|0}$ ($\star$), where the dot stands for partial derivative w.r.t. η. Using Eqs.(9) and ($\star$) we obtain([1])

$$k_{\mu}(k^{\mu})_{|0} = \frac{\dot{a}}{a}\,\frac{\omega^2}{c^2}\,F\,\frac{L_{FF}}{L_F} - \frac{1}{2}\,(\frac{\omega^2}{c^2}\,F\,\frac{L_{FF}}{L_F})_{|0}. \tag{11}$$

Now, $\dot{F} = -\,4\,\frac{\dot{a}}{a}\,F$, by recalling that $B^2 \propto a^{-4}$. Moreover, from the method of the effective metric, it can be shown that k^0 does not vary with time in the first order approximation [27] unlike $||\vec{k}||$. Hence $k_{\mu}(k^{\mu})_{|0} = -\,\frac{\omega}{c}\,\frac{\dot{\omega}}{c}$ ($\star\star$). By inserting relation ($\star\star$) in (11), and then expanding and arranging, one finds

$$\frac{\dot{\nu}}{\nu} = -\,\frac{\dot{a}}{a}\,\frac{Q + 2FQ_F}{1 - Q}. \tag{12}$$

where we have set $Q = F\,\frac{L_{FF}}{L_F}$ and $Q_F = \partial Q/\partial F$.

At present cosmological time (t) and for a duration very short as compared to the universe age, Eq.(12) reduces to $\frac{\dot{\nu}}{\nu} \simeq -\,H_0\,\frac{Q+2FQ_F}{1-Q}$ ($\dot{\nu}$ is the photon frequency t-derivative). $\dot{\nu} \neq 0$ if and only if a) the NLED contribution is non-null, i.e., $L_{\rm FF} \neq 0$, and b) F depends on time.

The NLED Lagrangian. – The explicit form of this general nonlinear Lagrangian (which simulates the effect of dark energy in Ref. [28]) reads

$$L = -\frac{1}{4}F + \frac{\gamma}{F}\,, \quad \text{or} \quad L = -\frac{1}{4}F + \frac{\gamma_n}{F^n}\,, \tag{13}$$

([1])By removing the NLED extraterm from Eq.(8), this reduces it to $g^{(local)}_{\mu\nu}k^{\mu}k^{\nu} = 0$ so that the photons would just see the local background metric.

where n is a strictly positive integer. From Eqs.(12,13), the time variation of the photon frequency, due to interaction with very weak $\mathbf{B}(t)$ fields, reads

$$\frac{\dot{\nu}}{\nu} = A_n\gamma_n \frac{4n\gamma_n - (2n+1)F^{n+1}}{(F^{n+1} + 4n\gamma_n)(F^{n+1} + 4n(n+2)\gamma_n)}. \tag{14}$$

with $A_n = 4H_0 n(n+1)$. Notice that γ_n should be negative in order to guarantee that the Lagrangian is bound from below (see [29], sections 27 and 93), $\gamma_n = -(B_n c)^{2(n+1)}$. Also, it is worth noticing that Eq.(14) in the nearly-zero field limit ($B \to 0$) would reduce to

$$\frac{\dot{\nu}}{\nu} = H_0 \frac{n+1}{n+2}, \tag{15}$$

which implies a blueshift.

Discussion and conclusion. – We stress that the NLED is a universal theory for the electromagnetic field, with $\gamma_{n=1} = \gamma$ in Eq.(13) being a universal constant, the value of which was fixed by Ref. [28] by using the CMB constraint. Setting $B_1 = \frac{1}{c}|\gamma|^{1/4}$, one finds $B_1 = 0.008 \pm 0.002$ μG [30]. But be awared of that a conclusive fashion of fixing γ should benefit of a dedicated laboratory experiment, as it was done, for instance, to fix the electron charge through Millikan's experiment.

Thence, to compute the effect (shift) on the Pioneer communication signal frequencies (uplink and downlink), we need only to introduce the value of the strength of the local supercluster $\mathbf{B}$-field: $B_{\rm LSC} \sim 10^{-8} - 10^{-7}$ G [31]. Now, the theory must be such that $L_{\rm F} < 0$ (hence, $F^{n+1} + 4n\gamma_n > 0$) for the energy density of the EM field be positive definite (see [16], appendix B), which entails $\mathbf{B} > B_1$. On the other hand, the good accordance of the Voyager 1/2 magnetometers data with Parker's theory constraints $B_{\rm LSC}$ to be less than 0.022 μG within the solar system up to the heliopause. Hence, we may conclude that 0.01 μG $< B_{\rm LSC} <$ 0.022 μG within the solar system. By recalling that the uplink frequency of Pioneer 10/11 spacecrafts is $\nu = 2.2$ GHz, one obtains for the median value $B_{\rm LSC} = 0.018$ μG (both expressions are normalized by $\frac{H_0}{70\ \rm km\ s^{-1}\ Mpc^{-1}}$, Eq.(15))

$$\frac{\dot{\nu}}{\nu} = 2.8 \times 10^{-18}\ \rm s^{-1}, \qquad \frac{d\Delta\nu}{dt} = 6 \times 10^{-9}\ \frac{\rm Hz}{\rm s}, \tag{16}$$

with $\Delta\nu$ the frequency discrepancy pointed out earlier.

A Note on interplanetary magnetic field and NLED effects.— It has been pointed out that the strength of the IPMF could severly minimize the NLED effects, because it will overrun the interstellar or intergalactic magnetic fields at heliocentric distances. Notwithstanding, the actual data from Voyager 1/2 spacecrafts of the IPMF average strength are both consistent with a non-zero local supercluster magnetic field (LSCMF) amounting up to 0.022 μG [32,33] (the accuracy of the measurements performed by Pioneer 10/11 magnetometers is at best 0.15 μG, and 0.022 μG for the low field system of Voyager 1/2 magnetometers [34]). Besides, it is just beyond the Saturn orbit, ~ 10 Astronomical Units (AU), that the anomaly begins to be clearly observed. Surprisingly, it is just after passing the Saturn orbit that the strength of the magnetic field vehiculated by the solar wind gets down the strength associated to the insterstellar and intergalactic magnetic fields, as one can verify by perusing Refs. [32, 33]. Thus, since a magnetic field cannot shield (or block) any another magnetic field (the stronger field can only reroute the weaker field, otherwise it would violate Maxwell's laws), then it follows that the LSCMF has its magnetic influence extended upto nearly the location of the Saturn orbit, and in this way it forces the photons being emitted by the Pioneer spacecrafts from larger heliocentric distances to get accelerated due to the NLED effects. Besides, notice that Ref. [35] also shows that the local cloud of interstellar gas in HII regions does not keep out the Galactic magnetic field.

In passing, we call to the reader's attention the fact that some workers in the field have claimed that the effect should have showed up already at the small distance corresponding to Mars, Jupiter or Saturn orbits, because of the high technology involved in the tracking of planet orbiting spacecrafts as Galileo and Cassini or the Mars' nonroving landers, which would allow to single out the anomaly at those heliocentric distances. However, as those spacecrafts are inside the region where the solar wind dominates, this definitely precludes the NLED photon acceleration effect to show up at those distances since the much higher magnetic field there would introduce a negligible NLED effect, and as stated below the solar pressure influence on the signal frequency is still large.

Finally, the new frequency shift that is predicted by NLED is not seen yet in the laboratory because of the following reasons: a) the most important, the strength of the Earth magnetic field is much larger than the one required in the NLED explanation of the anomaly for the effect to show up, and b) the coherence time $\tau = 1/\Delta\nu$ of EM waves in present atomic clocks (frequency width $\Delta\nu > 0.01$ Hz, or otherwise stated $c\tau < 0.2$ AU) is too short as compared to the time of flight of photons from Pioneer 10/11 spacecrafts past 20 AU. Nonetheless, if the conditions demanded by our model were satisfied this effect will certainly be disentangled in a dedicated experiment where, for instance, the Earth magnetic field is kept outside the case containing an experimental set up where a very weak magnetic field is maintained inside, a source of photons set to travel and a receiver data-collecting.

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