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An alternative method to obtain the muon charge ratio at sea level

H M Portella[1], *L C S de Oliveira*[2] and *C E C Lima*[2]

[1] Instituto de Física, Universidade Federal Fluminense,
Av. Litorânea s/n, Gragoatá,
24210-340, Niterói, RJ, Brazil

[2] Centro Brasileiro de Pesquisas Físicas,
Rua Dr. Xavier Sigaud 150,
Urca, 22290-180, Rio de Janeiro, RJ, Brazil

Abstract

The muon charge ratio generated from hadronic showers in the earth's atmosphere is obtained accurately in the energy range from a few GeV to several TeV and for some zenith angles between 0^o and 89^o. To solve the hadron diffusion equations we apply the analytical method based on depth-like ordered exponential operator used in our last paper. A comparison among our calculations with the measured ratio and Lipari's analytical calculation is made. Our differential muon fluxes are also compared with data at sea level. The agreement between them is in general very good ($> 96\%$).

1 Introduction

A theory of the passage of cosmic ray through a material medium requires a knowledge on several factors such as: the composition and the energy distribution of the incident particles, the characteristics of interactions of these particles with the nuclei of the medium, and the structure of the medium.

Nowadays, the main results concerning the diffusion of high energy cosmic ray particles through matter have been obtained by Monte Carlo methods due to the complexity of the variables involved in this kind of problem [1]. However, analytical methods are still worth to obtain approximated solutions of some of these problems, *e.g.*, cosmic ray diffusion in interstellar medium, Electromagnetic Cascade Theory, etc..

As mentioned in our last paper, these methods allow us to obtain in a simple way, relations among the different particle fluxes accurately and also to show the influence of the hypothesis used in the calculations on the hadron and lepton fluxes in the earth's atmosphere. They also provide directions to be followed by more sofisticated analysis like cascade simulations.

In this paper we calculated the muon fluxes and muon charge ratio for different zenith angles in a wide energy range (from GeV to TeV) originated from a hadronic shower in the earth's atmosphere. It is a continuation of our previous paper [2] and it is related to depth-like ordered exponential operators similar to those used by Feynman in some physical problems [3]. This method enable us to acquire compact solutions for any form of primary spectrum and to take into account non-scaling properties of the hadronic cross-sections [4].

The reason for these calculations is that we are interested in obtain the lepton-antilepton ratios whose play a very important role in the study of one of the most intriguing current problem, namely the Atmospheric Neutrino Anomaly [5].

This paper is divided as follows: in Section 2, we solve the nucleon diffusion equations in order to get the proton and neutron fluxes. In Section 3 and 4, we calculate the charged pion and kaon fluxes, respectively. In Section 5, we calculate the charged muon fluxes separately, taking into account energy losses and decay. In Section 6, we present comparisons of our muon fluxes with experimental data and our muon charge ratio with several experimental data and Lipari's calculation. Finally, we discuss and make some remarks about our results.

2 The nucleon diffusion equations

The diffusion equations for the nucleonic components of the cosmic radiation in earth's atmosphere can be written as

$$\frac{\partial}{\partial t}p(t,E) = -\frac{p(t,E)}{\lambda_p(E)} + \int_E^\infty \frac{p(t,E')}{\lambda_p(E')} f_{pp}(E,E')\frac{dE'}{E'} + \int_E^\infty \frac{n(t,E')}{\lambda_n(E')} f_{np}(E,E')\frac{dE'}{E'} \tag{1}$$

and

$$\frac{\partial}{\partial t}n(t,E) = -\frac{n(t,E)}{\lambda_n(E)} + \int_E^\infty \frac{n(t,E')}{\lambda_n(E')} f_{nn}(E,E')\frac{dE'}{E'} + \int_E^\infty \frac{p(t,E')}{\lambda_p(E')} f_{pn}(E,E')\frac{dE'}{E'} \tag{2}$$

where $p(t,E)$ and $n(t,E)$ are the proton and neutron fluxes at depth t in the energy range E and $E+dE$, $\lambda_p(E)$ and $\lambda_n(E)$ are the interaction mean free-path of protons and neutrons in the atmosphere and $f_{\alpha\beta}(E,E')$ are the energy distribution of the secondary nucleon $\beta(p$ or $n)$ originated from the α-th nucleon-air interaction.

Using the approximation $\lambda_p(E) = \lambda_n(E) = \lambda$ (interaction mean free-path of nucleons in air), and the isospin simmetry $f_{pp} = f_{nn}$, $f_{np} = f_{pn}$, the above equations can be uncoupled.

To do this we add and subtract these equations and we obtain the equations

$$\frac{\partial}{\partial t}N_i(t,E) = -\frac{N_i(t,E)}{\lambda} + \int_E^\infty \frac{N_i(t,E')}{\lambda} f^i_{NN}(E,E')\frac{dE'}{E'} \quad ; \quad i\text{=1 or 2} \tag{3}$$

where $N_1(t,E)$ stands for the nucleon flux and $N_2(t,E)$ for the difference between proton and neutron fluxes, and $f^1_{NN} = f_{pp} + f_{pn}$ and $f^2_{NN} = f_{pp} - f_{pn}$. Using the definition of the elasticity coefficient $\eta = \frac{E}{E'}$, the above equations become

$$\frac{\partial}{\partial t}N_i(t,E) = -\frac{N_i(t,E)}{\lambda} + \int_0^1 \frac{N_i(t,\frac{E}{\eta})}{\lambda} f^i_{NN}(\eta)\frac{d\eta}{\eta} \quad ; \quad i\text{=1 or 2.} \tag{4}$$

The solution of the equation (4) is subject to the boundary conditions

$$N_1(0,E) = p_0(E) + n_0(E) \tag{5}$$

and

$$N_2(0,E) = p_0(E) - n_0(E) \tag{6}$$

where $p_0(E)$ and $n_0(E)$ are the proton and neutron fluxes at the top of atmosphere.

To solve equation (4) we introduce the operator

$$\hat{A}_i = -\left(1 - \int_0^1 d\eta f^i_{NN}(\eta)\hat{\sigma}\right)\frac{1}{\lambda} \tag{7}$$

where the operator $\hat{\sigma}$ acts only on the energy function [6]

$$\hat{\sigma}F(t,E) = \frac{1}{\eta}F(t,\frac{E}{\eta}) \tag{8}$$

for $\eta \geq \eta_{min} > 0$.

So, the equation (4) takes the form

$$\frac{\partial}{\partial t}N_i(t,E) = \hat{A}_i N_i(t,E) \tag{9}$$

These equations must be integrated with the boundary conditions (5) and (6) supposed to be continuous positively bounded ($N_i(0,E) < M$ (positive real number)), which solutions are given by

$$N_i(t,E) = e^{t\hat{A}_i}N_i(0,E) \quad . \tag{10}$$

If the primary spectra $N_i(0,E)$ assume the usual power form, then the expression (10) takes the well-known form

$$N_i(t,E) = N_{0_i}E^{-(\gamma+1)}e^{-t/L_i} \tag{11}$$

where N_{0_1} and N_{0_2} is the sum and the difference between the proton and neutron fluxes at the top of the atmosphere, respectively, and $-1/L_i$ is the eigenvalue of the operator $\hat{A}_i$ acting on the energy function $N_{0_i}E^{-(\gamma+1)}$, with

$$L_i = \frac{\lambda}{1 - Z^i_{NN}} \tag{12}$$

and

$$Z^i_{NN} = \int_0^1 f^i_{NN}(\eta)\eta^\gamma d\eta \quad . \tag{13}$$

The fluxes of protons and neutrons are obtained from equations (11), and are put in terms of N_{0_1} and δ_0 (proton excess at the top of the atmosphere).

$$p(t,E) = \frac{N_{0_1}}{2}\left(e^{-t/L_1} + \delta_0 e^{-t/L_2}\right)E^{-(\gamma+1)} \tag{14}$$

and

$$n(t,E) = \frac{N_{0_1}}{2}\left(e^{-t/L_1} - \delta_0 e^{-t/L_2}\right)E^{-(\gamma+1)} \tag{15}$$

where

$$\delta_0 = \frac{N_{0_2}}{N_{0_1}} = \frac{p_0 - n_0}{p_0 + n_0} \tag{16}$$

3 The charged pion diffusion equations

The equation that describes the diffusion of the charged pions in the atmosphere, considering the validity of the Feynman scaling law is

$$\frac{\partial}{\partial t}\Pi^{\pm}(t,E,\theta^*) = -\frac{\Pi^{\pm}(t,E,\theta^*)}{\lambda_\pi(E)} - \frac{\Pi^{\pm}(t,E,\theta^*)}{\lambda^{\pi}_{decay}(E)} + \int_0^1 \frac{\Pi^{\pm}(t,\frac{E}{x},\theta^*)}{\lambda_\pi(\frac{E}{x})} f_{\pi^\pm\pi^\pm}(x)\frac{dx}{x} +$$
$$+\int_0^1 \frac{\Pi^{\mp}(t,\frac{E}{x},\theta^*)}{\lambda_\pi(\frac{E}{x})} f_{\pi^\mp\pi^\pm}(x)\frac{dx}{x} + \int_0^1 \frac{p(t,\frac{E}{x})}{\lambda(E)} f_{p\pi^\pm}(x)\frac{dx}{x} + \int_0^1 \frac{n(t,\frac{E}{x})}{\lambda(E)} f_{n\pi^\pm}(x)\frac{dx}{x} \tag{17}$$

where θ^* is the zenith angle in the pion production, $x \approx \frac{E}{E'}$ is the Feynman variable and $f_{\pi^\pm\pi^\pm}(x)$ and $f_{N\pi^\pm}(x)$ are respectively the secondary spectra of the pions produced in the meson-air nucleus and nucleon-air nucleus interactions. The decay lenght of the pion in the atmosphere is given by $\lambda^{\pi}_{decay}(E) = c\beta\tau_\pi\frac{E}{m_\pi}\rho(t,\theta^*)$ and we suppose the approximation $\lambda_\pi = \lambda_{\pi^+} = \lambda_{\pi^-}$.

The solutions of these equations are subjected to the boundary conditions

$$\Pi^{\pm}(0,E,\theta^*) = 0 \quad . \tag{18}$$

Considering the isospin symmetry, the interaction lenghts constant and adding and subtracting $\Pi^+(t,E,\theta^*)$ and $\Pi^-(t,E,\theta^*)$ we can uncouple the above equations and we obtain

$$\frac{\partial}{\partial t}\Pi_i(t,E,\theta^*) = -\frac{\Pi_i(t,E,\theta^*)}{\lambda_\pi} - \frac{\Pi_i(t,E,\theta^*)}{\lambda^{\pi}_{decay}} + \int_0^1 \frac{\Pi_i(t,\frac{E}{x},\theta^*)}{\lambda_\pi} f^i_{\pi\pi}(x)\frac{dx}{x} + \int_0^1 \frac{N_i(t,\frac{E}{x})}{\lambda} f^i_{N\pi}(x)\frac{dx}{x} \tag{19}$$

where $\Pi_1(t,E,\theta^*) = \Pi^+(t,E,\theta^*) + \Pi^-(t,E,\theta^*)$, $\Pi_1(t,E,\theta^*) = \Pi^+(t,E,\theta^*) - \Pi^-(t,E,\theta^*)$ and the energy spectrum $f^1_{\pi\pi}(x) = f_{\pi^+\pi^+}(x) + f_{\pi^+\pi^-}(x)$, $f^2_{\pi\pi}(x) = f_{\pi^+\pi^+}(x) - f_{\pi^+\pi^-}(x)$, $f^1_{N\pi}(x) = f_{p\pi^+}(x) + f_{p\pi^-}(x)$ and $f^2_{N\pi}(x) = f_{p\pi^+}(x) - f_{p\pi^-}(x)$.

As in the case of nucleons we introduce the following operators

$$\hat{B}_{N_i} = \left(\int_0^1 dx f^i_{N\pi}(x)\hat{\sigma}_{N_i}\right)\frac{1}{\lambda} \quad , \tag{20}$$

$$\hat{B}_{\Pi_i} = -\left(1 - \int_0^1 dx f^i_{\pi\pi}(x)\hat{\sigma}_{\Pi_i}\right)\frac{1}{\lambda_\pi} \quad , \tag{21}$$

$$\hat{G}_{\Pi}\Pi_i(t,E,\theta^*) = -\frac{1}{\lambda^{\pi}_{decay}}\Pi_i(t,E,\theta^*) \quad , \tag{22}$$

where $\hat{\sigma}_{N_i}$ and $\hat{\sigma}_{\pi_i}$ act on one energy function as defined in equation (8), and the operator $\hat{G}_\pi$ is defined by the eigenvalue equation above

By introducing these operators in equation (19) we obtain the symbolic operator equation

$$\frac{\partial}{\partial t}\Pi_i(t,E,\theta^*) = \hat{G}_{\Pi}\Pi_i(t,E,\theta^*) + \hat{B}_{\Pi_i}\Pi_i(t,E,\theta^*) + \hat{B}_{N_i}N_i(t,E) \tag{23}$$

which solutions satisfy the boundary conditions (5) and (18).

The general solution is

$$\Pi_i(t,E,\theta^*) = \int_0^t Exp\left[\int_z^t (\hat{G}_\Pi + \hat{B}_{\Pi_i})dz'\right] \hat{B}_{N_i} N_i(z,E)dz \quad , \tag{24}$$

where $Exp\left[\int_z^t (\hat{G}_\Pi \ + \ \hat{B}_{\Pi_i})dz'\right]$ is the expansional defined by a sum of multiple depth-ordered integrals (see the appendix in ref.[2]).

The above equation can be simplified using the decomposition properties of the expansional, to deal with the non-commutative operators $\hat{G}_\Pi$ and $\hat{B}_{\Pi_i}$, and assuming for the nucleon energy spectrum at the top of the atmosphere; $N_i(0,E) \ = \ N_{0_i} \ E^{-(\gamma+1)}$ (a similar calculation is shown in ref.[2]). The solution of the above equation assumes the compact form:

$$\Pi_i(t,E,\theta^*) = \int_0^t dz \frac{Z^i_{N\pi}}{\lambda} e^{-(t-z)/L_{\pi_i}(\gamma)} e^{-z/L_i(\gamma)} \left\{ \hat{T}_\Pi(t,z) + \right.$$

$$\left. + \int_z^t dz' \left[\hat{T}_\Pi(t,z')\hat{G}_\Pi(z')(t-z')(\hat{B}_{\Pi_i}(\gamma) - \hat{B}_{\Pi_i}(\gamma+1)\right] \hat{T}_\Pi(z,z') + ... \right\} N_{0_i} E^{-(\gamma+1)} \tag{25}$$

where

$$\hat{T}_\Pi(t,z) = Exp\left(\int_z^t dz' \hat{G}_\Pi(z')\right) \tag{26}$$

and

$$Z^i_{N\pi} = \int_0^1 x^\gamma f^i_{N\pi}(x)dx \tag{27}$$

with $-1/L_{\pi_i}(\gamma)$ and $-1/L_i(\gamma)$ being the eigenvalues of the operators $\hat{B}_{\Pi_i}(\gamma)$ and $\hat{B}_{N_i}(\gamma)$ acting on the function $N_{0_i} \ E^{-(\gamma+1)}$ and they are defined as

$$L_{\pi_1}(\gamma) = \frac{\lambda_\pi}{1 - Z_{\pi^+\pi^+} - Z_{\pi^+\pi^-}} \tag{28}$$

and

$$L_{\pi_2}(\gamma) = \frac{\lambda_\pi}{1 - Z_{\pi^+\pi^+} + Z_{\pi^+\pi^-}} \tag{29}$$

with

$$Z_{\pi^+\pi^-}(\gamma) = \int_0^1 x^\gamma f_{\pi^+\pi^-}(x)dx \tag{30}$$

In our numerical calculation we consider only the first term on the right-hand side of equation (25), because the contribution of the further terms were computed to be negligible. So, the expression (25) can be written in the following forms

$$\Pi_i(t,E,\theta^*) = \int_0^t dz \ \frac{Z^i_{N\pi}}{\lambda} e^{-(t-z)/L_{\pi_i}(\gamma)} e^{-z/L_i(\gamma)} \hat{T}_\Pi(t,z) N_{0_i} E^{-(\gamma+1)} \quad . \tag{31}$$

The charged pion fluxes are obtained by,

$$\Pi^\pm(t,E,\theta^*) = \frac{\Pi_1(t,E,\theta^*) \pm \Pi_2(t,E,\theta^*)}{2} \quad . \tag{32}$$

4 The charged kaon diffusion equations

The equations that describe the diffusion of the charged kaons in the atmosphere, under the Feynman scaling law, similar to that of pions are

$$\frac{\partial}{\partial t} K^\pm(t,E,\theta^*) = -\frac{K^\pm(t,E,\theta^*)}{\lambda_k(E)} - \frac{K^\pm(t,E,\theta^*)}{\lambda^k_{decay}(E)} + \int_0^1 \frac{K^\pm(t,\frac{E}{x},\theta^*)}{\lambda_k(\frac{E}{x})} f_{k^\pm k^\pm}(x)\frac{dx}{x} +$$

$$+\int_0^1 \frac{K^{\mp}(t,\frac{E}{x},\theta^*)}{\lambda_k(\frac{E}{x})} f_{k^{\mp}k^{\pm}}(x)\frac{dx}{x} + \int_0^1 \frac{\Pi^{\pm}(t,\frac{E}{x},\theta^*)}{\lambda_\pi(\frac{E}{x})} f_{\pi^{\pm}k^{\pm}}(x)\frac{dx}{x} + \int_0^1 \frac{\Pi^{\mp}(t,\frac{E}{x},\theta^*)}{\lambda_\pi(\frac{E}{x})} f_{\pi^{\mp}k^{\pm}}(x)\frac{dx}{x} +$$
$$+\int_0^1 \frac{p(t,\frac{E}{x})}{\lambda(E)} f_{pk^{\pm}}(x)\frac{dx}{x} + \int_0^1 \frac{n(t,\frac{E}{x})}{\lambda(E)} f_{nk^{\pm}}(x)\frac{dx}{x} \tag{33}$$

where the parameters and spectra appearing in the above equation are similar to those appearing in the preceeding section with the obvious change of π by k. We have used the approximation $\lambda_{k^+} = \lambda_{k^-} = \lambda_k$.

The solution of these equations must satisfy the boundary conditions

$$K^{\pm}(0,E,\theta^*) = 0 \quad . \tag{34}$$

In order to uncouple the equations (33), we need to add and subtract them as follows

$$\frac{\partial}{\partial t}K_i(t,E,\theta^*) = -\frac{K_i(t,E,\theta^*)}{\lambda_k} - \frac{K_i(t,E,\theta^*)}{\lambda^k_{decay}} + \int_0^1 \frac{K_i(t,\frac{E}{x},\theta^*)}{\lambda_k} f^i_{kk}(x)\frac{dx}{x} +$$
$$+\int_0^1 \frac{\Pi^+(t,\frac{E}{x},\theta^*)}{\lambda_\pi} f^i_{\pi^+k}(x)\frac{dx}{x} + \int_0^1 \frac{\Pi^-(t,\frac{E}{x},\theta^*)}{\lambda_\pi} f^i_{\pi^-k}(x)\frac{dx}{x} +$$
$$+\int_0^1 \frac{p(t,\frac{E}{x})}{\lambda} f^i_{pk}(x)\frac{dx}{x} + \int_0^1 \frac{n(t,\frac{E}{x})}{\lambda} f^i_{nk}(x)\frac{dx}{x} \tag{35}$$

where $f^1_{pk}(x) = f_{pk^+}(x) + f_{pk^-}(x)$, $f^2_{pk}(x) = f_{pk^+}(x) - f_{pk^-}(x)$. Analogously for $f^i_{nk}(x)$, $f^i_{\pi^\pm k}(x)$, changing p by n and $\pi^\pm$ in these energy distributions. Also, $f^1_{kk} = f_{k^+k^+} + f_{k^+k^-}$ and $f^2_{kk} = f_{k^+k^+} - f_{k^+k^-}$. If we neglect the strangeness changing contributions, $f_{k^\pm k^\mp} = 0$, supposing that $f_{k^+k^+} = f_{k^-k^-}$ and assuming $f_{\pi^+k^+} = f_{\pi^-k^-}$, $f_{\pi^+k^-} = f_{\pi^-k^+}$, the above equations can be written

$$\frac{\partial}{\partial t}K_i(t,E,\theta^*) = -\frac{K_i(t,E,\theta^*)}{\lambda_k} - \frac{K_i(t,E,\theta^*)}{\lambda^k_{decay}} + \int_0^1 \frac{K_i(t,\frac{E}{x},\theta^*)}{\lambda_k} f^i_{kk}(x)\frac{dx}{x} +$$
$$+\int_0^1 \frac{\Pi_i(t,\frac{E}{x},\theta^*)}{\lambda_\pi} f^i_{\pi k}(x)\frac{dx}{x} + \int_0^1 \frac{p(t,\frac{E}{x})}{\lambda} f^i_{pk}(x)\frac{dx}{x} + \int_0^1 \frac{n(t,\frac{E}{x})}{\lambda} f^i_{nk}(x)\frac{dx}{x} \quad . \tag{36}$$

Now introducing the operators

$$\hat{A}_{K_i} = -\left(1 - \int_0^1 dx f^i_{kk}(x)\hat{\sigma}_K\right)\frac{1}{\lambda_k} \quad , \tag{37}$$

$$\hat{B}_{\Pi_i} = \left(\int_0^1 dx f^i_{\pi k}(x)\hat{\sigma}_\Pi\right)\frac{1}{\lambda_\pi} \quad , \tag{38}$$

$$\hat{B}_{p_i} = \left(\int_0^1 dx f^i_{pk}(x)\hat{\sigma}_{p_i}\right)\frac{1}{\lambda} \quad , \tag{39}$$

$$\hat{B}_{n_i} = \left(\int_0^1 dx f^i_{nk}(x)\hat{\sigma}_{n_i}\right)\frac{1}{\lambda} \quad , \tag{40}$$

and as in the case of pions the operator $\hat{G}_K$ satisfies the eigenvalue equation

$$\hat{G}_K K_i(t,E,\theta^*) = -\frac{1}{\lambda^k_{decay}} K_i(t,E,\theta^*) \quad . \tag{41}$$

So, the equation (36) turns into the operator equation

$$\frac{\partial}{\partial t}K_i(t,E,\theta^*) = \hat{G}_K K_i(t,E,\theta^*) + \hat{A}_{K_i}K_i(t,E,\theta^*) + \hat{B}_{\Pi_i}\Pi_i(t,E,\theta^*) + \hat{B}_{p_i}p(t,E) + \hat{B}_{n_i}n(t,E) \quad . \tag{42}$$

The general solutions of the above equations wrote in terms of expansionals (see the appendix in ref.[2]) and submmited to the boundary conditions (5), (18) and (34) are

$$K_i(t,E,\theta^*) = \int_0^t Exp\left[\int_z^t (\hat{G}_K + \hat{A}_{K_i})dz'\right] \cdot \left[\hat{B}_{\Pi_i}\Pi_i(z,E,\theta^*) + \hat{B}_{p_i}p(z,E) + \hat{B}_{n_i}n(z,E)\right] dz \quad . \tag{43}$$

As in the pion case the expression (43) can be simplified using the decomposition properties of the expansional, assuming for the nucleon energy spectrum at the top of atmosphere $N_i(0,E) = N_{0_i}\ E^{-(\gamma+1)}$ and using for the pion fluxes the expression (31);

$$K_i(t,E,\theta^*) = \int_0^t dz\ e^{-(t-z)/L_{K_i}(\gamma)}\hat{T}_K(t,z)\left\{\frac{N_{0_1}E^{-(\gamma+1)}}{2\lambda}\left[(Z_{pk} \pm Z_{nk})e^{-z/L_1} + \right.\right.$$

$$\left. +\delta_0(Z_{pk} \mp Z_{nk})e^{-z/L_2}\right] + \int_0^z dz_1\ Z^i_{N\pi}e^{-(z-z_1)/L_{\pi_i}(\gamma)}e^{-z_1/L_i(\gamma)}\left[\frac{N_{0_i}E^{-(\gamma+1)}}{\lambda_\pi}(Z_{\pi^+k^+}(\gamma)\pm\right.$$

$$\left.\left.\pm Z_{\pi^-k^+}(\gamma)) + \frac{N_{0_i}E^{-(\gamma+1)}}{\lambda_\pi}(Z_{\pi^+k^+}(\gamma+1) \pm Z_{\pi^-k^+}(\gamma+1))\int_{z_1}^{z_2} dz_2\ \frac{b_\pi}{E\rho(z_2,\theta^*)}\right] + ...\right\} \tag{44}$$

where $Z_{pk} = Z_{pk^+} + Z_{pk^-}$, $Z_{nk} = Z_{nk^+} + Z_{nk^-}$, $Z_{\pi^+k^-}(\gamma+1) = \int_0^1 x^{(\gamma+1)}\ f_{\pi^+k^-}(x)\ dx$.

In the case of kaons originated from pions we used the definition of the expansional as a sum of multiple depth-ordered integrals [2].

The charged kaon fluxes are obtained by,

$$K^\pm(t,E,\theta^*) = \frac{K_1(t,E,\theta^*) \pm K_2(t,E,\theta^*)}{2} \quad . \tag{45}$$

5 The muon diffusion equations

The one-dimensional diffusion equations of the muons in the atmosphere are given by

$$\frac{\partial}{\partial t}\mu^\pm(t,E,\theta) = -\hat{G}_\mu\mu^\pm(t,E,\theta) + \frac{\partial}{\partial E}\left[\beta(E)\mu^\pm(t,E,\theta)\right] +$$

$$+\int_{E_{min}}^{E_{max}}(BR)_M\hat{G}_M M^\pm(t,E',\theta^*)f_{M\mu}(E,E')\frac{dE'}{E'} \tag{46}$$

with the boundary condition

$$\mu^\pm(0,E,\theta^*) = 0 \tag{47}$$

where $\mu^\pm(t,E,\theta)$ are the charged muon fluxes at atmospheric depth t and the first term on the right side of the equation represents the muon decay, the second the muon energy loss in the atmosphere and the last the meson source of muon. $(BR)_M$ is the branching ratio of the meson M in the channel $M \rightarrow \mu + \nu$.

$\hat{G}_\mu$ and $\hat{G}_M$ represents the decay operators of the muon and meson, respectively. For mesons they are defined in equations (22) and (41) and for the muons, the eigenvalue of $\hat{G}_\mu$ is

$$\lambda^\mu_{decay} \approx \frac{c\tau_\mu}{m_\mu}\epsilon(t-z,E)cos\theta^*(z)\rho(z) \quad . \tag{48}$$

The energy losses can be considered continuous for the energy range we are considering in this work, so in this case we can approximate the muon energy loss with the following parametrized form:

$$\beta(E) = -\frac{dE}{dt} = a + bE \tag{49}$$

where a represents the ionization and excitation losses and b the *bremsstrahlung*, pair-production and nuclear interaction losses. So, the solution of the inhomogeneous equation (46) is, like in the case of the mesons, given by a sum of a homogeneous and a particular part, with the final form

$$\mu^{\pm}(t,E,\theta) = \int_0^t exp\left[b(t-z_1) - \int_{z_1}^t \frac{1}{\lambda^{\mu}_{decay}(z,\epsilon,\theta^*(z))}dz\right] H(z_1,\epsilon(t-z_1,E),\theta^*(z_1))dz_1 \quad (50)$$

where

$$\epsilon(t-z_1,E) = E^{b(t-z_1)} + \frac{a}{b}\left(e^{b(t-z_1)} - 1\right) \quad (51)$$

represents the muon energy at depth z_1 in order to arrive at depth t with energy E, and the function $H(z_1,\epsilon(t-z_1,E),\theta^*(z_1))$ is

$$H(z,\epsilon(t-z_1,E),\theta^*(z_1)) = \int_{E_{min}}^{E_{max}} (BR)_M \hat{G}_M M^{\pm}(z_1,E',\theta^*(z_1)) f_{M\mu}(E,E')\frac{dE'}{E'} \quad (52)$$

$f_{M\mu}(E,E')$, E_{min} and E_{max} are obtained from the relativistic kinematics of two bodies in the final state, $\theta^*(z_1)$ is the zenith angle at the muon production point and the functions $M^{\pm}(z_1,E',\theta^*(z_1))$ are the meson fluxes obtained in equations (32) and (45).

6 Comparison with data

In order to make a comparison with the zenithal muon fluxes and the muon charge ratio at sea level, we need to take into account several factors, such as, the primary cosmic ray spectrum, the neutron/proton ratio at the top of the atmosphere, the hadronic Z-factors, the energy losses and decays, and the interaction lenghts. We will adopt the same parameters and distributions as suggested by T.K. Gaisser [7] and P. Lipari [8].

The vertical column density as a function of height, $x_{\mathrm{V}}(h)$, used in our calculation is taken from the fit of K. Maeda [9] for the average US Standard Atmosphere. This fit corresponds to choosing a constant temperature in the stratosphere ($h \geq 11$ km) and a linear dependence in the troposphere ($h < 11$ km).

A comparison of our calculations and the measured differential muon fluxes at sea level for $\theta = 0^o$ [10, 11, 12, 13, 14] and $\theta = 30^o$, 60^o, 80^o [11] and 89^o [15] are shown in the figures (1) and (2). The agreement is in general very good (greather than 96%).

Figure (3) shows a comparison of our calculation (solid line) with experimental data [10, 14, 16] and with Lipari's analytical calculation (dotted line) [8]. At low energy ($E_\mu < 1$ GeV) our calculated muon charge ratio is around 1.21. It rises slightly in the energy region (5 GeV $\leq E_\mu \leq$ 100 GeV) and becomes larger at high energy ($E_\mu > 1$ TeV). This is because the number of kaons is enhanced with respect to pions (the kaon decay contibutes with 5% of the muon flux at 10 GeV and rises to 35% at 10 TeV [2]). The agreement among them is quite remarkable (around 97%). It seems that our calculations overestimate somewhat the low and the high part of the experimental points. At low energy ($E_\mu < 1$ GeV) the discrepancy seems to be due to the non inclusion in our calculations of the geomagnetic effect. This effect decreases the number of primary protons below 10 GeV and consequently the muon charge ratio for energies below a few GeV. For energies between 1 and 20 GeV, our ratio is larger than the experimental data ($\approx 4\%$) as can be seen in the figure. This is due to the fact that $\Pi^{\pm}$ fluxes have a slightly different development in the atmosphere. The Π^- flux that receive a larger contribution from the neutron component propagates somewhat deeper in the atmosphere. Therefore, at a fixed energy the positive muon will have a major probability to decay than the negative muon. So, the μ^+ are more deplected than the μ^- and the calculated μ^+/μ^- decreases in this region. For $E_\mu > 1$ TeV the discrepancy is possibly due to the uncertanties in the kaon Z-factors determination.

Figure (4) shows a comparison of our calculations with experimental data [15, 17]. In this figure the upper and lower limits of the dashed region represent our analytical results for $\theta = 78^o$ and $\theta = 89^o$, respectively. The agreement between them is in general very good ($\geq$ 96%). We notice a tendency similar to that already observed in the figure (3). At low energy $E_\mu \approx 10$ GeV the muon charge ratio is around 1.30 and it rises to 1.46 at 10 TeV. This is explained, as in the previous figure, by the enhancement of the kaon fluxes for energies higher than 1 TeV.

In these figures we note an $\mu^\pm$ asymmetry. This is consequence of the asymmetries in the meson fluxes, $\Pi^\pm$ and $K^\pm$. For pions this is due to the excess of protons over neutrons in the primary cosmic rays and to the fact that $Z_{p\pi^+} > Z_{p\pi^-}$ reflecting the valence quark content of proton and pions. For kaons this asymmetry is stronger, showing that the $k^-(s\overline{u})$ does not receive any contribution from the valence quark content of the incident nucleon.

7 Conclusions

The integro-differential equations which describes the diffusion of nucleons and mesons in the earth atmosphere are integrated using a Feynman-like procedure of ordered exponential operators. Then, we obtain the muon numbers from these hadronic fluxes. In order to derive the μ^+/μ^- ratio at sea level we calculated separately the neutron and proton in the first stage. Then, we kept separately the positive and negative mesons ($\pi^\pm$, $K^\pm$), although this is not strictly necessary for the calculation of the muon spectrum. Our solutions are valid in large energy range, few GeV to PeV for zenith angle covering 0^o - 89^o and allows the possibility to investigate the effects of deviations of the primary spectrum from the power law form (*e.g.* at $E_\mu < 1$ GeV a time dependence is introduced by solar modulation).

Our calculated muon fluxes ($\theta = 0^o$, 60^o, 80^o and 89^o) agrees very well with the experimental data (almost 97%).

A comparison between our calculations and the measured vertical muon charge ratio shows an agreement, in general good, although some small differences can be seen. The largest disagreement is at the lowest ($E_\mu < 1$ GeV) and highest ($E_\mu > 1$ TeV) energies. In the first energy region the difference is about 5%. Our calculation is higher than the experimental data because we do not include the geomagnetic effects. This effect decreases the proton fluxes at the top of the atmosphere but do not change the neutron fluxes, so the μ^+/μ^- at sea level decreases. At higher energies the difference can be traced to the uncertainties about the properties of strange meson production (the Z-factors for kaon production in the nucleon-air interactions have probably an uncertainty of approximately 25%).

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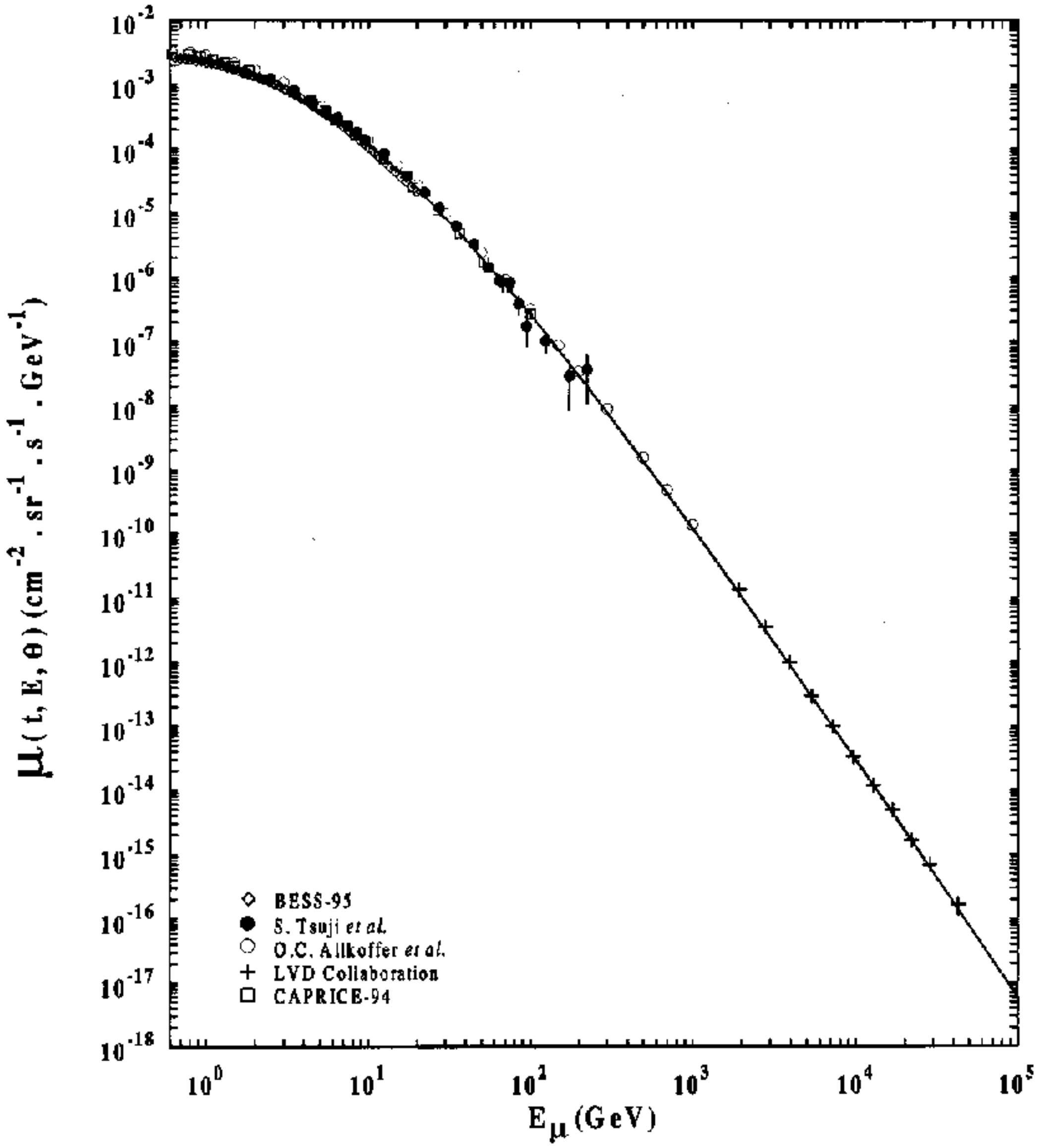

Figure 1

Figure 1 - Differential muon flux at sea level for $\theta = 0^o$ as a function of the energy. The solid line represents our calculation and the experimental data are indicated in the figure.

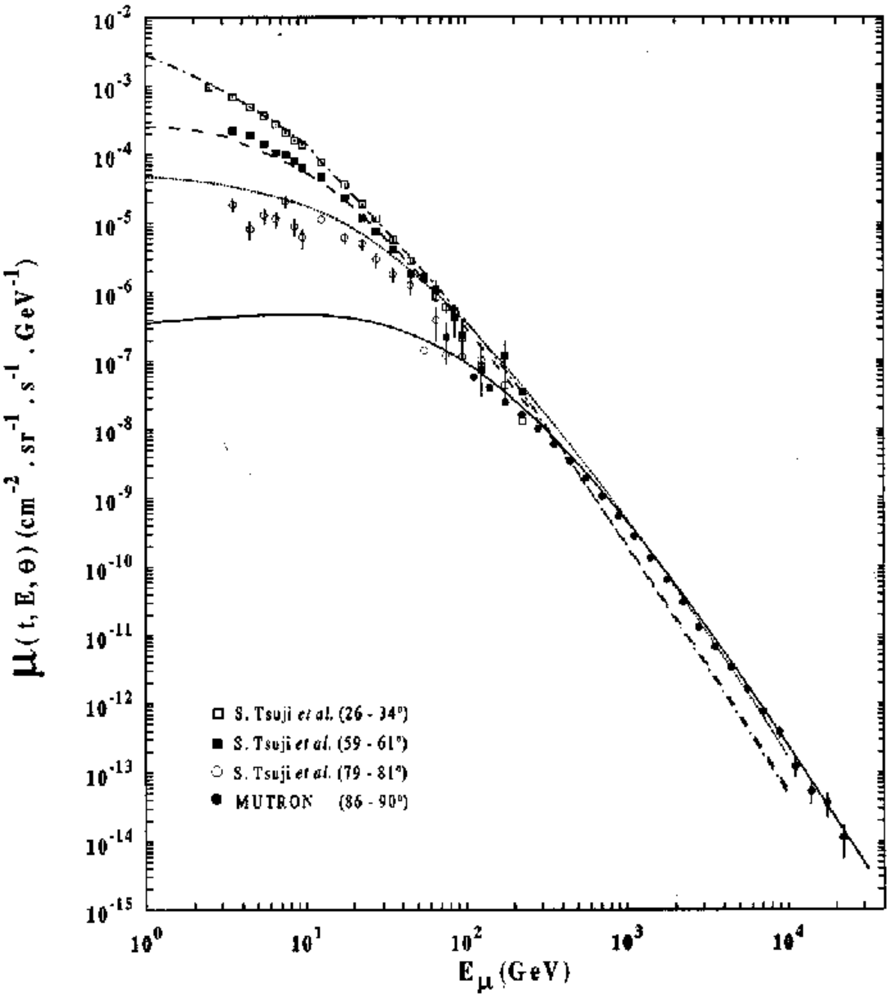

Figure 2

Figure 2 - Differential muon fluxes at sea level for zenith angles $\theta = 30^o$, 60^o, 80^o and 89^o as a function of the energy. Our calculations are represented by dotted-dashed, dashed, dotted and solid lines in a increasing order of zenith angles. The experimental data are indicated in the figure.

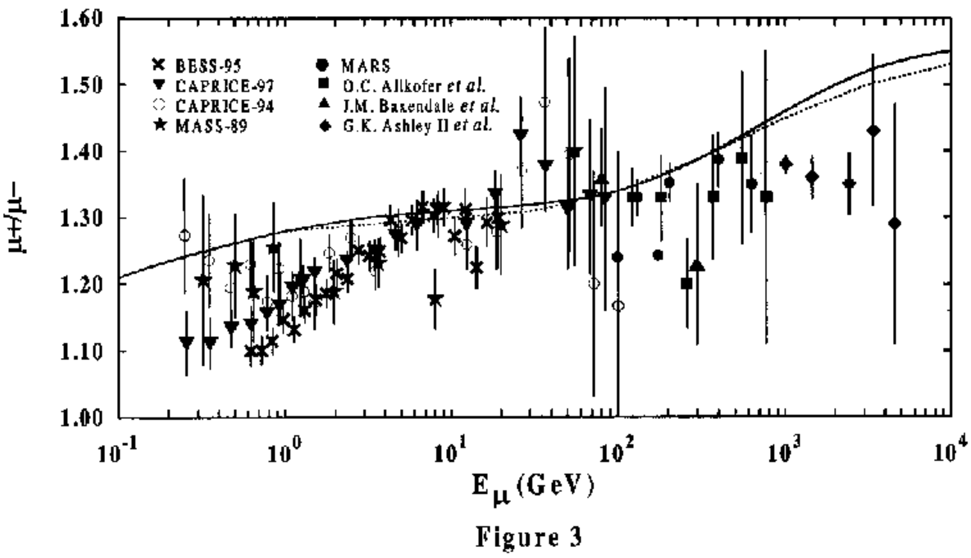

Figure 3 - Energy distribution of the muon charge ratio at sea level for $\theta = 0^o$. The solid line represents our calculation and the dotted line represents the Lipari's ones. The experimental data are indicated in the figure.

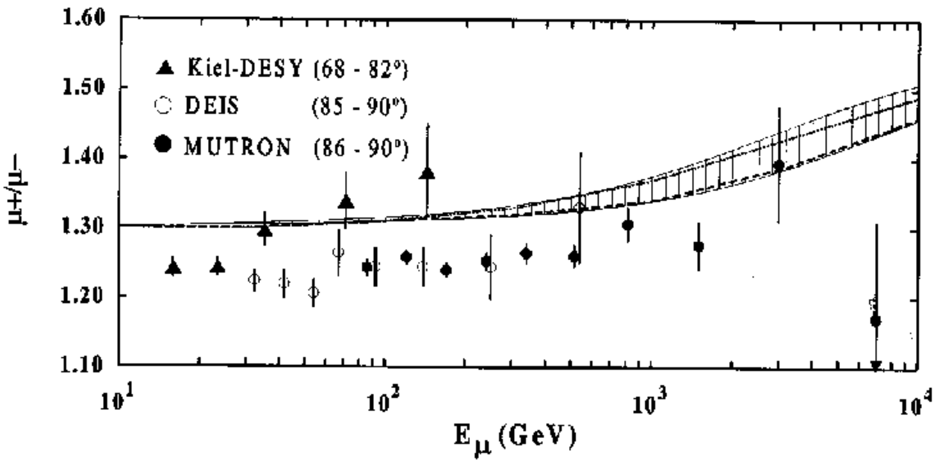

Figure 4

Figure 4 - Energy distribution of the muon charge ratio at sea level for $78^o \leq \theta \leq 89^o$ (dashed region). The dotted and the dashed lines represent the Lipari's calculation for $\theta = 78^o$ and 89^o, respectively. The experimental data are indicated in the figure.