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ABSTRACT

The nuclear Giant Dipole Resonance (GDR) energies are calculated using a macroscopic hydronamical model with two new features. The motion is treated as a combination of the usual Goldhaber-Teller (GT) and Steinwedel-Jensen (SJ) modes, and the restoring forces are all calculated using the Droplet Model. The A dependence of the resonance energies is well reproduced without any adjustable parameters, and the measured magnitude of the energies serves to fix the value of the effective mass m* used in the theory. The GDR is found to consist mainly of a GT-type motion with the SJ-mode becoming more important for heavy nuclei. The width Γ of the GDR is also estimated on the basis of an expression for one-body damping.

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# I. INTRODUCTION

The giant electric dipole resonance (GDR) is a beautiful example, among the vast variety of possible nuclear excitations, of a manifestly collective mode that can be understood, to a large extent, in terms of a macroscopic approach. It corresponds to the absorption of electric dipole radiation by the vibration of the neutrons against the protons and the subsequent damping of this motion into intrinsic excitation.

The GDR can be observed in every nucleus throughout the periodic table and very little structure is to be seen in the energy dependence of the absorption cross-section, except for the lightest nuclei.[1] The absorption cross-section for most nuclei follows a Lorentz curve whose mean energy $E_m$ (see Fig. 2) varies smoothly with mass number in a manner that shows little or no dependence on nuclear shell effects.[2]

On the basis of a few early experiments Goldhaber and Teller[3] published a list of three possible semi-classical explanations for the A dependence of the resonance energy. The first postulated an elastic binding of the neutrons to the protons that would result in a resonance energy independent of A. The second proposal, later elaborated by Steinwedel and Jensen[4] (the SJ-mode), was that the resonance might consist of density vibrations of the neutron and proton fluids against each other with the surfaces fixed. This kind of motion, which corresponds to the lowest acoustic mode in a spherical cavity, would result in a resonance energy proportional to $A^{-1/3}$. Their third suggestion, one that they chose to discuss in some detail (the GT-mode), was that the neutrons and protons might behave like two separate rigid but inter-penetrating density distributions. The resulting resonance, consisting of the harmonic displacement of these distributions with respect to each other, would be expected to have an energy dependence proportional to $A^{-1/6}$.

Because of the crude nature of the model and the severity of the assumptions needed to justify it, the GT-mode has received relatively little attention over the years. On the other hand, the SJ-mode, which also imposes a harsh and unrealistic constraint (that the vibration takes place in a rigid fixed spherical cavity) on the motion, has served as the basis for a vast literature dealing with the GDR. The SJ-mode has been widely applied and has been extended to deformed nuclei,[5] to include compressibility,[6] to include the coupling to surface vibrations[7,8] and other surface effects.[9,10]

Our interest in the GDR was revived when we realized that the development of the Droplet Model[11,12] (which explicitly identifies the energy associated with displacing the surface of the neutron distribution from that of the proton distribution) would permit a more realistic calculation of the restoring force for the GT-mode than the ad hoc procedure that was used in the original work. We also came to realize that a much more satisfactory macroscopic description of the resonance results if it is considered to be a superposition of GT- and SJ-modes. A moments reflection should serve to convince the reader that if all constraints were removed from a SJ-type density vibration then the inertia associated with the flow of neutrons and protons would tend to carry them beyond the location of the original surface when they pile up first on one side of the nucleus and then the other. This tendency of the neutron and proton distributions to undergo a harmonic displacement from each other is just the GT-mode.

The work that is to be described here contains these two new features. First, all the restoring forces are calculated in terms of the Droplet Model. Second, the motion is considered to be a superposition of GT- and SJ-modes with the relative magnitudes of the two modes being determined by the coupling between them and the associated forces and inertias. We find that the GDR is mainly a GT-mode, with the relative amount of SJ-mode increasing for heavier nuclei that are softer with respect to the neutron and proton compressions that are involved. We also find an A dependence for the resonance energy that is intermediate between that of the GT- and SJ-modes, in excellent agreement with the measured valves.

## II. DEGREES OF FREEDOM

To describe the motion we choose a spherical (polar) coordinate system with the z-axis, which is a symmetry axis, aligned along the direction of motion. The equation of motion will be solved subject to the constraint that the solution can be represented by a vector,

$$\underset{\sim}{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \tag{2.1}$$

times a harmonic time dependence, where the vector components $\alpha_1$ and $\alpha_2$ represent the relative amounts of the GT- and SJ-modes.

### A. GT-Mode

The GT-mode,[3] illustrated on the left side of Fig. 1, consists of a rigid displacement of the neutrons from the protons by an amount

$$d = \alpha_1 R \quad , \tag{2.2}$$

where R is the mean radius of the nucleus. The protons and neutrons are displaced from the origin by the amounts,

$$d_z = \frac{N}{A} d \quad \text{and} \quad d_n = - \frac{Z}{A} d \quad , \tag{2.3}$$

which leaves the center of mass fixed.

The dipole moment is given by

$$D_1 = Zed_z = \alpha_1 \left( \frac{NZ}{A} \right) eR \quad , \tag{2.4}$$

where e is the unit of electronic charge.

The flow fields for the protons and neutrons in the GT-mode are given by

$$\underset{\sim}{v}_{1z} = \frac{N}{Z}\underset{\sim}{v}_1 \quad \text{and} \quad \underset{\sim}{v}_{1n} = -\frac{Z}{A}\underset{\sim}{v}_1 \quad , \tag{2.5}$$

where

$$\underset{\sim}{v}_1 = \dot{\alpha}_1 R(\cos\theta\, \hat{e}_r - \sin\theta\, \hat{e}_\theta) \quad . \tag{2.6}$$

B. SJ-Mode

In the SJ-mode,[4] illustrated on the right side of Fig. 1, the protons and neutrons vibrate against each other in a fixed spherical cavity in such a way that their density variations are given by

$$\delta\rho_z = \frac{N}{A}\rho_z\delta\eta \quad \text{and} \quad \delta\rho_n = \frac{Z}{A}\rho_n\delta\eta \tag{2.7}$$

where

$$\delta\eta = \alpha_2 C j_1(kr)\cos\theta \quad , \tag{2.8}$$

where

$$kR = a = 2.08 \tag{2.9}$$

and

$$C = 2a/j_0(a) = 9.93 \tag{2.10}$$

The expressions $j_0$ and $j_1$ are spherical Bessel functions. Equation (2.9) serves to determine the value of k in terms of the nuclear radius R in order to satisfy the boundary condition of a fixed surface, and it also serves to define the quantity a which is useful in some of the expressions that follow. Equation (2.10) gives an expression for the quantity C, whose value has been chosen so that the expression for the dipole moment in the SJ-mode is

$$D_2 = \alpha_2 \frac{NZ}{A} eR \quad , \tag{2.11}$$

in analogy with Eq. (2.4). This normalization is important since it establishes a scale against which the relative contributions of GT- and SJ-modes to the GDR can be measured.

The velocity fields for the protons and neutrons in the SJ-mode are given by

$$\underset{\sim}{v}_{2z} = \frac{N}{A}\underset{\sim}{v}_2 \quad \text{and} \quad \underset{\sim}{v}_{2n} = -\frac{Z}{A}\underset{\sim}{v}_2 \quad , \tag{2.12}$$

where

$$\underset{\sim}{v}_2 = \dot{\alpha}_2 \frac{C}{k}\left[j_1'(kr)\cos\theta\,\hat{e}_r - \frac{1}{kr}j_1(kr)\sin\theta\,\hat{e}_\theta\right] \quad . \tag{2.13}$$

## III. EQUATION OF MOTION

The homogeneous equation of motion for harmonic vibrations of the system is

$$(\omega^2 \underset{\approx}{B} - \underset{\approx}{C})\, \underset{\sim}{\alpha} = 0 \quad , \tag{3.1}$$

where $\underset{\approx}{B}$ and $\underset{\approx}{C}$ are the inertia and stiffness matricies defined in terms of the kinetic energy T and the potential energy V by the expressions,

$$T = \frac{1}{2}\, \dot{\underset{\sim}{\alpha}} \cdot \underset{\approx}{B} \cdot \dot{\underset{\sim}{\alpha}} \quad \text{and} \quad V = \frac{1}{2}\, \underset{\sim}{\alpha} \cdot \underset{\approx}{C} \cdot \underset{\sim}{\alpha} \tag{3.2}$$

### A. The Inertia Matrix, $\underset{\approx}{B}$

The total kinetic energy of the system can be written

$$T = \frac{1}{2}\, m \int_{Vol} \rho_z(\underset{\sim}{v}_{1z} + \underset{\sim}{v}_{2z})^2 + \rho_n(\underset{\sim}{v}_{1n} + \underset{\sim}{v}_{2n})^2 \quad , \tag{3.3}$$

where the $\rho$'s are particle number densities and m is the nucleon mass. If one substitutes from Eqs. (2.5) and (2.12), performs the indicated integrals and then compares the resulting expression with Eq. (3.2), the components of $\underset{\approx}{B}$ are found to be

$$\begin{aligned} B_{11} &= B \quad , \\ B_{12} &= B_{21} = B \quad , \\ B_{22} &= \frac{a^2 - 2}{2}\, B \quad , \end{aligned} \tag{3.4}$$

where

$$B = m r_o^2 \left(\frac{NZ}{A^2}\right) A^{5/3} \quad , \tag{3.5}$$

and the quantity $(a^2 - 2)/2 = 1.17$.

## B. The Stiffness Matrix, $\underset{\approx}{C}$

In analogy with the determination of $\underset{\approx}{B}$ in the previous section, the values of the components of the stiffness matrix $\underset{\approx}{C}$ can be determined by calculating the Droplet Model potential energy as a function of $\alpha_1$ and $\alpha_2$ and matching the coefficients of the quadratic terms to the corresponding terms in Eq. (3.2).

The Droplet Model expression for the dependence of the potential energy on $\underset{\sim}{\alpha}$ is contained in the expression,[12]

$$V = \text{const} + \frac{1}{\frac{3}{4}\pi r_0^2} \int_{\text{vol}} J\delta^2 + \frac{1}{4\pi r_0^2} \int_{\text{surf}} \left(H\tau^2 + 2P\tau\delta_s - G\delta_s^2\right) . \tag{3.6}$$

The quantity $\tau$ is the distance between the equivalent sharp surface of the proton distribution and that of the neutron distribution (the "neutron skin thickness"), in units of $r_0$. The nuclear asymmetry $\delta$ is defined by the expression

$$\delta = (\rho_n - \rho_z)/\rho \tag{3.7}$$

and $\delta_s$ is simply the value of $\delta$ at the surface. The Droplet Model coefficients J, H, P and G serve to define the response of the system to variations in $\tau$ and $\delta$.

Of course, $\tau$ is a function of $\alpha_1$, and position on the surface through the expression

$$\tau = \tau_0 - \alpha_1 A^{1/3} \cos\theta \quad . \tag{3.8}$$

and $\delta$ is a function $\alpha_2$ and position through the expression

$$\delta = \delta_o - 2\alpha_2\left(\frac{NZ}{A^2}\right) Cj_1(kr) \cos\theta \quad , \tag{3.9}$$

which is obtained from Eq. (3.7) by substituting in Eqs. (2.7) and (2.8). The quantity $\delta_s$ is obtained from Eq. (3.9) by letting $r = R$ so that $kr \rightarrow a$.

The intergrals in Eq. (3.6) can be performed after substituting in Eqs. (3.8) and (3.9). The resulting expression in $\alpha_1$ and $\alpha_2$ can be compared with Eq. (3.2) in order to establish that the components of $\underset{\approx}{C}$ are,

$$C_{11} = \frac{2}{3} HA^{4/3} \quad ,$$

$$C_{12} = C_{21} = \frac{4}{3} PA\left(\frac{NZ}{A^2}\right) a^2 \quad , \tag{3.10}$$

$$C_{22} = JA\left(\frac{NZ}{A^2}\right)^2 4a^2(a^2 - 2) - \frac{8}{3} GA^{2/3}\left(\frac{NZ}{A^2}\right)^2 a^4 \quad .$$

Note that $C_{11}$, which is the coefficient describing the restoring force in the GT-mode, is proportional to the Droplet Model coefficient H rather than being proportional to J (the volume symmetry energy coefficient) as was assumed in the original work.[3] The coefficient H describes the resistance against the formation of a neutron skin. Another point to note is that the coefficient $C_{22}$, corresponding to the SJ-mode, consists not only of the usual volume term proportional to J, but also of a "surface term" proportional to the Droplet Model coefficient G. In fact, a good fit to the measured GDR energies has been obtained using only the SJ-mode and a Droplet Model-motivated, surface-dependent symmetry energy like that in Eq. (3.10).[2] The symmetry energy coefficients determined in this way were somewhat larger than those determined by a fit to nuclear masses.

The off-diagonal terms $C_{12}$ and $C_{21}$ provide the potential energy coupling between the modes because of the joint dependence of the surface energy on $\tau$ and $\delta_s$. In addition the two modes are inertially coupled through the terms $B_{12}$ and $B_{21}$ in the inertia matrix.

## IV. COMPARISON WITH EXPERIMENT

Equation (3.1) has solutions only when the determinant of the coefficients vanishes,

$$\det(\omega^2 \underset{\approx}{B} - \underset{\approx}{C}) = 0 \quad , \tag{4.1}$$

which leads to the expression,

$$\omega_\pm = \sqrt{\frac{D \pm \sqrt{D^2 - 4\det(\underset{\approx}{B}) \cdot \det(\underset{\approx}{C})}}{2\det(\underset{\approx}{B})}} \tag{4.2}$$

where

$$D = B_{11}C_{22} - 2B_{12}C_{12} + B_{22}C_{11} \quad . \tag{4.3}$$

The resulting eigen-energy of the system is

$$E_m = E_- = \hbar\omega_- \tag{4.4}$$

where $E_m$ represents the predicted mean energy of the GDR. The energy $E_+ = \hbar\omega_+$ corresponds to a higher lying mode (40-50 MeV) that does not couple strongly to the electromagnetic field because the GT- and SJ-modes are out of phase and the dipole moment of the mode nearly vanishes.

In order to compare the predictions of Eq. (4.4) with the measured values of $E_m$, we must first choose an appropriate set of values for the nuclear constants appearing in $\underset{\approx}{B}$ and $\underset{\approx}{C}$. From a recent Droplet Model fit to nuclear masses, fission barriers, etc,[13] we know that,

$$\begin{aligned} J &= 36.8 \text{ MeV} \quad , \\ Q &= 17 \text{ MeV} \quad , \\ r_0 &= 1.18 \text{ fm} \quad . \end{aligned} \tag{4.5}$$

The quantities H, P and G do not appear separately in the mass formula but only in the combination Q defined by[11,12]

$$Q = H/\left[1 - \frac{2}{3}\frac{P}{J}\right] \tag{4.6}$$

Another relationship exists among these coefficients, which is

$$G = \frac{3}{2}\frac{J}{Q}P \quad . \tag{4.7}$$

Consequently, the value of one of these coefficients can still be chosen arbitrarily.

Since the correlation which exists in the mass formula among the coefficients H, P and G which causes them to collapse into the single quantity Q did not seem to be present in our description of the GDR we hoped to use this model to determine their separate values. Unfortunately, the GDR energies $E_m$ also depend almost completely on Q, and only very broad limits can be set on the values of the other coefficients.

To see the effect of varying the values of the Droplet Model coefficients consider the cases given in Table I and the corresponding curves in Fig. 2. First, for guidance let us list the values of the coefficients obtained in a self-consistant Thomas-Fermi calculation of the properties of the nuclear surface. These are,[11]

$$\begin{aligned} H &= 9.42 \text{ MeV} \quad , \\ P &= 17.55 \text{ MeV} \quad , \\ G &= 45.4 \text{ MeV} \quad , \end{aligned} \tag{4.8}$$

which were calculated under the assumption that J = 28.1 and Q = 16. No other estimate of these quantities exists as far as we know.

In the three cases (a), (b) and (c) in Fig. 2, we have chosen arbitrarily to fix P while varying Q. These and other similar studies have served to convince us that the predicted values of $E_m$ are mainly dependent on Q and almost independent of the values of the other coefficients. The exception to this general observation has to do with the break in the energy curve where it drops abruptly to zero. This behavior is caused by the vanishing of $C_{22}$ because of the negative contribution to the symmetry energy. The vanishing of this term apparently is an artifact caused by the neglect of higher order terms in $C_{22}$. Still, we can assume that the model is sufficiently accurate to allow us to reject a set of coefficients like (d) that causes the energy to vanish in the moddle of the known mass region.

Figure 2 shows us that the general trend in the resonance energies $E_m$ is well reproduced using the value of Q from Eq. (4.5) and values of the other coefficients close to Eq. (4.8) as in case (b). The substantial difference in the magnitude of the measured and predicted values of $E_m$ is probably due to the fact that the curve (b) has been calculated using the nucleon mass m = 939 MeV/$c^2$ rather than an effective mass $m^* < m$.

Due to the presence of exchange forces in the nucleon-nucleon interaction it is necessary to use an effective mass $m^*$ when discussing the movement of neutrons against protons.[15-19] The effect is most clearly seen in the integrated absorption cross-sections, which are as much as 1.4 times as large as the sum-rule limits. This apparent discrepancy has been resolved by recognizing the need for an effective mass in the cross-section calculations, but the need for an effective mass has usually been ignored when the resonance energy is discussed.[20] Probably, the harsh constraints inherent in the GT- or SJ-modes when used alone served to artifically force the predicted energy upward giving a spurious agreement with experiment so that the use of an effective mass did not seem to be required.

If we replace m by m* = 0.7 m in $\underset{\approx}{B}$ and use the set of Droplet Model coefficients corresponding to case (b) in Table I the predicted values of $E_m$ correspond to the solid curve in Fig. 3. The dashed curves are given to indicate the best fits that can be obtained with the GT- or SJ-modes alone. The excellent agreement between our calculations and the measured values is obtained (in essence) without any adjustable parameters except for the value of m*, and even this is comparable to the value that is required by the integrated cross-sections.

The ratio of the relative contributions of the JS- and GT-modes to the GDR is plotted in Fig. 4. One can see from this figure, that even though the two modes are coupled in this model, the GT-mode dominates over the whole nuclear mass region. For the light nuclei the GDR is predicted to be an almost pure GT-mode. Only for heavier nuclei (that are softer with respect to neutron and proton density variations) does the SJ-mode begin to become important. The only other calculation we know about that considers the GDR to be a mixture of GT- and SJ-modes is that of Bertsch and Stricker[21] who do a microscopic RPA calculation with a variation on the transition charge density. Their results support our conclusion about the dominance of the GT-mode. This work casts serious doubt on much of the work done on lighter nuclei which assumes a hydrodynamical model of the SJ-type.

## V. ESTIMATE OF THE WIDTH

The width of the GDR can be estimated on the biasis of the one-body damping expression of Swiatecki, et al.[22,23] which can be written,

$$\dot{E} = \rho\bar{v} \oint_{surf} \dot{n}^2 \quad . \tag{5.1}$$

This expression is based on a semi-classical approach to the damping of collective motion into intrinsic states and is obtained from ordinary kinetic theory by considering volume conserving, but non-adiabatic, motion of the walls of a system of non-interacting particles in a container. Even though the expression is obtained on the basis of classical considerations alone, its motivation comes from the observation that the microscopic origin of damping is concentrated in the surface region.

Equation (5.1) states that the rate of energy absorption $\dot{E}$ is equal to the product of the nuclear density $\rho$ and the average particle velocity $\bar{v}$ with an integral over the surface of the square of the normal surface velocity $\dot{n}^2$. Notice that there are no adjustable parameters such as one encountered in ordinary hydrodynamic viscosity or in formulations that postulate "frictional" forces between interpenetrating density distributions in relative motion with respect to each other.[24,25]

The SJ-mode is unaffected by this damping mechanism since there is (by definition) no motion normal to the surface. However, the GT-mode is damped since it consists of neutrons and protons vibrating back and forth against the walls of their common potential. The normal velocities of the protons and neutrons are,

$$\dot{n}_z = -\frac{N}{A}\cos\theta\,\dot{\alpha}_1 \quad ,$$
$$\dot{n}_n = \frac{Z}{A}\cos\theta\,\dot{\alpha}_1 \quad . \tag{5.2}$$

Substitution of Eq. (5.2) into Eq. (5.1) and subsequent integration yields

$$\dot{E} = D_{11}\dot{\alpha}_1^2 \quad , \tag{5.3}$$

where

$$D_{11} = mA\left(\frac{NZ}{A^2}\right)\bar{v}R \quad . \tag{5.4}$$

The quantity $D_{11}$ is an element in the damping matrix $\underset{\approx}{D}$, where $D_{12} = D_{21} = D_{22} = 0$, and the equation of motion can be written,

$$\underset{\approx}{B}\underset{\sim}{\ddot{x}} + \underset{\approx}{D}\underset{\sim}{\dot{x}} + \underset{\approx}{C}\underset{\sim}{x} = 0 \quad . \tag{5.5}$$

For a pure GT-mode the damping width is given by,

$$\Gamma_1 = \hbar\frac{D_{11}}{B_{11}} = \hbar\left(\frac{\bar{v}}{R}\right) \quad , \tag{5.6}$$

where $\bar{v} = \frac{3}{4} v_{Fermi}$ and R is the nuclear radius. We might have guessed this result ahead of time since is is the simplest expression one can formulate that has the correct dimensions.

Of course, the actual GDR motion is not a pure GT-mode. The width can be estimated by calculating the importance of the damping term in Eq. (5.5) when the motion is constrained to be that of the unperturbed eigen-mode. In that case

$$\Gamma = \hbar\,\frac{\langle \underset{\approx}{D} \rangle}{\langle \underset{\approx}{B} \rangle} \quad , \tag{5.7}$$

where

$$\langle \underset{\approx}{D} \rangle = \underset{\sim}{\alpha}_- \cdot \underset{\approx}{D} \cdot \underset{\sim}{\alpha}_- \quad ,$$

$$\langle \underset{\approx}{B} \rangle = \underset{\sim}{\alpha}_- \cdot \underset{\approx}{B} \cdot \underset{\sim}{\alpha}_- \quad , \qquad (5.8)$$

and $\underset{\sim}{\alpha}_-$ is the eigen-vector corresponding to $\omega_-$ which is the motion we have identified as the GDR.

The values of Γ predicted by Eq. (5.7) are compared with the measured values in Fig. 4. We see that only about half of the observed width is accounted for by this mechanism.

## VI. DISCUSSION

One of the factors that limited the amount of serious interest applied to the GT-mode in the past was the unsatisfactory procedure by which the restoring force (which is clearly a surface phenomena) was related to the volume symmetry energy coefficient. This objection does not apply to our work since we use the Droplet Model, where the symmetry dependence of the surface energy is explicitly displayed, to calculate the restoring forces. (See also Ref. 9, where a Thomas-Fermi model is used to estimate the actual surface displacement restoring forces.)

Even the SJ-mode descriptions of the GDR have been somewhat deficient in the past because of their neglect of the influence of neutron excess at the surface on the surface energy. (This effect is included implicitly in the work of Brennan and Werntz[10] and explicitly in an estimate made by Berman.[2]) As above, we find that this important effect is automaticly included when the Droplet Model is used.

But the importance of the work described here is not in the improvements that have been made in estimating the restoring forces, rather it is in the recognition that a macroscopic description of the GDR should contain aspects of both the GT- and SJ-modes. We find (as many microscopic calculations have suggested)[21,26-28] that the motion in light nuclei corresponds closely to a pure GT-mode. Since our model includes both possible modes, we find that the SJ-mode becomes more important for heavier nuclei that are less resistant to variations in the neutron and proton density distributions. Recognition of the composite nature of the motion has a number of important consequences. One of these is the lowering of the predicted energies as a consequence of the reduction of the number of constraints on the motion. The energy is lowered to an extent that cannot be corrected by any reasonable readjustment of the restoring forces. Consequently, one is able to clearly see the need for the use of an effective mass in calculating the energies. (As was already recognized in the calculation of the total absorption cross-section.) Another important consequence is that many of the results obtained by coupling the motion to surface degrees of freedom (such as in the "dynamic collective model")[7,8] need to be reconsidered in light of the fact the motion is probably not a pure SJ-mode as has often been assumed.

The success of this model in describing the GDR raises the exiting prospect of a more refined treatment of other giant-resonance phenomena as well. The methods used here (1. Recognition of the neutron skin thickness as a degree-of-freedom, and the use of Droplet Model restoring forces, and 2. The treatment of the collective motion as a combination of neutron and proton surface displacements with bulk hydrodynamic modes), can be applied to monopole, quadrupole and higher modes as well. Of course, our experience with the GDR leads us to expect the surface displacements to dominate for light nuclei with the hydrodynamic bulk density variations becoming more important with increasing nuclear mass number.

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FIGURE CAPTIONS

Fig. 1. Schematic drawings that serve to illustrate the general features of the Goldhaber-Teller[3] (GT) and Steinwedel-Jensen[4] (JS) dipole modes. For each case, one-half cycle of the vibration is shown as a function of time. In the GT-mode a uniform proton distribution (dashed) vibrates against the neutron distribution (solid). In the SJ-mode the neutrons tend to pile up first on one side of the nucleus and then the other. The protons (not shown) move in the opposite direction so the total density remains constant.

Fig. 2. The measured values of the mean energy $E_m$ of the GDR are plotted against the mass number A. The dots are from the Lorentz curve fits of Ref. 2 and the crosses are from Refs. 1 and 14. The four curves labeled (a), (b), (c) and (d) correspond to the different sets of Droplet Model coefficients given in Table I, and discussed in the text. The curves all lie below the measured values because they were calculated (for purpose of illustration) with an effective mass $m^* = m$.

Fig. 3. The measured values of $E_m$ are the same as those in Fig. 2. The solid curve corresponds to the set of coefficients labeled (b) in Table I, and an effective mass $m^* = 0.7$ m. The two dashed lines (which are normalized to pass through the same point at A = 100) serve to illustrate the $A^{-1/6}$ and $A^{-1/3}$ behavior that is characteristic of the GT- and SJ-modes when they are considered separately.

Fig. 4. The predicted ratio of the SJ-component of the GDR to the GT-component is plotted against the nuclear mass number A. The resonance is seen to be an almost pure GT-mode for light nuclei while the SJ-mode begins to play a more important role with increasing mass.

Fig. 5. The measured width Γ of the GDR (obtained from the Lorentz curve fits of Ref. 2) is plotted against the mass number A. The dots correspond to single Lorentz curve fits to (presumedly) spherical nuclei, while the crosses correspond to mean values for deformed nuclei calculated from the expression $\Gamma = \sqrt[3]{\Gamma_1 \Gamma_2^2}$. The solid curve corresponds to the predictions of Eq. (5.7) which is based on the concept of one-body damping.[22] The dahsed curve shows how the width would be increased if the motion consisted entirely of the GT-mode, which is the only one effected by this type of damping.

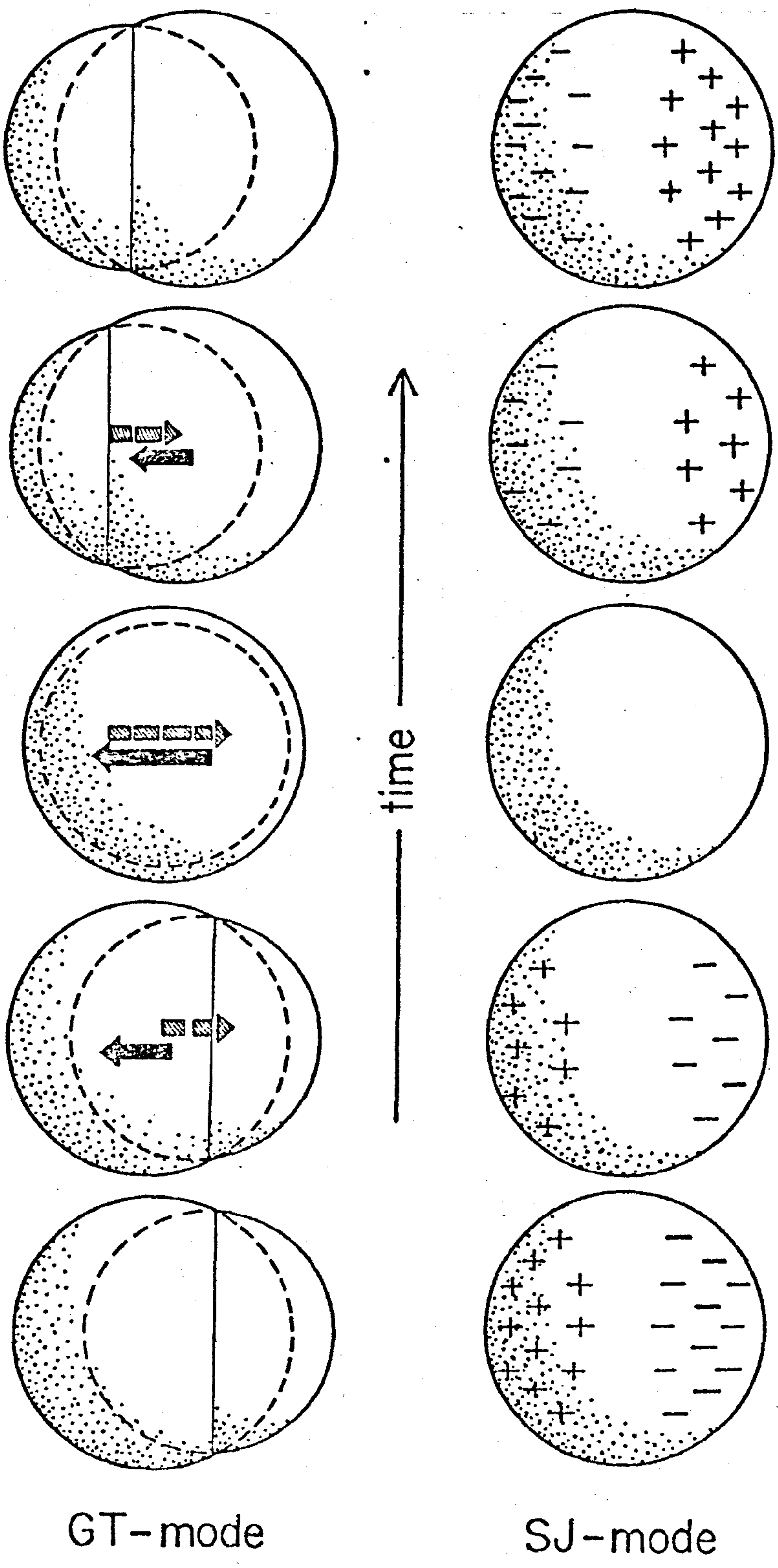


Fig. 1

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Table I. Different sets of Droplet Model coefficients (in MeV) corresponding to the curves shown in Fig. 2.

| Curve | Q | H | P | G |
|---|---|---|---|---|
| (a) | 20 | 14.12 | 16.26 | 44.81 |
| (b) | 17 | 12 | 16.26 | 52.72 |
| (c) | 14 | 9.88 | 16.26 | 64.01 |
| (d) | 17 | 10 | 22.73 | 73.80 |

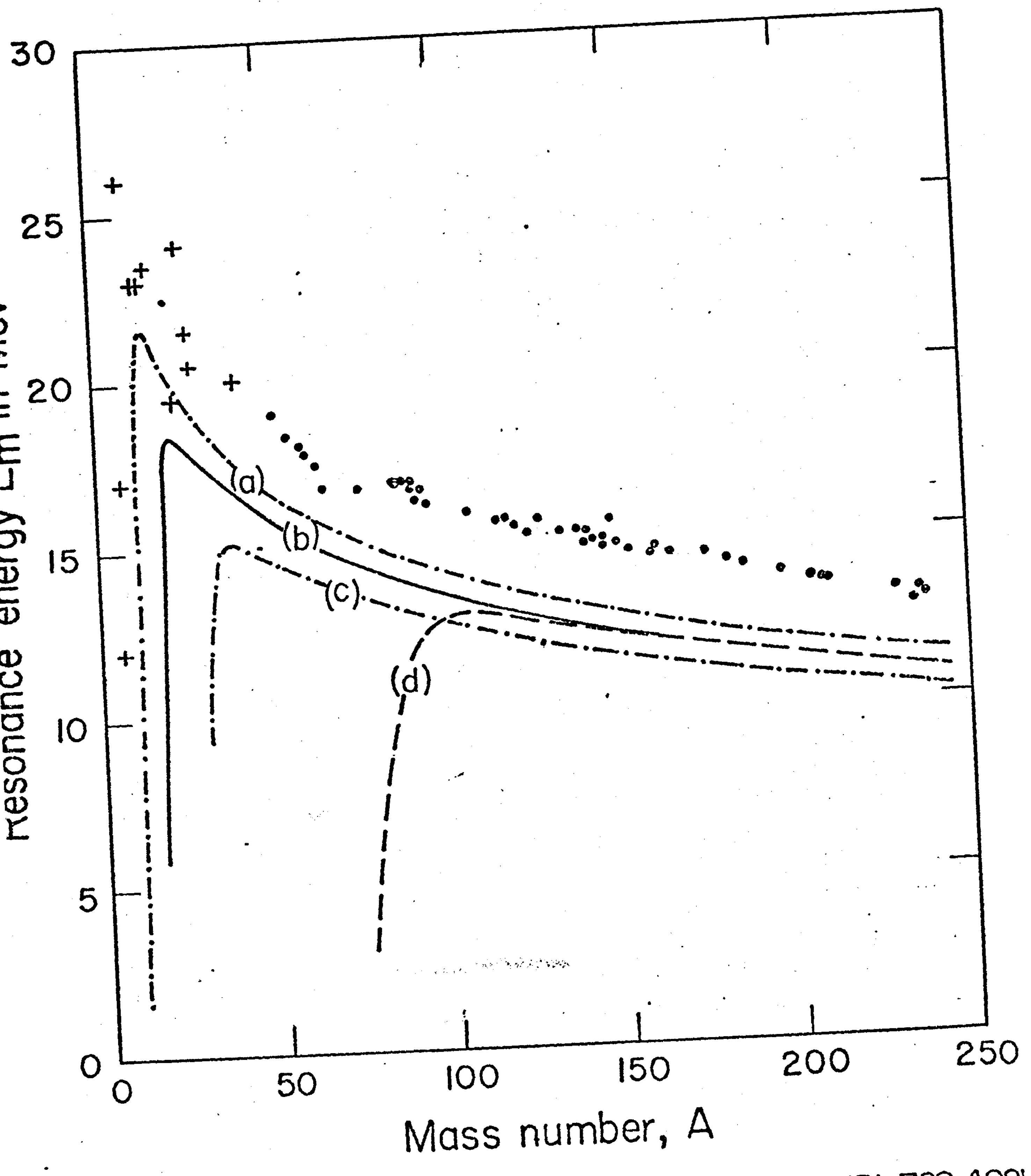
Resonance energy Em in MeV
Mass number, A
0
50
100
150
200
250
5
10
15
20
25
30
(a)
(b)
(c)
(d)
XBL 769 4021

Fig. 2.

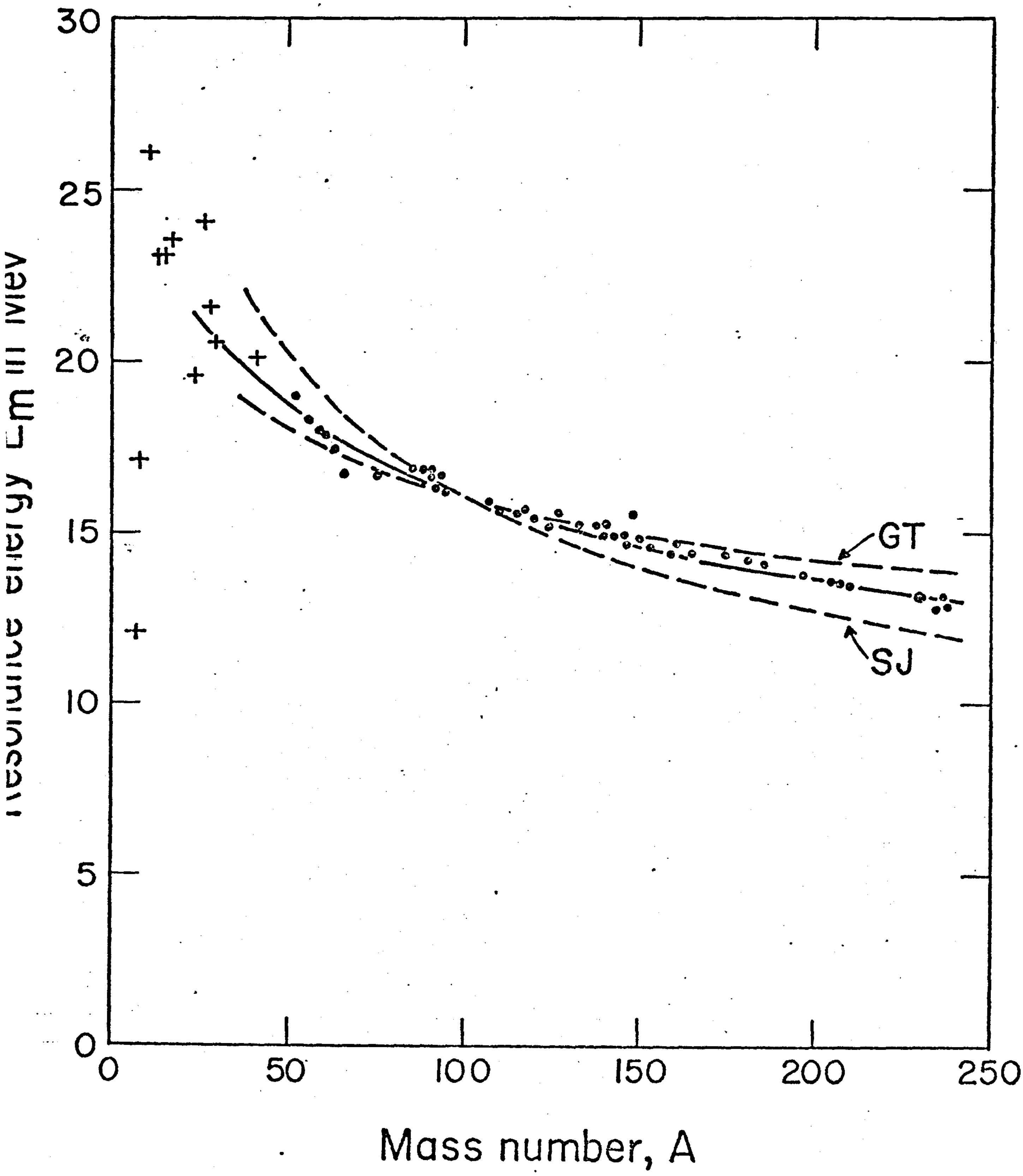


Fig. 3

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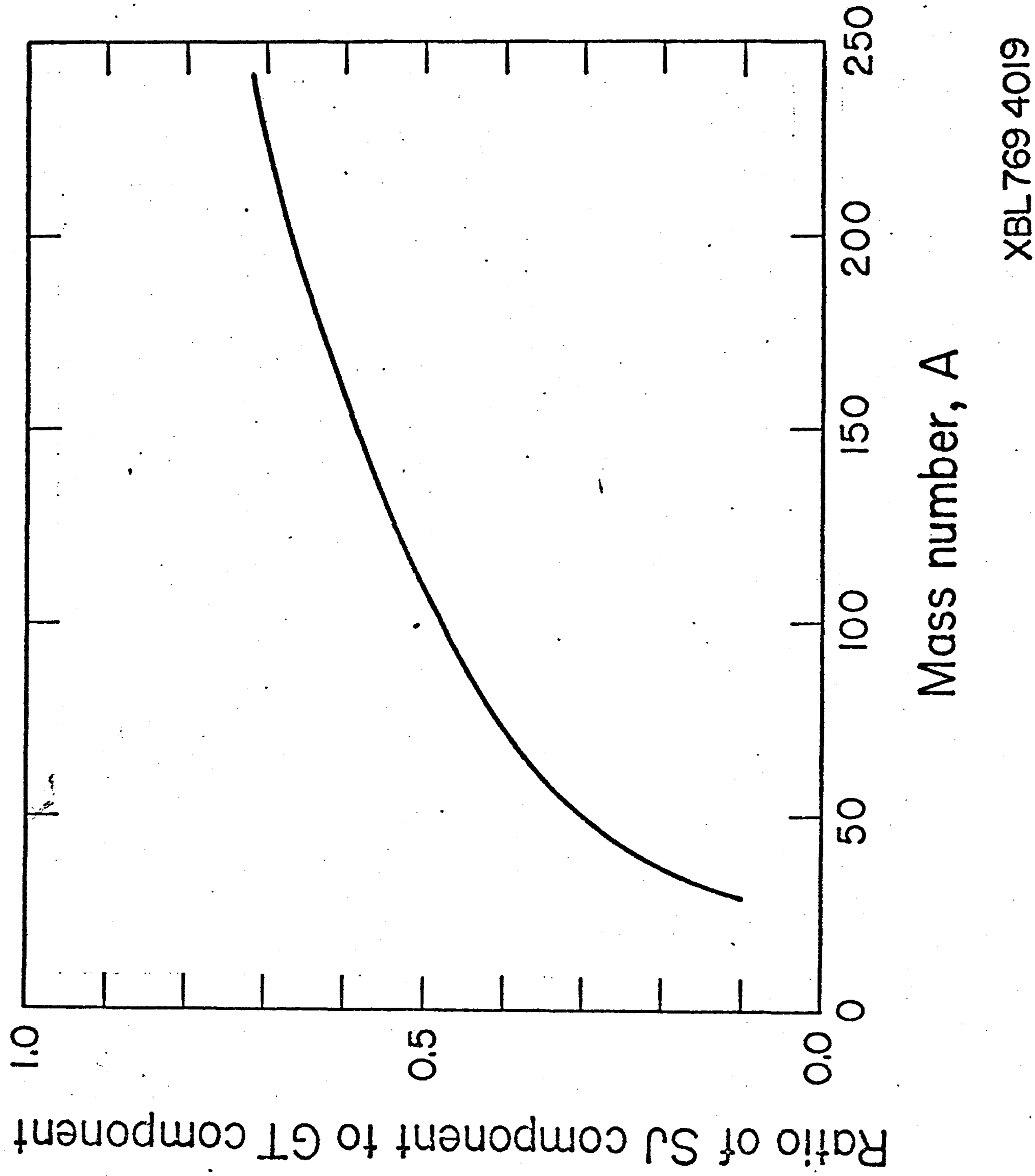
Ratio of SJ component to GT component
1.0
0.5
0.0
0
50
100
150
200
250
Mass number, A
XBL 769 4019

Fig. 4

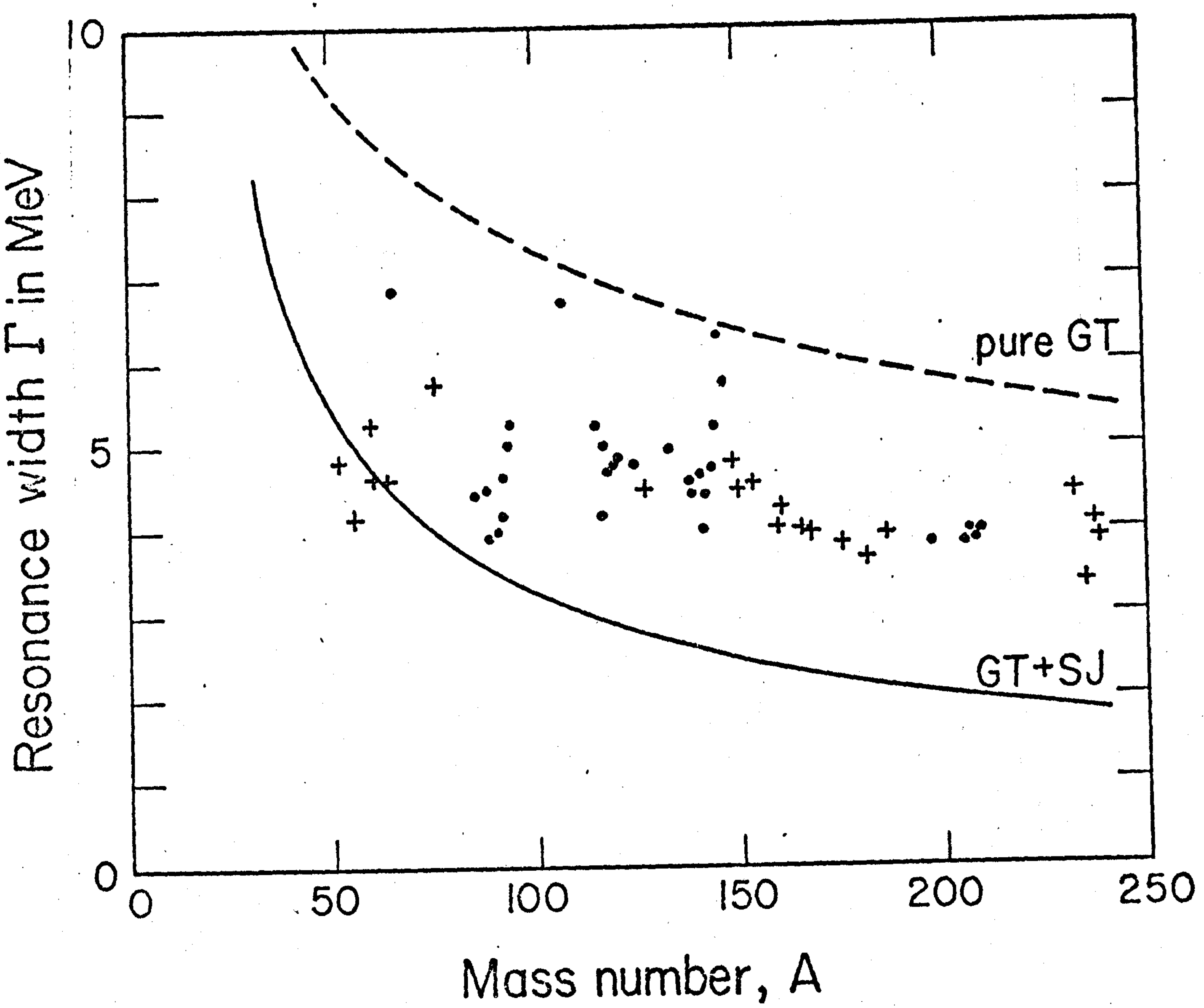


Fig. 5

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