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The Quantum Mechanics of a "Spinor-twin" Type II Superparticle

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ABSTRACT

Ten dimensional supersymmetric Yang-Mills theory may be described, in the light-cone gauge, in terms of either a vector or spinor superfield satisfying certain projection conditions (type I or II). These have been presented in a $SO(9,1)$ form, and used to construct spinning superparticle theories in extended spaces. This letter presents the covariant quantisation of a "spinor-twin" type II superparticle theory by using the standard techniques of Batalin and Vilkovisky. The quantum action defines a quadratic field theory, whose ghost-independent BRST cohomology class gives the spectrum of N=1 super Yang-Mills.

Key-words: Superparticles; BV method; Field antifield formalism; Supersymmetry.

Discussions of the mechanics of particles with spin shows that these can be described by either a particle theory with local world-line supersymmetry [1], or by a local fermionic symmetry[2]. This was generalised to superpace, to obtain a number of *spinning* superparticle theories satisfying certain constraints whose spectrum were precisely those of the ten-dimensional supersymmetric Yang-Mills theory. The quantum mechanics of a free superparticle in a ten-dimensional space-time is of interest because of its close relationship to ten-dimensional super Yang-Mills theory, and this corresponds to the massless sector of type I superstring theory. The $SO(9,1)$ covariant superfield formulation of super Yang-Mills which reduces to $SO(8)$ ones may be obtained by either an $SO(9,1)$ vector or spinor superfields. These superfields are chosen to satisfy either rotational quadratic ("Type I") or linear ("Type II") constraints that restricts their field content to the physical propagating fields. The constraints are imposed by an explicit projection operator, constructed out of supercovariant derivatives, acting on unconstrained superfields. These were explicitly given on its $SO(8)$ form in [3], and presented on its $SO(9,1)$ form in [2]. The spinor and vector superfields are related by $\gamma^i_{\dot{a}b}\Psi_b = (1/8)D_{\dot{a}}\Psi^i$. Yet, it is well known the abstruse quantisation of superparticle models in a covariant manner [4], there are by now several formulations which can be covariantly quantized. These superparticle theories with spectra coinciding with that of the super Yang-Mills are constructed by adding appropriate Lagrange multiplier terms to certain superparticle actions, some of them leading to these type I (II) constraints.

This letter presents the covariant quantisation of a "spinor-twin" type II superparticle model, by using the Batalin-Vilkovisky formalism [5]. The methods of [6,7] are used to argue that the zero ghost-number BRST cohomology class in the reduced formalism is exactly the same as the zero ghost-number cohomology class in the full formalism with an infinite number of ghosts.

We begin by briefly reviewing the description of ten-dimensional "spinor-twin" type I superparticle models [2]. A spinor wavefunction can be obtained either from a spinning particle with local world-line supersymmetry, or from a particle action with local fermionic symmetry. In [2], it was seen that super Yang-Mills theory in ten-dimensions is described by precisely such wavefunctions subject to certain extra super-covariant constraints. The quantum mechanics of the spinor-twin type I superparticle theory was given in [7]. This superparticle action is formulated in an extended ten-dimensional superspace with coordinates $(x^\mu, \theta_A, \phi^A)$ where θ_A and ϕ^A are anti-commuting Majorana-Weyl spinors.♠ The physical

♠ A Majorana spinor Ψ corresponds to a pair of Majorana-Weyl spinors, Ψ_A and Ψ^A. The 32×32

states are described by a superspace wavefunction satisfying [2]

$$p^2\Psi_A=0,\qquad \not{p}_{AC}D^C\Psi_B=0,\qquad \not{p}^{AB}\Psi_B=0,\\ D^AD^B\Psi_C+8(\gamma^\mu)^{E[A}(\Gamma_\mu\not{p})^{B]}{}_C\Psi^D=0. \tag{1}$$

which leaves a superfield $\Psi_a(x^i,\theta^{\dot{a}})$ satisfying a quadratic projection condition which is precisely the $SO(8)$ constraint of ten dimensional super Yang-Mills theory. The covariant quantisation of this superparticle was briefly discussed in [7] in the gauge $e=1$ with the other gauge fields set to zero. Covariant quantisation requires the methods of Batalin and Vilkovisky [5] since the gauge algebra only closes on shell, and requires an infinite number of ghost fields since the symmetries are infinitely reducible. Following the BV procedure leads to a gauge-fixed quantum action which, after field redefinitions and integrating out all non-propagating fields, takes the form [7]

$$S_Q=\int d\tau\left[p_\mu\dot{x}^\mu-\frac{1}{2}p^2+i\hat{\theta}\dot{\theta}+i\hat{\phi}\dot{\phi}+\hat{c}\dot{c}+\hat{\kappa}\dot{\kappa}+i\hat{v}\dot{v}+i\hat{\rho}\dot{\rho}+\hat{\zeta}\dot{\zeta}\right], \tag{2}$$

where $\hat{\theta}=d-\not{p}\theta-4\hat{c}\kappa$. As the quantum action defines a free field theory, it is easy to quantize by imposing canonical commutation relations on the operators corresponding to the variables $(p_\mu,x^\mu,\hat{\theta},\theta,\hat{\phi},\phi,\hat{c},c,\hat{\kappa},\kappa,\hat{v},v,\hat{\rho},\rho,\hat{\zeta},\zeta)$. It proves useful to choose a Fock space representation for the ghost and define a ghost vaccum $|0>$ which is annihilated by each of the antighosts ($\hat{\kappa}|0>=0,\hat{c}|0>=0,\hat{v}|0>=0,\hat{\rho}|0>=0,\hat{\zeta}|0>=0$). It also proves useful to define a *twisted* ghost vacuum $|0>_g$, where for each ghost g in the subscript, that ghost is an annihilation operator and the corresponding anti-ghost is a creation operator. The physical states on both twisted and untwisted Fock space should be the same, as they are *dual* representations of the same spectrum. It is then viewed the superspace coordinates x^μ ,$\hat{\theta}_A$ and ϕ^A as hermitian coordinates while $p_\mu=-i\partial/\partial x^\mu$, $\hat{\theta}_A=\partial/\partial\theta_A$ and $\hat{\phi}^A=\partial/\partial\phi^A$, and consider states of the form $\Phi(x,\theta,\phi)M|\Omega>$ with wavefunction Φ, where M is some monomial constructed from the (anti-)ghost and $|\Omega>$ is one of the ghost vacua. It was found then that the ghost-independent state $\Phi(x,\theta,\phi)|0>$ gives the physical spectrum consisting of the eight bosons and eight fermions which form the Yang-Mills multiplet together with the zero-momentum ground state which is a supersymmetry singlet.

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matrices $C\gamma^\mu$ (where C is the charge conjugation matrix) are block diagonal with 16×16 blocks $\gamma^{\mu AB}$, $\gamma^\mu{}_{AB}$ which are symmetric and satisfy $\gamma^{\mu AB}\gamma^\nu{}_{BC}+\gamma^{\nu AB}\gamma^\mu{}_{BC}=2\eta^{\mu\nu}\delta^A_C$. In this notation the supercoordinates has components θ_A, $\theta\gamma^\mu\theta=\theta_A\gamma^\mu{}_{AB}\theta_B$, $\not{p}_{AB}=p^\mu\gamma^\mu{}_{AB}$, etc.

I shall now describe a ten-dimensional spinor-twin type II superparticle model with a spinor super-wavefunction satisfying

$$\begin{gathered} p^2\Psi_A = 0, \qquad \not{p}_{AC}D^C\Psi_B = 0, \qquad \not{\phi}^{AB}\Psi_B = 0, \\ (\gamma^{\mu\nu\rho\sigma})_A{}^B D^A\Psi_B = 0, \end{gathered} \tag{3}$$

which is equivalent to constraints (1). The model is formulated in an extended ten-dimensinal superspace with coordinates $(x^\mu, \theta_A, \phi^A)$ where θ_A and ϕ^A are anticommuting Majorana-Weyl spinors, and to describe super Yang-Mills we wish to impose the extra constraints $d^A(\Gamma^{\mu\nu\rho\sigma})^B{}_A = 0$ and $\hat{\phi}\hat{\phi} = 0$ which can be done by adding appropriate lagrange multiplier terms.

The spinor-twin type II superparticle action is then given by the sum of [2]

$$S_0 = \int d\tau \Big[p_\mu \dot{x}^\mu + i\hat{\theta}\dot{\theta} + i\hat{\phi}\dot{\phi}\Big], \tag{4}$$

and

$$\begin{aligned} S'' = \int d\tau \Big[-\frac{1}{2}ep^2 + i\psi\not{p}d + i\varphi\not{p}\hat{\phi} + i\Lambda_{\mu\nu\rho\sigma} d\Gamma^{\mu\nu\rho\sigma}\hat{\phi} \\ - i\beta(\phi\hat{\phi} - 1) + \frac{1}{2}\hat{\phi}\omega\hat{\phi}\Big], \end{aligned} \tag{5}$$

where, as usual, p_μ is the momentum conjugate to the space-time coordinate x^μ, d^A is a spinor introduced so that the Grassmann coordinate θ_A has a conjugate momentum $\hat{\theta}^A = d^A - \not{p}^{AB}\theta_B$, ϕ^A is a new spinor coordinate and $\hat{\phi}_A$ is its conjugate momentum. The fields e, ψ^A, φ_A, $\Lambda_{\mu\nu\rho\sigma}$, β and $\omega^{AB} = -\omega^{BA}$ are all Lagrange multipliers (which are also gauge fields for corresponding local symmetries) imposing the following constraints

$$\begin{gathered} p^2 = 0, \qquad \not{p}d = 0, \qquad \not{p}\hat{\phi} = 0, \\ \hat{\phi}_A\hat{\phi}_B = 0, \qquad \phi^A\hat{\phi}_A - 1 = 0, \qquad d^A(\Gamma^{\mu\nu\rho\sigma})^B{}_A\hat{\phi}_B = 0. \end{gathered} \tag{6}$$

The action (4)-(5) is invariant under the global space-time supersymmetry transformations [2]

$$\delta\theta = \epsilon, \qquad \delta x^\mu = i\epsilon\Gamma^\mu\theta, \tag{7}$$

(where ϵ_A is a constant Grassmann parameter) together with a number of local symmetries.♠ These include world-line reparameterization which, when combined with a *trivial* symmetry,

♠ The symmetries divide into two types [8]. Symmetries of the *first* type are those under which a gauge field transforms into the derivative of a gauge parameter, while of the *second* type are those which involves only gauge fields.

gives the $\mathcal{A}$ transformations

$$\delta e = \dot{\xi}, \qquad \delta x^{\mu} = \xi p^{\mu}, \tag{8}$$

the other fields being inert. There are also two fermionic symmetries of the first kind, $\mathcal{B}$ and $\mathcal{B}$', with fermionic spinor parameters $\kappa^A(\tau)$ and $\varphi_A(\tau)$ given by

$$\begin{gathered} \delta\psi = \dot{\kappa}, \qquad \delta\theta = \not{p}\kappa, \qquad \delta e = 4i\dot{\theta}\kappa, \\ \delta x^{\mu} = id(\Gamma^{\mu})\kappa + i\theta(\Gamma^{\mu})\not{p}\kappa, \end{gathered} \tag{9}$$

and

$$\delta\varphi = \dot{\zeta} + \beta\zeta, \qquad \delta\phi = \zeta\not{p}, \qquad \delta x^{\mu} = i\hat{\phi}(\Gamma^{\mu})\zeta \tag{10}$$

where ζ_A is a spinor parameter. The bosonic symmetries associated with the gauge fields β and ω^{AB} (the $\mathcal{C}$ and $\mathcal{C}$' symmetries) are defined by

$$\begin{gathered} \delta\beta = \dot{\eta}, \qquad \delta\hat{\phi} = \eta\hat{\phi}\ , \qquad \delta\phi = -\eta\phi, \\ \delta\omega = -2\eta\omega, \qquad \delta\Lambda_{\mu\nu\rho\sigma} = \eta\Lambda_{\mu\nu\rho\sigma}\ , \qquad \delta\varphi = -\eta\varphi\ , \end{gathered} \tag{11}$$

and

$$\delta\omega = \dot{\Upsilon} + 2\beta\Upsilon, \qquad \delta\phi = i\Upsilon\hat{\phi}, \tag{12}$$

where η is a bosonic parameter and $\Upsilon^{AB} = -\Upsilon^{BA}$ is a bosonic bispinor parameter. There is also a tensor symmetry associated with the gauge field $\Lambda_{\mu\nu\rho\sigma}$ (referred to as $\mathcal{F}$ symmetry) with bosonic parameter $\Sigma_{\mu\nu\rho\sigma}$ and given by

$$\begin{aligned} &\delta\Lambda_{\mu\nu\rho\sigma} = \dot{\Sigma}_{\mu\nu\rho\sigma} + \beta\Sigma_{\mu\nu\rho\sigma}, \qquad \delta d = -2\hat{\phi}\not{\Sigma}\not{p}, \\ &\delta\theta = -\hat{\phi}\not{\Sigma}\ , \qquad \delta\phi = d\not{\Sigma}, \qquad \delta e = 4i\hat{\phi}\not{\Sigma}\psi, \\ &\delta x^{\mu} = i\hat{\phi}\not{\Sigma}(\Gamma^{\mu})\theta\ , \qquad \delta\omega = 4i\not{\Sigma}\not{p}\not{\Lambda}. \end{aligned} \tag{13}$$

The gauge algebra of the symmetries $\mathcal{A}$, $\mathcal{B}$, $\mathcal{B}$', $\mathcal{C}$, $\mathcal{C}$' and $\mathcal{F}$ closes on shell. In this situation, the Batalin and Vilkovisky procedure can be used in order to determine the quantum action and the BRST charge. The approach which is in principle the most straightforward, but turns out to be technically the most complicated, involves introducing ghost fields corresponding to each of the symmetries $\mathcal{A}$, $\mathcal{B}$, $\mathcal{B}$', $\mathcal{C}$, $\mathcal{C}$' and $\mathcal{F}$. The minimal set of fields that enter the BV quantisation scheme is determined by the classical gauge symmetries, together with the requirement that the BRST transformations of the classical fields and the ghost should be on-shell nilpotent. This procedure fixed much of the structure of the master action.

We introduce ghosts $(c, \kappa_1, \zeta_1, \eta_1, v_1, \not{\Psi}_1)$ corresponding to the classical symmetries (8)-(13). These ghost fields have opposite Grassmann parity to the set of classical gauge parameters $(\xi, \kappa, \varphi, \beta, \omega, \Sigma)$. The BRST transformations are

$$\begin{aligned}
&se = \dot{c} + 4i\dot{\theta}\kappa_1 + 4i\hat{\phi}\not{\Psi}_1\psi\ , \qquad s\psi = \dot{\kappa}_1\ , \qquad s\beta = \dot{\eta}_1, \\
&sx^\mu = cp^\mu + id(\Gamma^\mu)\kappa_1 + i\theta(\Gamma^\mu)\not{p}\kappa_1 + i\hat{\phi}(\Gamma^\mu)\zeta_1 + i\hat{\phi}\not{\Psi}_1(\Gamma^\mu)\theta \\
&s\theta = \not{p}\kappa_1 - \hat{\phi}\not{\Psi}_1\ , \qquad s\varphi = \dot{\zeta}_1 + \beta\zeta_1 - \eta_1\varphi\ , \qquad s\hat{\phi} = \eta_1\hat{\phi}\ , \\
&s\phi = \zeta_1\not{p} - \eta_1\phi + iv_1\hat{\phi} + d\not{\Psi}_1\ , \qquad s\omega = -2\eta_1\omega + \dot{v}_1 + 2\beta v_1 + 4i\not{\Psi}_1\not{p}\not{X}\ , \\
&s\Lambda_{\mu\nu\rho\sigma} = \eta_1\Lambda_{\mu\nu\rho\sigma} + \dot{\Sigma}_{(1)\mu\nu\rho\sigma} + \beta\Sigma_{(1)\mu\nu\rho\sigma}\ , \qquad sd = -2\hat{\phi}\not{\Psi}_1\not{p}, \\
&s\zeta_1 = -\eta_1\zeta_1\ , \qquad sv_1 = -2\eta_1 v_1 + 2i\not{\Psi}_{(1)}\not{p}\not{\Psi}_{(1)}\ , \qquad s\kappa_n = i^n\not{p}\kappa_{(n+1)}\ , \\
&s\eta_1 = \eta_1\eta_1, \qquad s\not{\Psi}_{(1)} = \not{\Psi}_{(1)}\eta_1, \qquad sc = -2i\kappa_1\not{p}\kappa_1 + 4i\kappa_1\not{\Psi}_{(1)}\hat{\phi}
\end{aligned} \tag{14}$$

The minimal set of fields Φ^A_{min} consists of all classical and ghost fields that furnish a representation of the BRST algebra,

$$\Phi^A_{min} = \Big\{x, p, e, c; \theta, d, \psi, \kappa_1, \ldots, \kappa_n; \phi, \hat{\phi}, \varphi, \zeta_1; \beta, \eta_1; \omega, v_1; \Lambda_{\mu\nu\rho\sigma}, \Sigma_{(1)\mu\nu\rho\sigma}\Big\}. \tag{15}$$

In addition to the above minimal set of fields, gauge fixing requires the introduction of anti-ghosts, extra-ghosts and Nakanishi-Lautrup (NL) auxiliary fields

$$\Phi^A_{non-min} = \Big\{\hat{c}, \hat{\kappa}_1, \ldots, \hat{\kappa}_n, \hat{\zeta}_1, \hat{\eta}_1, \hat{v}_1, \hat{\not{\Psi}}_1; \kappa^m_n; \pi^m_n\pi_c, \pi_1, \ldots \pi_n, \pi^m_n\pi_\zeta, \pi_\eta, \pi_v, \pi_\Sigma\Big\}. \tag{16}$$

For each field Φ^A the BV method requires the introduction of a corresponding anti-field $\Phi^\star_A$ of opposite Grassmann parity. Next, we need to find a solution $S(\Phi^A, \Phi^\star_A)$ to the master equation $(S,S) = 0$ subject to the boundary condition $S|_{\Phi^\star_A=0} = S_0 + S''$. Yet, care must be taken, as the grading of the fields plays an important role. The solution to the master equation for the minimal set of field Φ^A_{min} is

$$S_{min} = S_0 + S'' + S_1 + S_2, \tag{17}$$

where $S_0 + S''$ is the classical action of the spinor-twin type II superparticle (4)-(5). The term linear in anti-fields, S_1, is

$$\begin{aligned} S_1 = \int d\tau \Big[& x^\star_\mu (cp^\mu + i\theta\gamma^\mu \not{p}\kappa_1 + id\gamma^\mu\kappa_1 + i\hat{\phi}\gamma^\mu\zeta_1 - i\hat{\phi}\not{\Sigma}_1\gamma^\mu\theta) \\ & + \theta^\star(\not{p}\kappa_1 - \hat{\phi}\not{\Sigma}_1) - d^\star(2\hat{\phi}\not{\Sigma}_1\not{p}) + e^\star(\dot{c} + 4i\dot{\theta}\kappa_1 + 4i\hat{\phi}\not{\Sigma}_1\psi) \\ & + \psi^\star(\dot{\kappa}_1) + \hat{\phi}^\star(\eta_1\hat{\phi}) + \phi^\star(\not{p}\zeta_1 + iv_1\hat{\phi} - \eta_1\phi + d\not{\Sigma}_1) \\ & + \varphi^\star(\dot{\zeta}_1 + \beta\zeta_1 - \eta\varphi) + \not{A}^\star(\eta_1\not{A} + \dot{\not{\Sigma}}_1 + \beta\not{\Sigma}_1) + \beta^\star(\dot{\eta}_1) \\ & - \zeta^\star_1(\eta_1\zeta_1) + \eta^\star_1(\eta_1\eta_1) + \omega^\star(-2\eta_1\omega + \dot{v}_1 + 2\beta v_1 + 4i\not{\Sigma}_1\not{p}\not{A}) \\ & + \kappa^\star_n(i^n\not{p}\kappa_{(n+1)}) + v^\star_1(-2\eta_1 v_1 + 2i\not{\Sigma}_1\not{p}\not{\Sigma}_1) + \not{\Sigma}^\star_1(\not{\Sigma}_1\eta_1) \\ & + c^\star(-2i\kappa_1\not{p}\kappa_1 + 4i\kappa_1\not{\Sigma}_1\hat{\phi})\Big], \end{aligned} \tag{18}$$

while the term quadratic in antifields, S_2, is

$$\begin{aligned} S_2 = \int d\tau \Big[& 2e^\star\Big(i\theta^\star\kappa_2 + \sum_{n=1}^{+\infty} i^{2n+1}\kappa^\star_n\kappa_{n+2} + x^\star_\mu(i\kappa_1\gamma^\mu\kappa_1 - \theta\gamma^\mu\kappa_2) \\ & + 4c^\star\kappa_1\kappa_2\Big) + 4\varphi^\star c^\star\kappa_2\not{\Sigma}_1 + ix^\star_\mu(\psi^\star\gamma^\mu\kappa_2 + 2\omega^\star\not{\Sigma}_1\gamma^\mu\not{\Sigma}_1)\Big]. \end{aligned} \tag{19}$$

The full master action in then given by adding to S_{min} the following action for the non-minimal fields

$$S_{non-min} = \int d\tau \Big[\hat{c}^\star\pi_c + \sum_{n=1}^{+\infty}\sum_{m=0}^{+\infty} i^n \hat{\kappa}^{m\star}_n \pi^m_n + \hat{\zeta}^\star_1\pi_\zeta + \hat{\eta}^\star_1\pi_\eta + \hat{v}^\star_1\pi_v + \hat{\not{\Sigma}}^\star_1\pi_\Sigma\Big]. \tag{20}$$

The corresponding quantum action S_Q is then given by substituting $\Phi^\star_A = \partial\Psi/\partial\Phi^A$. The gauge fermion $\Psi(\Phi^A)$ is implemented by imposing gauge conditions on the gauge fields rather than on the coordinates. The simplest gauge is

$$\Psi(\Phi^A) = \int d\tau \Big[\hat{c}(e-1) + \hat{\kappa}\psi + \hat{\zeta}_1\varphi + \hat{\eta}\beta + \hat{\not{\Sigma}}_1\not{A} + \frac{1}{2}\hat{v}_1\omega\Big], \tag{21}$$

where $\hat{c}$, $\hat{\kappa}$, $\hat{\zeta}_1$, $\hat{\eta}$, $\hat{\not{\Sigma}}_1$ and $\hat{v}_1$ are anti-ghosts fields. This leads to the following quadratic gauge-fixed quantum action.

$$S_Q = \int d\tau \Big\{p_\mu\dot{x}^\mu + i\hat{\theta}\dot{\theta} + i\hat{\phi}\dot{\phi} + \frac{1}{2}p^2 + \hat{c}\dot{c} + \hat{\kappa}_1\dot{\kappa}_1 + \hat{\zeta}_1\dot{\zeta}_1 + \hat{\not{\Sigma}}_1\dot{\not{\Sigma}}_1 + \hat{\eta}_1\dot{\eta}_1 + \hat{v}_1\dot{v}_1\Big\}. \tag{22}$$

where $\hat{\theta} = d - \not{p}\theta - 4i\hat{c}\kappa_1$. This is invariant under the modified BRST transformations given by $\hat{s}\Phi^A = \partial_l S/\partial\Phi^\star_A\Big|_{\Phi^\star_A = \partial\Phi/\partial\phi^A}$, which are generated by the following conserved ($\dot{Q}_{BRST} = 0$)

and nilpotent ($Q_{BRST}^2 = 0$) BRST charge

$$
\begin{aligned}
Q_{BRST} = & \frac{1}{2} c p^2 + 2id\not{p}\kappa_1 + 2i\hat{\phi}\not{p}\zeta_1 - 2id\hat{\phi}\not{\nabla}_1 - \hat{\phi}v_1\hat{\phi} \\
& - \hat{\phi}\eta_1\phi + \hat{\zeta}_1\zeta_1\eta_1 + \hat{\not{\nabla}}_1\not{\nabla}_1\eta_1 + \hat{\eta}_1\eta_1\eta_1 + 2\hat{v}_1 v_1\eta_1 - 2\hat{c}\hat{\theta}\kappa_2 \\
& - 2i\hat{c}\kappa_1\not{p}\kappa_1 + i\hat{\kappa}_1\not{p}\kappa_2 + 4\hat{c}\hat{\zeta}_1\kappa_2\not{\nabla}_1 + 2i\hat{\kappa}_1\hat{c}\kappa_3 + 2i\hat{v}\not{\nabla}_1\not{p}\not{\nabla}_1 \ .
\end{aligned}
\tag{23}
$$

As the action (22) is free, the model can be quantized canonically by replacing each of the fields by an operator and imposing canonical (anti-) commutation relations on the conjugate pairs (p_μ, x^μ), $(\hat{\theta}, \theta)$, $(\hat{\phi}, \phi)$, $(\hat{c}, c)$, $(\hat{\kappa}_1, \kappa_1)$, $(\hat{\zeta}_1, \zeta_1)$, $(\hat{\not{\nabla}}_1, \not{\nabla}_1)$, $(\hat{\eta}_1, \eta_1)$ and $(\hat{v}_1, v_1)$. Then a state is physical if it is annihilated by the BRST charge. The cohomology classes can be classified according to their total ghost number and the physical states are taken to be the cohomology class of some definite ghost number. We consider two distinct Fock space representations of the ghost system, the *untwisted* one in which the ghost ground state $|0>$ is annihilated by each of the antighosts ($\hat{\kappa}_1|0>=0, \hat{c}|0>=0, \hat{v}_1|0>=0, \hat{\eta}_1|0>=0, \hat{\not{\nabla}}_1|0>=0, \hat{\zeta}_1|0>=0$), and the *twisted* one in which antighosts are creation operators and ghosts are annihilation operators. It is viewed x, θ and ϕ as hermitian coordinates while $p_\mu = -i\partial/\partial x^\mu$, $\hat{\theta}_A = \partial/\partial\theta_A$ and $\hat{\phi}^A = \partial/\partial\phi^A$, and consider states of the form $\Phi(x, \theta, \phi)\ M\ |\Omega>$ with wavefunction Φ, where M is some monomial constructed from (anti-)ghosts and $|\Omega>$ is one of the ghost ground state. The wave function $\Phi(x, \theta, \phi)$ satisfies the following conditions

$$
\begin{aligned}
& p^2\Phi = 0, \qquad \not{p}d\Phi = 0, \qquad \not{p}\hat{\phi}\Phi = 0, \\
& \hat{\phi}\hat{\phi}\Phi = 0, \qquad (\phi\hat{\phi} - 1)\Phi = 0, \qquad d\Gamma^{\mu\nu\rho\sigma}\hat{\phi}\Phi = 0,
\end{aligned}
\tag{24}
$$

which are precisely the the ones discussed in [2]. Therefore, the BRST cohomology class with no ghost dependence gives a physical spectrum consisting of 8 bosons and 8 fermions which form the super Yang-Mills multiplet. The monomial M cohomology classes have to be investigated in both spinor-twin type I and type II models.

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