

# Asymptotic Conformal Invariance in a Non-Abelian Chern-Simons-Matter Model<sup>1</sup>

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## Abstract

One shows here the existence of solutions to the Callan-Symanzik equation for the non-Abelian  $SU(2)$  Chern-Simons-matter model which exhibits asymptotic conformal invariance to every order in perturbative theory. The conformal symmetry in the classical domain is shown to hold by means of a local criteria based on the trace of the energy-momentum tensor. By using the recently exhibited regimes for the dependence between the several couplings in which the set of  $\beta$ -functions vanish, the asymptotic conformal invariance of the model appears to be valid in the quantum domain. By considering the  $SU(n)$  case the possible non validity of the proof for a particular  $n$  would be merely accidental.

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Topological field theories<sup>2</sup> are a class of gauge models which has deserved much attention, for they have an interesting ultraviolet behavior that renders easier the task of getting non-perturbative results. In particular, the Chern-Simons model in three dimensions [2] has been deeply studied in the lastest years. They are sometimes presented as possible applications in super-conductivity and 3D-gravity models [3]. The relevant property of this model lies on a very interesting perturbative behavior, namely, its ultraviolet finiteness [3, 4]. This is rigorously proven for the  $D = 1 + 2$  Chern-Simons theories in the Landau gauge [5] and for the Chern-Simons-Yang-Mills theory in [6]. The Abelian Chern-Simons gauge field coupled to scalar matter field is shown to have trivial  $\beta$ -function in [7]. In [8], from the algebraic renormalization approach [9], the non-Abelian theory coupled to both scalar and spinorial matter fields is checked

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<sup>2</sup>See [1] for a general review and references.

to have the Chern-Simons coupling constant kept unrenormalizable. The scale invariance of non-Abelian Chern-Simons minimally coupled to matter has already been discussed in the two-loop approximation in ref. [10]. Recently, in ref. [11], the non-Abelian-Chern-Simons gauge model minimally coupled to matter, with matter-matter interaction terms compatible with the power-counting renormalizability criterion is shown to exhibit asymptotic scale invariant solutions to the Callan-Symanzik equation. A discussion on the asymptotic conformal invariance is presented in [12].

In this work, the  $SU(2)$  Chern-Simons-matter model minimally coupled to matter, with matter-matter interaction terms compatible to the power-counting renormalizability criterion is considered. Dimensionful couplings are also allowed. The existence of asymptotic conformal invariant solutions to the Callan-Symanzik for the model is ascertained. Conformal symmetry is obtained in the classical domain by means of a suitable criterion [13] based in a local trace relation of the energy-momentum tensor. Such a criterion simplifies the task of extending the proof of the symmetry to the quantum domain. This extension is obtained under the light of the algebraic renormalization approach [9] and based in the results of the paper [8]. Finally, from the recently obtained result, [11], namely, the existence of solutions to the Callan-Symanzik equation for which the several  $\beta$ -functions vanish, one proves the existence of conformal asymptotic invariant solutions to the Callan-Symanzik equation.

The Chern-Simons-matter BRS covariant model in the Landau gauge has the following action,

$$\begin{aligned} \Sigma = \int d^3x \left\{ \kappa \varepsilon^{\mu\nu\rho} (A_\mu^a \partial_\nu A_\rho^a + \frac{1}{3} f_{abc} A_\mu^a A_\nu^b A_\rho^c) + (ie \bar{\Psi}_j \gamma^\mu \mathcal{D}_\mu \Psi_j - m_\Psi \bar{\Psi}_j \Psi_j) + \right. \\ \left. + \frac{1}{2} (e \mathcal{D}_\mu \varphi_i^* \mathcal{D}^\mu \varphi_i - m_\varphi \varphi_j^* \varphi_j) - \frac{1}{2} \lambda_1 \bar{\Psi}_j \Psi_j \varphi_k^* \varphi_k - \frac{1}{2} \lambda_2 \bar{\Psi}_j \Psi_k \varphi_k^* \varphi_j + \frac{1}{6} \lambda_3 (\varphi_i^* \varphi_i)^3 \right. \\ \left. + (\text{“dim’ful” couplings}) + e g^{\mu\nu} (\partial_\mu b_a A_\nu^a + \partial_\mu \bar{c}_a \mathcal{D}_\nu c^a) + \sum_{\Phi=A_\mu^a, c^a, \Psi_j, \varphi_i} \Phi^{\dagger s} \Phi \right\}. \end{aligned} \quad (1)$$

The  $e$  denotes the determinant of the dreibein  $e_\mu^m$ . The gauge field  $A_\mu^a(x)$  lies in the adjoint representation of the gauge group  $SU(n)$ , with Lie algebra  $[X_a, X_b] = if_{ab}^c X_c$ . The scalar matter fields,  $\varphi_i(x)$ , and the spinor matter fields,  $\Psi_j(x)$ , are in the fundamental representation of  $SU(n)$ . The generators are respectively the matrices  $T_a^{(\varphi)}$  and  $T_a^{(\Psi)}$ . The fields  $c^a$ ,  $\bar{c}^a$  and  $b^a$  are the ghost, the antighost and the Lagrange multiplier fields. The  $A_{a\mu}^\dagger$ ,  $c_a^\dagger$ ,  $\Psi_j^\dagger$ ,  $\varphi_i^\dagger$  are the “antifields” coupled to the nonlinear variations under BRS transformations. The diffeomorphic form of the action is considered in order to allow the determination of the Belifant tensor. Indeed, the result is obtained in flat space-time limit. The generalized covariant derivative is defined by

$$\mathcal{D}_\mu \Phi(x) = (\partial_\mu - i A_\mu^a(x) T_a^{(\Phi)} + \frac{1}{2} \omega_\mu^{mn} \Omega_{mn}) \Phi(x). \quad (2)$$

The vanishing torsion makes the spin connection,  $\omega_\mu^{mn}(x)$ , dependent on the dreibein  $e_\mu^m(x)$ . One considers a manifold with asymptotically flat curvature and topology equivalent to the flat

$\mathcal{R}^3$ . The various symmetries of the model, manifested through its Ward identities are BRS, local Lorentz transformations, diffeomorphism and rigid gauge invariance, besides the constraints, Landau gauge condition, ghost and antighost equations. They are all shown in the work of ref. [8].

The symmetric energy-momentum tensor,  $\Theta_\nu^\mu$ , can be defined as

$$\Theta_{\mu\nu} = e^{-1} e_{(\mu}^m \frac{\delta \Sigma}{\delta e^{\nu)m}} . \quad (3)$$

It is well-known that every space-time symmetry may be related to the energy-momentum tensor. In particular, the trace  $\Theta_\mu^\mu$  is deeply connected to both dilatation and conformal symmetries [13]. By integrating, one obtains the Ward identity for the dilation invariance,

$$\begin{aligned} \int d^3x e \Theta_\mu^\mu &= \int d^3x e_\mu^m \frac{\delta \Sigma}{\delta e_\mu^m} = \mathcal{N}_e \Sigma \sim \\ &\sim \underbrace{(\mathcal{N}_\Psi + \mathcal{N}_{\bar{\Psi}} + \frac{1}{2} \mathcal{N}_\varphi + \mathcal{N}_b + \mathcal{N}_{\bar{c}} - \mathcal{N}_{\Psi^\dagger} - \mathcal{N}_{\bar{\Psi}^\dagger} - \frac{1}{2} \mathcal{N}_{\varphi^\dagger}) \Sigma}_{\int d^3x w^{(\Phi)}(x) \Sigma}, \end{aligned} \quad (4)$$

where the definition of the counting operator,  $\mathcal{N}_\Phi \Sigma = \int d^3x \Phi \frac{\delta \Sigma}{\delta \Phi}$  with ( $\Phi =$  any field) is used and “ $\sim$ ” means equality up to mass terms and dimensionful couplings. The difference between the integrands of the expression above must be of the form

$$e \Theta_\mu^\mu(x) \sim w^{(\Phi)}(x) \Sigma + \partial_\mu \Lambda^\mu(x) , \quad (5)$$

where, from [8],

$$\Lambda^\mu = e i \bar{\Psi} \gamma^\mu \Psi + e \varphi \nabla^\mu \varphi - s (e g^{\mu\nu} \bar{c} A_\nu) . \quad (6)$$

In what concerns the establishment of the conformal invariance in the classical domain, one can not ignore the local character of the symmetry. There are interesting criteria [14, 15] based on the symmetries manifested in the diffeomorphic version of the theory, but somewhat difficult to be interpreted at the quantum level. In the previous analysis [8, 11] of the model (1), the local trace identities, related to the Callan-Symanzik equation take a relevant place. Hence, it would be suitable to use a criterion [13] which points in such a direction. In focusing the conservation of energy-momentum,  $\partial^\mu T'_{\mu\nu} = 0$ , and definition of the charge  $P_\mu = \int d^{d-1}x T_{0\mu}$ , the tensor can admit different definitions [13, 16, 17]. In considering space-time transformations in the form of general coordinate transformations (GCT)  $x'^\mu = x^\mu + \varepsilon^\mu(x)$ , the complete ten-parameters  $(1+2)D$  conformal transformations can be mapped into GCT by the relation [18],

$$\partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu = \frac{2}{d} \eta_{\mu\nu} (\partial \cdot \varepsilon) \implies \partial^2 (\partial \cdot \varepsilon) = 0 . \quad (7)$$

The second order solutions are the special conformal group. The first order are the Poincaré group and the zeroth order are the dilatations. The current associated to the complete conformal group can be written as

$$j^\mu = \varepsilon^\nu(x) T_\nu^\mu(x) + (\partial \cdot \varepsilon) K^\mu + \partial_\nu (\partial \cdot \varepsilon) L^{\mu\nu} + (\partial_\nu \varepsilon_\rho - \partial_\rho \varepsilon_\nu) M^{[\nu\rho]\mu} , \quad (8)$$

where  $K, L$  and  $M$  local quantities depending on the fields and its first derivatives. The usual forms [16, 17] can be obtained by imposing the various forms of  $\varepsilon_\nu(x)$ , with respect to the subgroups of the whole conformal group. In [13], it is shown that in considering the dilatations  $\varepsilon_\nu(x) = \lambda x_\nu$ , under the requirement  $\partial_\mu j^\mu = 0$ , there arises the condition  $T_\mu^\mu = -\partial_\mu K^\mu$ , and that, for the restricted conformal group  $\varepsilon_\nu(x) = a_\nu x^2 - 2(a \cdot x)x_\nu$ , the requirement  $\partial_\mu j^\mu = 0$  furnishes both  $T_\mu^\mu = -\partial_\mu K^\mu$  and  $K^\mu = -\partial_\mu L^{\mu\nu}$ . Hence, the condition for conformal invariance is shown to be stronger than the condition for scale invariance. The conditions for conformal invariance taken together yield the sufficient condition

$$T_\mu^\mu = \partial_\mu \partial_\sigma L^{\mu\sigma}. \quad (9)$$

If the requirements above are satisfied, there follows the Ward identity for the conformal group. Redefinitions on  $T^{\mu\nu}$  imply similar redefinitions on  $K^\mu$  and  $L^{\sigma\rho}$ . The traceless  $T^{\mu\nu}$  is a particular case obtained for the improved tensor. The existence of a traceless  $T^{\mu\nu}$  depends on the theory and is a particular case of the present criterion. Then, it can be stated, [13], that the existence of a traceless stress tensor is equivalent to the conformal invariance.

From now on, the study will be restricted to the flat space limit. By focusing the attention on the r.h.s of the expression (5) and taking the flat limit space, one notes that the first term is a set of equations of motion which vanish on-shell. The second term in (5) can be studied in the expression (6). In (6), the third term is a total BRS variation and is again physically trivial, being non-observable on-shell. Hence, the trace (5), up to terms which vanish on-shell is given by

$$\Theta_\mu^\mu \sim i\partial_\mu(\bar{\Psi}\gamma^\mu\Psi) + \frac{1}{2}\partial_\mu\partial_\sigma\eta^{\mu\sigma}(\varphi^2). \quad (10)$$

By invoking the global gauge symmetry both  $\bar{\Psi}\gamma^\mu\Psi$  and  $\varphi^2$  show to be conserved quantities, what causes the vanishing of the trace of the energy-momentum tensor  $\Theta_\mu^\mu \sim 0$  up to mass terms and dimensionful couplings. Therefore, the condition for the asymptotic on-shell conformal invariance holds in the classical domain.

The possible validity of the conformal invariance for (1) into the quantum domain would depend on the existence, or not, of anomalies. This matter was examined by [8] in the framework of the algebraic renormalization method [9]. In such an approach, the study of the renormalizability was carried out through the analysis of the extension of the Slanov-Taylor (ST) identity to the quantum domain. The absence of gauge anomalies for diffeomorphisms and local Lorentz invariant functionals in this class of manifolds, [19, 20], in addition to the power-counting renormalizability criterion, restrict that analysis to the space of the diffeomorphism and local Lorentz invariant integral functionals  $\Delta$  of the various fields having integrand of dimension 3. The gauge condition, ghost, antighost equations and rigid gauge invariance are shown to be non-anomalous [9]. The extension of the validity of the various symmetries to the vertex functional,  $\Gamma$ , is restricted to the analysis of the basis generated by the problem posed by the ST identity,  $B_\Sigma\Delta = 0$ ,  $B_\Sigma$  being the ST operator, with conditions imposed by the diffeomorphism, local

Lorentz and rigid gauge symmetries, together with the constraints given by the Landau gauge condition, ghost and antighost equations. The solution of this given problem generates then a basis in functional integral space where various studies, renormalizability, non-renormalization of the parameters of the theory, anomalies could take place. The solutions are divided in two parts:  $\Delta = \Delta_{\text{cohom}} + B_{\Sigma}\hat{\Delta}$ . Because the nilpotency,  $B_{\Sigma}^2 = 0$ , the second term is the trivial part, and is associated to non-physical field redefinitions. The  $\Delta_{\text{cohom}}$  part is associated to the renormalization of physical parameters. The space of the solutions are also divided in sectors, depending on the ghost number. The ghost number-0 is associated the arbitrary invariant counterterms,

$$\begin{aligned} \Delta^{(0)} &= \overbrace{\left( \kappa z_{\kappa} \frac{\partial}{\partial \kappa} + \lambda_1 z_{\lambda_1} \frac{\partial}{\partial \lambda_1} + \lambda_2 z_{\lambda_2} \frac{\partial}{\partial \lambda_2} + \lambda_3 z_{\lambda_3} \frac{\partial}{\partial \lambda_3} \right) \Sigma}^{\Delta_{\text{cohom}}^{(0)}} \\ &+ \underbrace{\left( z_A \mathcal{N}_A + z_{\Psi} \mathcal{N}_{\Psi} + z_{\varphi} \mathcal{N}_{\varphi} \right) \Sigma}_{B_{\Sigma} \hat{\Delta}^{(0)}}. \end{aligned} \quad (11)$$

This corresponds to an expansion in the integral of the elements of the basis

$$\{\varphi^6, \varphi^2 \bar{\Psi} \Psi, n_A = \mathcal{B}_{\Sigma}(\hat{A}^{\dagger} A), n_{\Psi} = \mathcal{B}_{\Sigma}(\bar{\Psi}^{\dagger} \Psi + \bar{\Psi} \Psi^{\dagger}), n_{\varphi} = \mathcal{B}_{\Sigma}(\varphi^{\dagger} \varphi)\}. \quad (12)$$

The nontrivial solutions belonging to the ghost number-1 sector,  $\Delta_{\text{cohom}}^{(1)}$ , are the possible anomalies. However, as it was argued in [8] that in view of [19, 20], the cohomology of this sector is empty or at most contains Abelian ghosts, which based on [21], do not contribute to gauge anomalies. This was sufficient [8] to conclude the absence of gauge anomalies and the validity of ST identities to all orders of the vertex functional  $S(\Gamma) = 0$ . Hence, there followed the renormalizability of the model. The quantum version of (5) can be written as bellow:

$$\Theta_{\mu}^{\mu}(x) \cdot \Gamma \sim w^{\Phi}(x) \Gamma + \partial_{\mu}[\Lambda^{\mu}(x) \cdot \Gamma] + \Delta \cdot \Gamma, \quad (13)$$

where the term  $\Delta \cdot \Gamma$  represents the breaking due to radiative correction. From the absence of gauge anomalies, it can be expanded [8] by using the quantum version of basis (12). Once it represents the scale breaking, its coefficients are the  $\beta$ -functions and the anomalous dimensions,

$$\begin{aligned} e \Theta_{\mu}^{\mu}(x) \cdot \Gamma &\sim \overbrace{\left\{ \beta_{\lambda_1} n_{\lambda_1} + \beta_{\lambda_2} n_{\lambda_2} + \beta_{\lambda_3} n_{\lambda_3} - \gamma_A n_A - \gamma_{\Psi} n_{\Psi} - \gamma_{\varphi} n_{\varphi} \right\} \cdot \Gamma}^{\Delta \cdot \Gamma} \\ &+ w^{\Phi}(x) \cdot \Gamma + \partial_{\mu} [\Lambda^{\mu}(x) \cdot \Gamma], \end{aligned} \quad (14)$$

where  $n_{\lambda_k} \cdot \Gamma : \int d^3x n_{\lambda_k} \cdot \Gamma = \frac{\partial \Gamma}{\partial \lambda_k}$ . The absence of a  $\beta_{\kappa}$ -function corresponding to the Chern-Simons action in the basis indicates the vanishing of the  $\beta_{\kappa} = 0$  what proves the non-renormalization of the Chern-Simons term.

In order to extend the proof of the conformal symmetry to the quantum level, it is necessary to verify whether the properties of the trace (5) hold for (14). Hence, it must be a vanishing

trace or, at most, be in agreement with the form (9). By observing the r.h.s. of the quantum version of the trace (14), one finds that, up to terms in dimensionful couplings and mass, the term  $w^{(\Phi)}(x) \cdot \Gamma$  vanishes on-shell. The same occurs for the terms in  $\Delta \cdot \Gamma$  which contains the factors in the form  $\gamma_\Psi$ ,  $\gamma_A$  and  $\gamma_\phi$ , because they are physically trivial. There remain the  $\beta$ -functions terms together with the dimension 3 insertion  $\partial_\mu[\Lambda^\mu \cdot \Gamma]$ . Once the breaking are supposed to be in the term  $\Delta \cdot \Gamma$ , the divergence term must keep the same symmetries of the classical domain. Therefore, on-shell, the breaking is due to the set of terms in the  $\beta$ -functions. By integrating (14), we obtain the Ward identity for anomalous dilatation invariance, i. e., the Callan-Symanzik equation in terms of trace identity

$$\int d^3x [\Theta_\mu^\mu(x) \cdot \Gamma] \sim (\beta_{\lambda_1} \partial_{\lambda_1} + \beta_{\lambda_2} \partial_{\lambda_2} + \beta_{\lambda_3} \partial_{\lambda_3} - \gamma_A \mathcal{N}_A - \gamma_\Psi \mathcal{N}_\Psi - \gamma_\phi \mathcal{N}_\phi) \cdot \Gamma. \quad (15)$$

In a recent paper [11], one imposed a dependence between the several coupling constants of (1) on the Chern-Simmons coupling  $\kappa$  [22, 23].

$$\lambda_i = \lambda_i(\kappa). \quad (16)$$

This induces in the Callan-Symanzik equation (15) the reduction condition

$$\beta_{\lambda_i} = \beta_\kappa \frac{\partial \lambda_i}{\partial \kappa}. \quad (17)$$

The dependence of (16) in  $\kappa$  was chosen to be polynomial. The dependence between the several couplings is not rigid being corrected order by order. The various 2-loop  $\beta$ -functions [24] were taken in (17) under the requirement (16). Due to the vanishing of the  $\beta_\kappa$ , the lower order solution of (16) are the roots  $f_k^{(0)}$  of the set of polynomials in  $\kappa$ . The real solutions which keep positive the coefficient of  $(\varphi_i^* \varphi_i)^3$  in (1) are the physically meaningful. The  $m$ -th order solution (16) for the dependence of the couplings on  $\kappa$ , (17), can be written as

$$M_{kk'}(f^{(0)}) \chi_{k'}^{(m)} = \vartheta_k(\chi^{m-1}, \chi^{m-2}, \dots). \quad (18)$$

The matrix in the l.h.s. depends on the lower-order solution and the non-homogeneous term in the r.h.s. depends on the earlier solutions only. The existence of all orders solutions to (16) is given by the condition  $\det M_{kk'}(f^{(0)}) \neq 0$ , which is shown to hold in [11]. There followed the existence of regimes in the dependence of the couplings for which the whole set of  $\beta$ -functions vanish, keeping the coupling constants unrenormalized.

Turning back to the equation (13), one can see that on-shell, in the flat space limit, the remaining non-vanishing terms were exactly those which depend on the  $\beta$ -functions. Therefore, one can see that the conformal symmetry can be extended to the quantum domain. From the renormalizability of the model, it can be stated that the non Abelian Chern-Simons-matter model exhibits asymptotic conformal invariant solutions to the associated Callan-Symanzik equation.

Hence, one can draw a major conclusion. A suitable criterion [13], based on a local trace condition on the energy-momentum tensor, was used to show the asymptotic conformal symmetry

in the classical domain. Such a criterion showed to be a shortcut for extend the proof to the quantum level. From the results of ref. [8], namely, the renormalizability of the model (1), the trace of the energy-momentum tensor with the scale breaking and the nonrenormalizability of the Chern-Simons coupling constant, the framework to the proof in the quantum domain could be established. A quantum version of the local trace relation could be written which showed to have breaking due to radiative corrections. In a previous work, [11], it was shown for (1), the existence of regimes in the dependence of the several couplings in  $\kappa$ , in which the whole set of  $\beta$ -functions vanish yielding the trace relation to vanish on shell in the asymptotic limit. The proof for cases other than  $SU(2)$  would be identical; the questionable argument is the existence of regimes where the dependence between the couplings lead to vanishing  $\beta$ -functions. However, the absence of physically meaningful solutions, expressed by the determinant condition, would be accidental. Nevertheless, it could occur for a particular value of  $n$ .

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