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CAUSALITY AND RELATIVISTIC EFFECTS IN INTRANUCLEAR CASCADE CALCULATIONS

by

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ABSTRACT

Two main goals are pursued in this work. The first one is concerned with the relativistic effects in high energy nuclear collisions, when non-invariance of simultaneity is taken into account. It is shown that the time ordering of nucleon-nucleon collisions is quite different for different observers, giving in some cases non-invariant final results for intranuclear cascade (INC) calculations. In particular, we have shown an example of such a case, in which the INC simulation, depending on the reference frame, presents a kind of density instability caused by a specific time ordering of collision events. The second one is to propose a new INC calculation, using a causality preserving scheme, which minimizes this kind of relativistic effect. It is verified that the causality preserving INC prescription essentially recovers the relativistic invariance.

I. INTRODUCTION

Relativistic heavy-ion collisions (RHIC) are considered to be a promising site for the appearance of a large variety of phenomena which are expected to reveal new properties of nuclear matter. Such an expectation is based on the fact that, in RHIC, a great number of degrees of freedom are involved and a huge amount of energy is available for excitation of the system.

As a matter of fact, theoretical predictions were made for the existence of density isomers, pion condensation, shock waves in nuclear matter, etc. References on these and other topics may be found in Ref. 1 and in the recent comprehensive review article by Nagamiya and Gyulassy[2]. However, in spite of the considerable experimental efforts made in the past few years, no evidence of these novel phenomena have been observed in any RHIC studied so far. Of course, this does not necessarily mean that these features of nuclear matter do not occur at all, but only say that their signatures could be masked by the mass of other events. Furthermore, it is well known that most of the present experimental data in RHIC are of inclusive character, so when it is averaged, e.g. over all values of the impact parameter, some important information may be irremediably lost. It is worthwhile to mention here that the observation of anomalously short mean free paths of secondary tracks in nuclear emulsion[3] suggests the possibility of finding exciting new information from exclusive RHIC measurements.

A rigorous treatment of RHIC requires a dynamical description of relativistic quantum mechanical finite systems, which is still unfortunately beyond our present theoretical and computational abilities (see, however, Boguta[4]). Therefore simplified models have been proposed to describe at least some aspects of the RHIC mechanism. These models enhance either macroscopic or microscopic features of the collision process. The fireball and firestreak models, and the relativistic hydrodynamics calculation belong to the first group, while the Boltzmann equation approach, the classical equation of motion method and the intranuclear cascade (INC) model, with its many variations, are examples of microscopic treatments[2,5].

All of these models are more or less equivalent in reproducing the present inclusive experimental data, in spite of their very different initial assumptions. Nevertheless, INC calculations have the advantage of being free from any adjustable phenomenological parameter. Their basic ingredients are the use of the on-shell nucleon-nucleon cross section to represent their interaction, and the assumption of straight-line motion for the nucleons between collisions. It is legitimate, therefore, to expect INC to serve as an unbiased background against which novel phenomena, if they occur, should stand out. Because of these features, INC calculations are widely used for extracting dynamical information of RHIC.

In order to take into account other realistic degrees of freedom, several processes such as pion production and barionic resonances at high energies[6], as well as mean field effects at lower energies can be incorporated into INC calculations[7]. On the other hand, besides the quantum effects normally neglected, there are still some aspects of RHIC that are usually excluded from INC, e.g. few particle correlation and relativistic effects. The first one is due to the finite range of the nucleon-nucleon interaction which promotes dynamical clustering mechanisms inside the nuclear matter. In fact, it has been recently shown that an interaction time of the order of 1 fm/c causes non-binary processes which are far from being negligible[8]. As for the second one, relativistic kynematics is normally used to treat nucleon-nucleon collisions, but there are other kinds of relativistic effects that have been left out. They appear due to the non-invariance of simultaneity in the time-dependent description of two (or more) body collisions. More especifically, any INC calculation intrinsically requires a well defined instant of nucleon-nucleon collision and their time ordering. But the non-invariance of simultaneity implies that it is impossible to associate a single time to two colliding nucleons and, in consequence, the time ordering of collisions in general differs from one frame to another. Thus the whole results of INC should in principle depend on the system of reference used.

Such relativistic effects have not been taken into account, presumably in the hope that they smear out after ensemble-averaging[7]. However, this is not at all trivial, especially if certain kind of biases and/or correlations are caused by some specific model of nucleon-nucleon collisions. In such a case this frame of reference dependence effect is expected to be enhanced, and to become non-negligible at sufficiently high energies.

Therefore, in order to clarify these points, we find it important to investigate the following two items:

1) Analyze carefully the role of the correlations during the RHIC in reinforcing the non-invariance of INC calculations;

2) How to modify the INC scheme in order to minimize

the non-invariance nature of INC procedure.

For the sake of simplicity and to make our argument more transparent, we consider only nucleon degrees of freedom. Thus we neglect π-meson production and nucleon resonance wholeout the present paper. We also neglect the Fermi momentum of nucleons in the initial states.

In Sec. II, we show how a dynamical correlation arises for a particular choice of the nucleon-nucleon collision style, even under the sequential binary collision assumption. We demonstrate that, in such a case, the time ordering of collision events is in fact relevant, hence this relativistic effect is important. In Sec. III, we present one possible way to reduce the non-invariance effects by preserving relativistic causality throughout the process. Sec. IV is dedicated to discussion and concluding remarks.

II. CORRELATION AND NON-INVARIANCE EFFECTS

The basic point of a conventional cascade calculation is the assumption of sequential instantaneous ($\Delta t = 0$) binary collisions. In such calculations, nucleons are assumed to move along straight lines with constant momenta until they collide. A collision between two nucleons occurs if the closest distance is smaller or equal to $\sqrt{\sigma_{tot}(s)/\pi}$, where σ_{tot} is the total free nucleon-nucleon cross section, and $\sqrt{s}$ is the CM energy of the colliding nucleon system. The collision instant is defined as the time of this closest approach. At each collision, the final momenta are randomly selected taking into account the experimental differential nucleon-nucleon cross section with total energy and momentum conserved. The INC ends when no more collisions take place. The procedure is repeated until sufficient statistics is reached for the quantities one is interested in.

It has been recently shown by Kodama et al.[8] that the fundamental assumption of INC calculations, i.e, the binary character of the collisions is not quite justified. In effect, it was found that the number of non-binary processes is far from being negligible compared to that of binary collisions. These non-binary processes were shown to take place because, during the time evolution of the INC, a number of nucleons ($m > 2$) become close together enough so that the collisions among them can not be treated any longer as simple sequential binary collisions. These multiparticle collisions may enhance certain dynamical correlations which manifest themselves as a collective aspect of RHIC. However, as we do not know how to treat multiparticle collisions, and as our present purpose is to study the non-

-invariant nature of conventional INC procedure, we maintain here the approximation $\Delta t = 0$.

The INC scheme develops according to a single time coordinate common to all nucleons. The instant at which two nucleons collide is taken to be the time when the two nucleons are seen at their closest relative distance. In other words, nucleon-nucleon collision times are calculated in terms of the time coordinate of an observational system common to all nucleons rather than in each nucleon-nucleon system. Consequently, the time ordering of collision events is defined for a given observational frame.

On the other hand, the collision criterion (whether two nucleons collide or not) should be expressed in terms of the impact parameter b_{n-n} defined in the nucleon-nucleon reference system rather than through the minimum distance in the observational system. This distance of closest approach does not necessarily coincide with the impact parameters b_{n-n}, unless we work in a system where the momenta of the colliding nucleons are coplanar.

Specifically, denoting coordinate and momentum of a nucleon at the observational time t by $\vec{r}$ and $\vec{p}$, respectively, we take the collision instant of the (i,j)-pair of nucleons as

$$t_c(i,j) = t - (\vec{v}_{ij}.\vec{r}_{ij})/|\vec{v}_{ij}|^2 \tag{1}$$

whereas their impact parameter is given by

$$b_{ij}^2 = |\vec{r}_{ij}|^2 + \{s_{ij}(\vec{p}_i.\vec{r}_{ij})(\vec{p}_j.\vec{r}_{ij})-m^2[(\vec{p}_i+\vec{p}_j).\vec{r}_{ij}]^2\}/s_{ij}(s_{ij}-4m^2) \tag{2}$$

In the above equations, $\vec{r}_{ij} = \vec{r}_j - \vec{r}_i$, $\vec{v}_{ij} = \vec{p}_j/E_j - \vec{p}_i/E_i$, with $E = (p^2+m^2)^{1/2}$ and $s_{ij} = (E_i+E_j)^2 - (\vec{p}_i+\vec{p}_j)^2$, where m is the nucleon rest mass. Note that the impact parameter, by definition, is a Lorentz scalar.

The collision criterion is now expressed as

$$b_{ij}^2 \leq \sigma_{tot}(s_{ij})/\pi \quad (3)$$

which is obviously relativistically invariant, while the one frequently used[6-8], based on the minimum distance of approach between nucleons, is not invariant. Therefore, in this procedure, the non-invariance of INC calculation may arise only due to the definition of the collision instant, Eq. (1). This non-invariance means that the history of nucleon collisions calculated in the INC scheme does not coincide for two different coordinate systems. However, we may expect that, if there exist no systematic correlations in the cascade processes, any differences would smear out after averaging over many samples of nucleus-nucleus collisions.

In order to verify the above statement, we performed an INC calculation using the formulas (1) and (3) for $^{12}C + {}^{12}C$ head-on reactions at various incident energies. The initial nucleon configuration is determined according to a Gaussian-type distribution. We took $\sigma_{tot}(s) = 40$ mb and the angular distribution was fitted to the experimental p-p cross section[9] as

$$\frac{d\sigma}{d|t|} = 0.5\,\sigma_{tot}\,\alpha\,\cosh\alpha(|t|-t_m/2)/\sinh(\alpha t_m/2) \quad (4)$$

where

$$|t| = 2p^2\,(1-\cos\theta)$$

with

$$p^2 = (s/2)^2 - m^2 \quad ,$$

$$t_m = 4p^2$$

and

$$\alpha \simeq 7.554\ (s-s_0)^{0.1933} \quad , \quad s > s_0 \quad ,$$

where s_0 = 2.126 GeV^2. For $s \leq s_0$, α is set to zero, i.e, the angular distribution is taken to be isotropic. Eq. (4) simulates the experimental forward peaked distributions found at high energies.

Our results are shown in Fig. 1, where the mean number of collisions per nucleon $\langle n_{col} \rangle$, calculated in the CM and the Lab systems, are plotted as a function of the incident energy. We can see that the result is almost invariant thanks to our invariant collision criterion Eq. (3). This means that the conventional procedure of nucleon-nucleon collision does not generate in this case any systematic correlation in the INC dynamics and that the non-invariance which may come from the use of Eq. (1) is seen to be very small indeed.

On the other hand, it is interesting to see how a change in the nucleon-nucleon collision style induces a systematic correlation in the collisions and, hence, a non-invariance of the INC scheme. For this purpose, instead of randomly choosing the final momenta of nucleons, we calculate the scattering angle as a function of impact parameter b_{n-n}. The scattering angle θ is obtained from the equation

$$\int_0^{|t|} \frac{1}{\sigma_{tot}} \frac{d\sigma}{d|t|}\, d|t| = 2 \int_b^{b_{max}} bdb/b_{max}^2 \qquad (5)$$

where $\pi b_{max}^2 = \sigma_{tot}$.

Substituting Eq. (4) into Eq. (5), we get

$$\cos\theta = -(2/\alpha t_m)\sinh^{-1}\{[1-2(b/b_{max})^2]\sinh(\alpha t_m/2)\} \quad (6)$$

Furthermore, the azimuthal angle ϕ is taken such that the collision process takes place in a plane and has an attractive character. To calculate explicitly this azimuthal angle, we consider the rest frame of nucleon j choosing the z-axis along the momentum of nucleon i. In this frame, the azimuthal scattering angle of nucleon i is taken as

$$\phi_i = \tan^{-1}(\tilde{y}/\tilde{x}) + \pi \quad (7)$$

where $\tilde{x}$ and $\tilde{y}$ are coordinates of nucleon i in this frame. This gives also the azimuthal angle of nucleon i in the nucleon-nucleon CM system (the momentum of nucleon i is in the positive direction of the z-axis and that of j, in the negative direction). Let us call such a collision style as a deterministic and attractive one.

In this case, some unusual phenomena appear in the INC results. In Fig. 2, we plotted again the mean number of collisions per nucleon, for the same reaction as in Fig. 1, calculated in the Lab system (dotted line) and in the CM system (solid line). To avoid the undesirable formation of clusters at t = 0, we took care of previously homogenizing the initial configuration, in the sense that no two nucleons are allowed to stay less than some given relative distance apart in the nucleus rest frame. We used as the cutting value the rms value of the proton times two (≅ 1.5 fm).

We see that for energies above 2 GeV/A, the Lab curve tends to diverge. The reason for this is found to be that some nucleons are trapped into many cyclic collisions among them, contributing in this manner to unusually increase the mean number of collisions.

These trapped nucleons may be interpreted as local density instabilities, probably caused by the non-conservation of total angular momentum in the nucleon-nucleon collision treatment. However, the crucial point is that such instabilities occur only in the Lab system. This spurious effect is provoked by the definition of collision time. Eq. (1), combined with the particular collision style, i.e, deterministic and attractive one.

The above specific example illustrates the fact that we have to be careful in applying the conventional INC calculation, especially when correlations in the collision scheme are present.

III. CAUSALITY PRESERVING INC

As mentioned in previous sections, in all INC calculations performed so far a proper treatment of relativistic simultaneity was not considered. This is a crucial point in any cascade calculation since the collision criterion depends on how simultaneity is treated. So, the question is: Can any INC calculation be relativistically invariant ? The answer seems to be somewhat pessimistic. Indeed, the no-interaction theorem of Sudarshan[10] states that the covariance conditions for a many-particle Hamiltonian system are so restrictive that no interaction among particles is allowed. In other words, without introducing field degrees of freedom, it is impossible to construct a covariant many-body theory. This seems to cast an intrinsic obstacle to the INC approach. In practice, this question is bypassed by the argument that, if the incident energy is not too high ($E_{in}/A \lesssim 1$ GeV), the non-relativistic description is well justified in the CM system. However, this is not at all satisfactory, specially when INC applications at higher energies are envisaged.

In order to formulate invariant criteria to define collision events, which are characterized by the two different world points,one for each nucleon, we face a much more difficult situation than in the non relativistic case, where a common collision time could be assigned to both colliding particles. Furthermore, any INC calculation intrinsically requires a well defined ordering of collision events since one computational step is obtained from the results of the previous one. Unfortunately, it is impossible to completely order them in time in an invariant way because the collision events are associated to different particles. In spite of this fact, we can still give a partial time ordering by imposing that causality is preserved. For example, in Fig. 3, we can assert that event B precedes A, and also that C precedes A, although we cannot give the time order between B and C. Such a partial time ordering is obviously invariant. We will show in the following that by introducing this partial time ordering and by giving the appropriate collision criterion, Eq. (3), relativistic invariance of INC calculation scheme is essentially recovered.

First of all, we should express nucleon histories as a succession of world events, in order that causality may be appropriately treated. Thus we must associate to a nucleon-nucleon collision process <u>two</u> world events, one for each nucleon. To define these world events, we adopt the collision instant and collision criterion similar to those given through Eqs. (1) and (3), but now the collision time must be calculated separately in the rest frame of each nucleon, in order to get the proper times associated to the collision event. More explicitly, we calculate the world events as follows: Let x and p be the four-coordinate and four-momenta in an arbitrary observational system (O-system). The event which corresponds to the last collision of a given nucleon i is specified by x_i and its final momenta by p_i. Let t_i^* be the proper time of i corresponding to this event. The proper time interval $\delta t_i^*(j)$, which elapsed from the last collision until the instant when i collides with j is defined as

$$\delta t_i^*(j) = t_c^* - t_i^*$$

where t_c^* is the collision instant, analogous to that given by Eq. (1),

$$t_c^* = t_i^* - (\vec{v}_{ij}^{\,*}.\vec{r}_{ij}^{\,*})/|\vec{v}_{ij}^{\,*}|^2 \tag{8}$$

In Eq. (8), the asterisks represent quantities in the rest frame of the nucleon i. We obtain

$$\delta t_i^*(j) = p_i - (x_j - x_i)/m - B_{ij}/A_{ij} \tag{9}$$

with

$$A_{ij} = 1 - (m/E^*)^2$$

and

$$B_{ij} = p_i(x_j - x_i)/m - p_j.(x_j - x_i)/E^*$$

where

$$E^* = p_i.p_j/m$$

In this definition, the collision (i,j) occurs only if

$$B_{ij}^2 - A_{ij}(C_{ij} - \sigma/\pi) \geq 0 \tag{10}$$

and

$$\delta t_i^*(j) \quad , \quad \delta t_j^*(i) > 0 \tag{11}$$

where

$$C_{ij} = (x_j - x_i)^2 + \{p_i \cdot (x_j - x_i)/m\}^2 \quad .$$

Note that condition (10) expresses the same collision criterion as condition (3). Condition (11) is imposed by causality. In this condition, $\delta t_j^*(i)$ is given by Eq. (9) exchanging indices i and j. We also observe that the above collision criterion is covariant, since all quantities involved are either constant or scalar products of four-vectors.

This covariant definition introduces multiple time coordinates, i.e, one for each nucleon. The difficulty of handling these multiple time coordinates in a dynamical INC calculation requires a scheme different from the usual procedure of step-by-step time evolution. As one alternative way, we propose here a causal progressive description. The basic idea is to determine at each stage of the calculation of nucleon-nucleon collisions, the time ordering in the O-system without violating causality in the proper time of each nucleon. As we pointed out previously, it is impossible to obtain a time ordering of spacelike events in a invariant manner, i.e, independent of a particular O-system used. As a matter of fact, by introducing the time ordering, whatever it is, using one particular O-system, the results of INC calculation may depend on the system choosen especially in the presence of correlations. This is the very limitation for INC method. Nevertheless, it seems that the situation is not so pessimistic. In fact, we may minimize this effect of reference frame dependence, if we establish a time ordering which preserves causality. Once causality is respected, the remaining ambiguity in the time ordering appears only in those binary collisions which are space-like separated. The more separation there is, the less the ordering becomes important. Thus, we expect the non-invariance effect to be reduced.

We proceed with the causality-respecting time ordering as follows. Suppose that we are given, at a certain stage of the calculation, for all nucleons i, the four-coordinates x_i and the proper time t_i^*, when this nucleon acquired the present value of its four-momentum p_i, due to its latest collision. In the next stage, the first collision which seems to happen to the i-th nucleon is the one which has the minimum positive value of $\delta t_i^*(\ell)$, $\ell = 1,\ldots,A$; $\ell \neq 1$, where the positiveness condition is due to causality (A is the total number of nucleons in the system). The minimum occurs, let us say, for $\ell = j$. Then, for the i-th nucleon, the next collision would be with the j-th nucleon. However, it could well happen that the next collision for the j-th nucleon is not with the i-th nucleon. In other words, $\delta t_j^*(i)$ is not necessarily the minimum among the positive $\delta t_j^*(\ell)$, $\ell = 1,\ldots$ .., A; $\ell \neq j$. If there existed k such that $0 < \delta t_j^*(k) < \delta t_j^*(i)$, and if the collision (j,k) actually took place, then the predicted collision (i,j) would not exist, since the momentum of j after the collision (j,k) is no longer the same as that used in the prediction of collision (i,j).

Thus we must consider only those nucleons pairs $\{(i_m,j_m),\ m = 1,\ldots\}$ which satisfy

$$\delta t_{i_m}^*(j_m) = \mathrm{Min}\ \{\delta t_{i_m}^*(\ell) > 0,\ \ell = 1,\ldots,A\ \ ;\ \ \ell \neq i_m\}$$

and

$$\delta t^{*}_{j_m}(i_m) = \text{Min}\,\{\delta t^{*}_{j_m}(\ell) > 0 \quad , \ell = 1,\ldots,A \ ; \ \ell \neq j_m\}$$

if we require an internal consistency of causality among collisions.

Now, among these pairs, we select, as the collision to be processed at this stage, the pair for which, in the O-system, the collision starts first. By repeating the above procedure step-by-step, we get the desired causality-respecting time ordering, which is expected to minimize the non-invariance effects.

In Fig. 4, are shown the results of the causality preserving INC, according to the above prescription for the case of $^{12}C + {}^{12}C$ head-on reactions. The solid curve gives the values of $\langle n_{col}\rangle$ for the case of deterministic and attractive collision style, whereas the dashed one corresponds to the case of the standard collision style, viz, the random choice of final momenta. In both cases, the CM and Lab results coincide. In other words, our causality preserving INC scheme is invariant.

IV. DISCUSSION AND CONCLUDING REMARKS

In this work, we discussed the role of time ordering in INC calculations. Because of the non-invariant nature of simultaneity in a relativistic system, the time ordering of nucleon--nucleon collision events is quite different from one system of reference to another. Since the INC scheme is based on this time ordering, the whole history of nucleon collision in each sample run of a nucleus-nucleus collision may deviate completely from, for example, the CM calculation to the laboratory one. Fig. 5-a shows how the collision history of nucleons calculated in the conventional scheme differs from the CM system to the Lab system. In this figure, the nucleon-nucleon collisions are displayed according to the order in which they appear in each calculation.

We observe that the histories of nucleon collisions in the CM and Lab systems differ from each other not only by the ordering of collisions but also by the appearance of a number of unmatched collisions. This clearly shows why the time ordering could cause the non-invariance effect.

On the other hand, we proposed here an INC prescription in which a more appropriate time ordering is taken into account. In effect, it is verified that our causality preserving INC is invariant. In Fig. 5-b, the collision histories are shown for the CM and Lab systems, calculated by the causality preserving INC. Although the ordering of collision events is very different, it is clear that the two calculations correspond to an identical physical situation, since all the collisions that take place in both systems are exactly the same.

We have shown that the non-invariance of the conventional INC is masked by the statistical averaging over many sample runs. In other words, under the hypothesis of zero collision time ($\Delta t = 0$) and of stochastic feature of nucleon-nucleon collisions, and due to the invariant criterion of Eq. (3), this non-invariance of individual runs is averaged out.

It is doubtful, however, that such an apparent-invariance--through-averaging situation would remain if some of the above INC assumptions are modified. In particular, we have shown through one specific example that, if the stochastic behaviour of nucleon--nucleon collisions is replaced by a deterministic one, a large net (i.e, after averaging) non-invariance effect is obtained. In our example, such a large non-invariance appears to be due to a series of collisions among nearby nucleons. If the zero collision time assumption is dropped, each of these groups of nucleons must be treated as a cluster which final state configuration should be determined through multiparticle interactions. However, these clusters are dynamical ones and should be treated as extended objects in space-time. The non-invariance observed in our example can be understood as the result of non-invariant simulation of the clustering processes under the assumption $\Delta t = 0$.

It is important to recall that the collision time $\Delta t = 0$ is not at all realistic, especially at high energies. Indeed, this is only a good approximation in the limit of a dilute gas. Our results suggest that the relativistic effects, calculated in the $\Delta t = 0$ picture, may give a measure of the deviation of this approximation from the actual case ($\Delta t \neq 0$) in the presence of correlations.

It is a challenging problem to develop a theoretical approach to RHIC in which the nucleon clustering effect can be appropriately treated. Work along this direction is currently in progress. In this case, our idea of using a causality preserving description is expected to be still helpful to compatibilize a relativistically covariant treatment with the clustering processes.

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FIGURE CAPTIONS

Fig. 1: Mean number of collisions per nucleon for the $^{12}C+^{12}C$ head-on reaction, plotted against the incident energy. The results calculated in the CM system are represented by solid lines, whereas those in the Lab system, by dashed lines. The curves indicated as "closest approach criterion" are the results of the calculation using the collision criterion[6]

$$\mathbf{b}_{ij}^2 + \gamma_{CM}^2(\hat{\mathbf{v}}_{CM}\cdot\mathbf{b}_{ij})^2 \leq \sigma_{tot}/\pi$$

where $\mathbf{b}_{ij}$ is the relative coordinate vector of the closest approach, and our "present calculation" based on Eq. (3).

Fig. 2: The same as the Figure 1 for the deterministic and attractive collision style.

Fig. 3: Schematical representation of the nucleon world trajectories: A, B and C indicate collision events among the specific nucleons.

Fig. 4: Mean number of collisions per nucleon for the $^{12}C+^{12}C$ head-on reaction, calculated by the causality preserving algorism. The upper curve is for the deterministic--attractive nucleon-nucleon collision style, whereas the lower curve stands for the stochastic one. In both cases, the CM and Lab results coincide.

Fig. 5: Collision History Diagram - In each vertical line, nucleon-nucleon collision events of a typical sampling run are indicated according to the order in which they take place. Identical collisions (same pair with same momenta) in the CM and Lab systems are connected by solid line. Collisions with the same pair but with different momentum configuration are connected by dashed line. Distinct collisions are specified by bold dots. The results using the conventional INC scheme are shown in (a), and those by the causality preserving algorism, in (b), both for the same initial conditions.

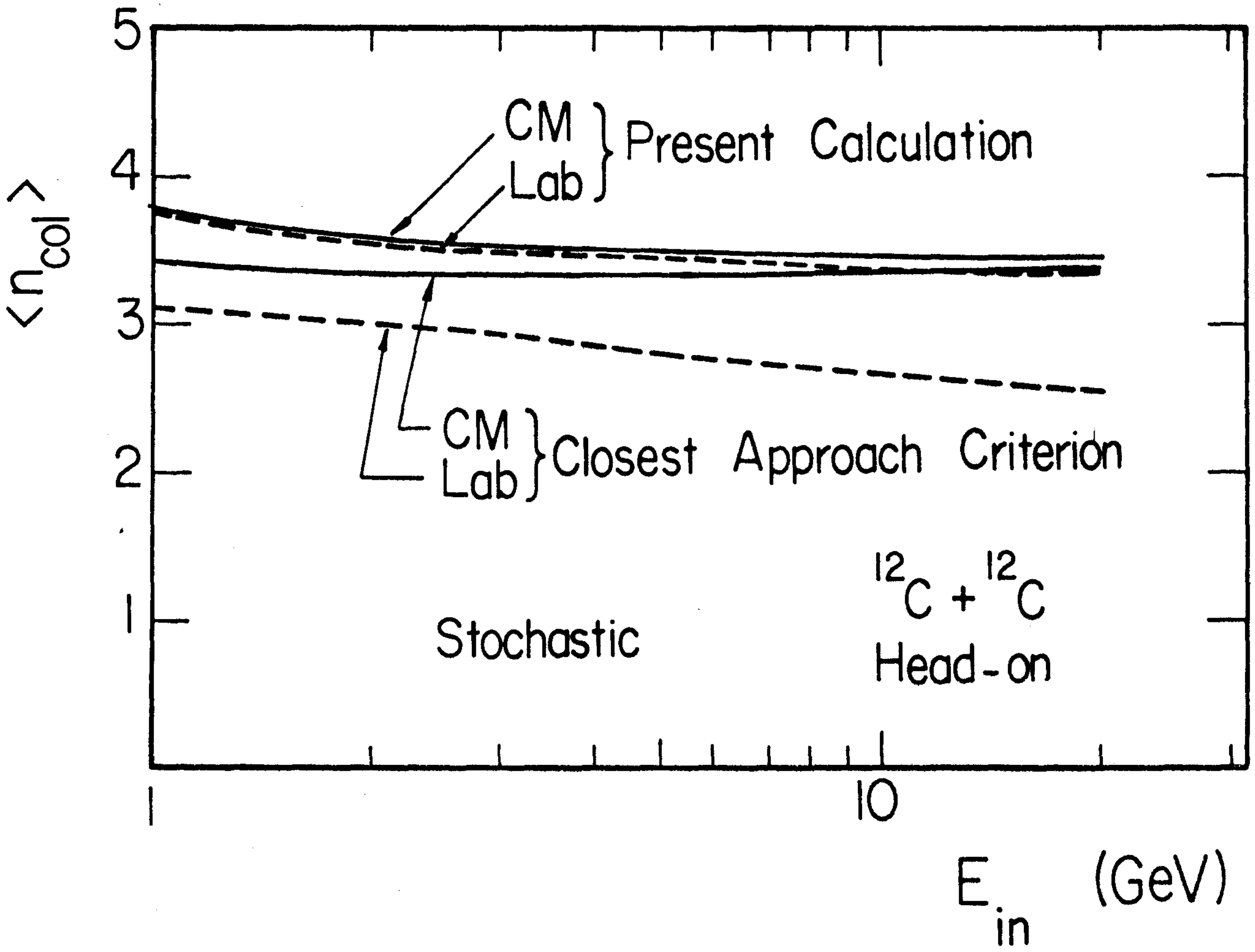
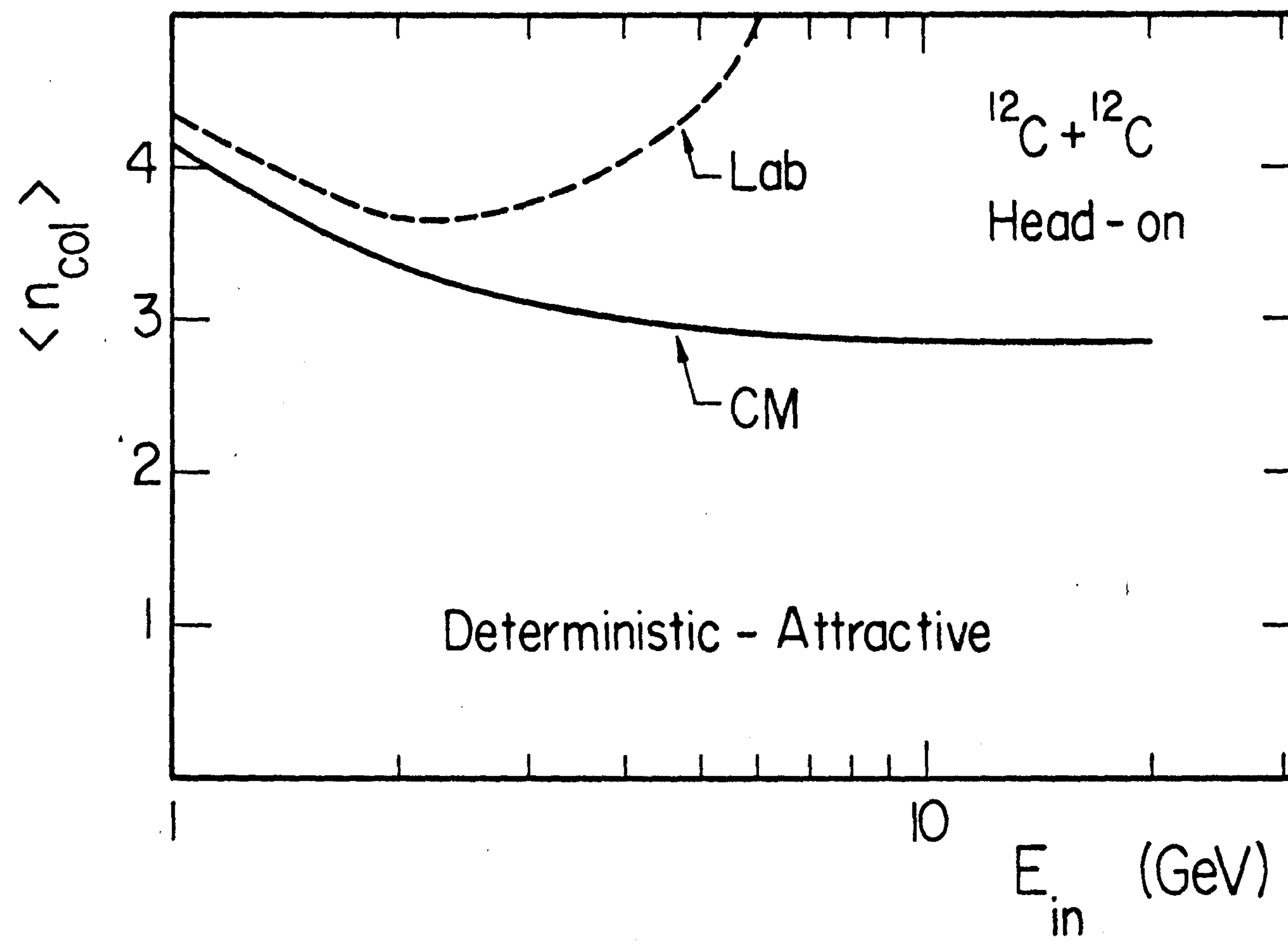

5
4
3
2
1
CM
Lab
Present Calculation
CM
Lab
Closest Approach Criterion
Stochastic
12C + 12C
Head-on
1
10
E in (GeV)
<n col>

FIG. 1



4
3
2
1
$\langle n_{col} \rangle$
$^{12}C + ^{12}C$
Head - on
Lab
CM
Deterministic - Attractive
1
10
E_{in} (GeV)

FIG. 2

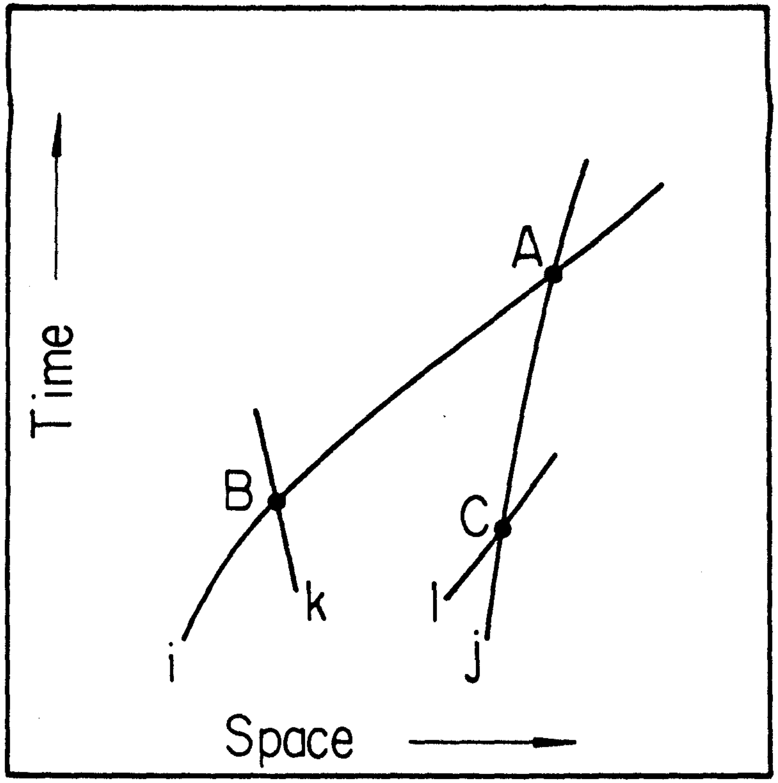


FIG. 3

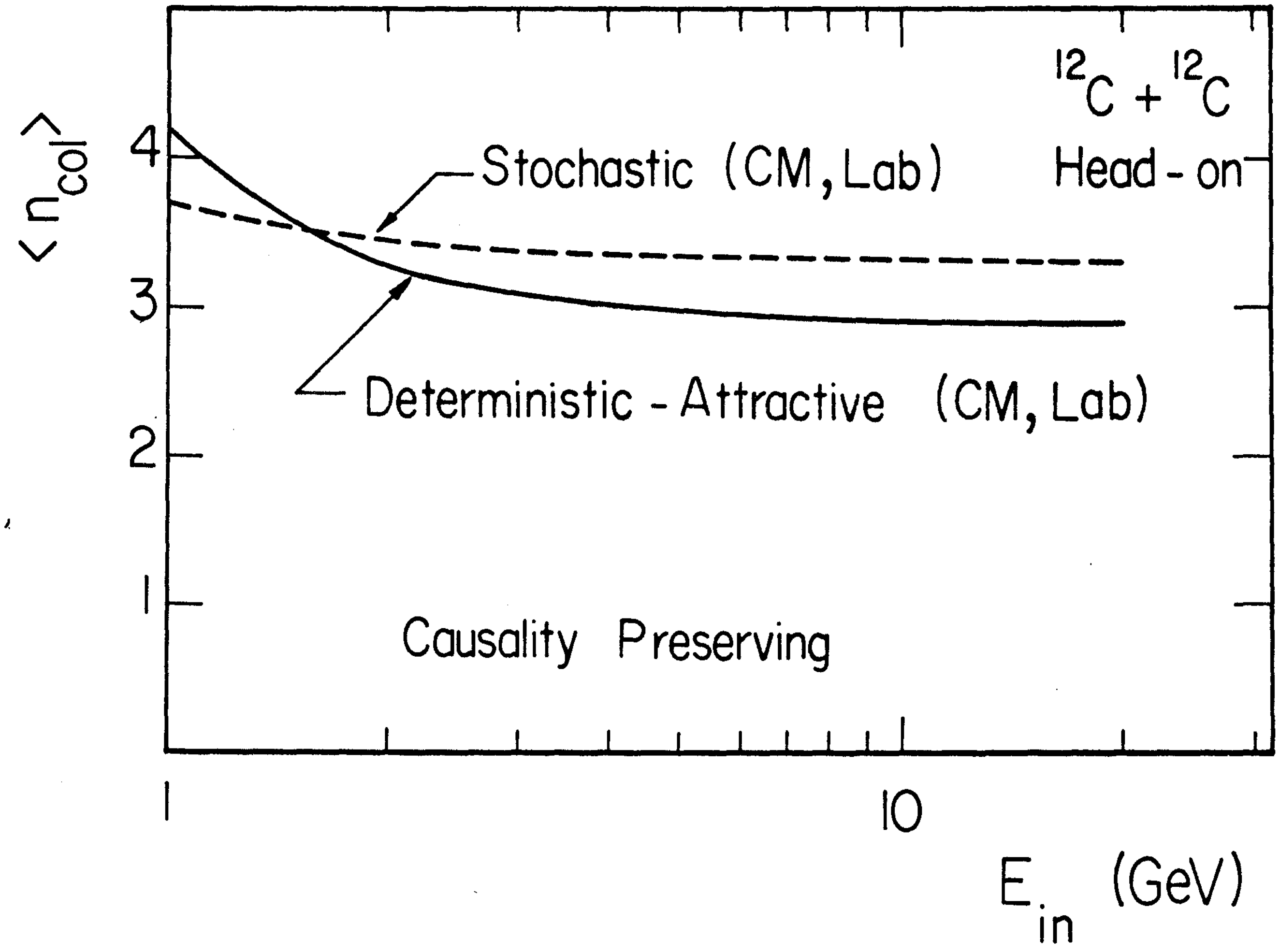
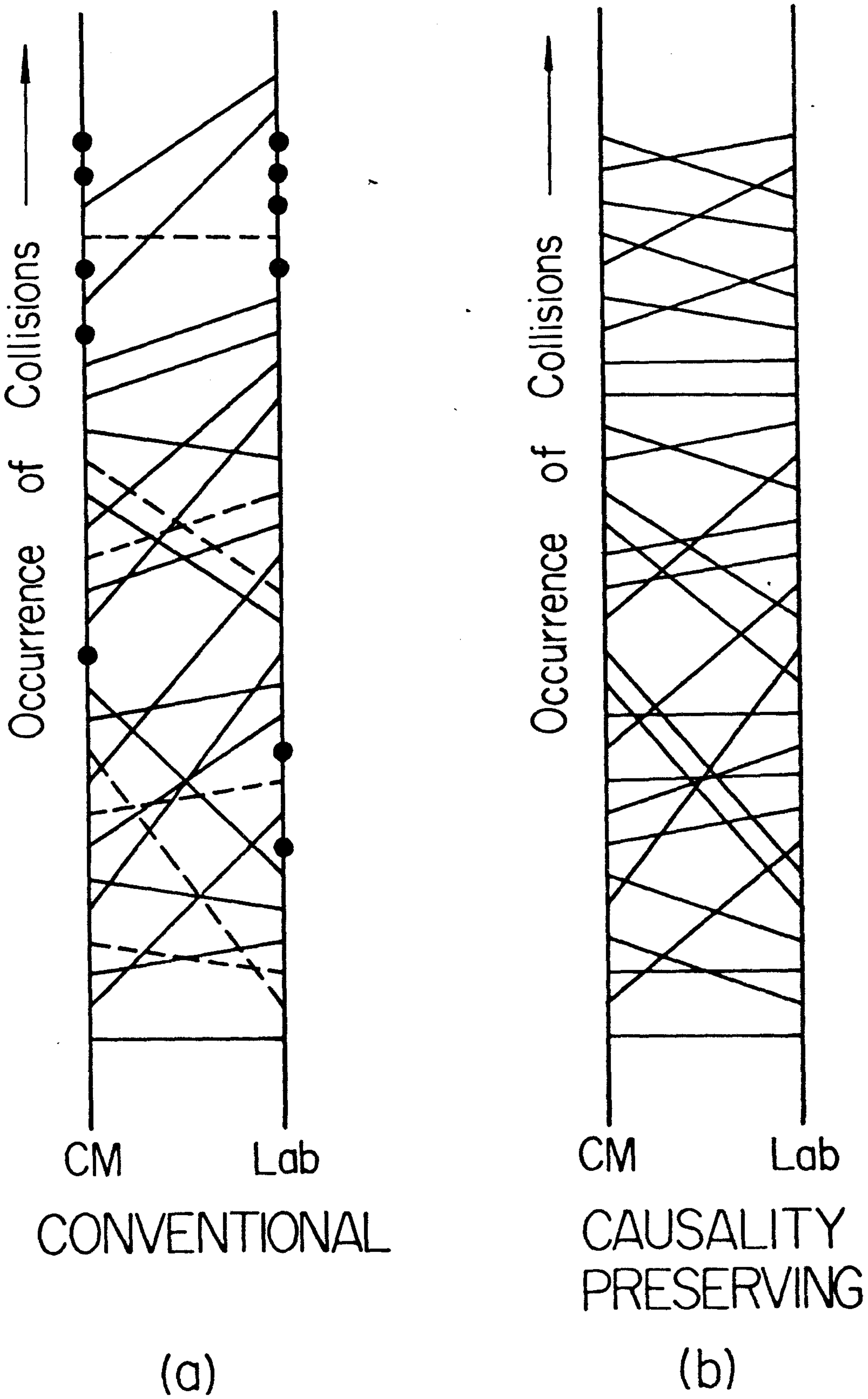

$^{12}C + {^{12}C}$
Head - on
Stochastic (CM, Lab)
Deterministic - Attractive (CM, Lab)
Causality Preserving
$\langle n_{col} \rangle$
4
3
2
1
1
10
E_{in} (GeV)

FIG. 4



Occurrence of Collisions
CM
Lab
CONVENTIONAL
(a)
Occurrence of Collisions
CM
Lab
CAUSALITY PRESERVING
(b)

FIG. 5