

# One-dimensional $\sigma$ -models with $N = 5, 6, 7, 8$ off-shell supersymmetries

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December, 19, 2008

## Abstract

We computed the actions for the  $1D$   $N = 5$   $\sigma$ -models with respect to the two inequivalent  $(2, 8, 6)$  multiplets. 4 supersymmetry generators are manifest, while the constraint originated by imposing the 5-th supersymmetry automatically induces a full  $N = 8$  off-shell invariance. The resulting action coincides in the two cases and corresponds to a conformally flat  $2D$  target satisfying a special geometry of rigid type.

To obtain these results we developed a computational method (for *Maple 11*) which does not require the notion of superfields and is instead based on the nowadays available list of the inequivalent representations of the  $1D$   $N$ -extended supersymmetry. Its application to systematically analyze the  $\sigma$ -models off-shell invariant actions for the remaining  $N = 5, 6, 7, 8$   $(k, 8, 8 - k)$  multiplets, as well as for the  $N > 8$  representations, only requires more cumbersome computations.

CBPF-NF-026/08

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# 1 Introduction

In recent years the representation theory of the  $1D$   $N$ -extended supersymmetry algebra has been substantially clarified, see [1, 2, 3, 4, 5]. Its knowledge has been used to investigate  $N$ -extended  $1D$   $\sigma$ -models, both for linear [6], as well as for non-linear realizations (see e.g. [7]) of the extended supersymmetry. The main, but not exhaustive reference for the linear  $N = 8$   $1D$   $\sigma$ -models is [6] (further developments have also been investigated in [8]). An  $N = 8$ -superfield description was given for the whole list of  $(k, 8, 8 - k)$  multiplets (where  $k = 0, 1, 2, \dots, 8$  is the number of coordinates of the target manifold, 8 is the number of fermionic fields, while  $8 - k$  is the number of auxiliary fields). A corresponding list of  $N = 8$  off-shell invariant actions was also produced. The [6] results, however, are not complete for two independent reasons. The  $N = 8$  off-shell actions there listed are not the most general ones (at least for some values of  $k$ ). Furthermore, no discussion was made concerning a non-maximal number of extended supersymmetries, namely the most general off-shell invariant actions associated to  $N = 5, 6, 7$  supersymmetry generators (they are also irreducibly represented in the  $(k, 8, 8 - k)$  multiplets; on the other hand  $N = 8$  is the maximal number of supersymmetry generators that such multiplets carry). For what concerns the first point, in [2] it was produced the most general  $N = 8$  off-shell action for  $k = 1$  (the  $(1, 8, 7)$  multiplet), later studied also in [9]. The second point is very specific of the limitations of the superfield formalism, not very suitable to deal with such issues (no  $N = 5, 6, 7$  superfield formalism is available). The situation is further complicated by the [3] observation that inequivalent supersymmetry transformations acting on the same field-content (the  $(k, 8, 8 - k)$  multiplets for a fixed  $k$ ) can be encountered. In [4] the classification of such cases (for  $N \leq 8$  they only exist for  $N = 5, 6$ ) was given in terms of the inequivalent connectivities of the graphs associated to the supersymmetry transformations. In [5] it was further proven that the  $N = 5$  inequivalent connectivities are in consequence of the different decompositions in terms of the  $N = 4$  subalgebras. In this work we investigate for the first time the question whether the inequivalent  $N = 5$  supersymmetry transformations induce inequivalent  $N = 5$  off-shell actions.

We developed a general method (concretely implemented in a computational package for Maple 11, a version which allows performing algebraic computations with grassmann fields). Our framework allows to impose constraints, in sequence, arising from the 5-th, 6-th, 7-th and 8-th supersymmetry, studying whether an  $N = 5$  off-shell invariance automatically induces an  $N = 8$  invariance. This issue is related with deep properties of supersymmetry. In [10] it was shown that, under a twist, the action of a  $D = 4$   $N = 2$  Super-Yang-Mills is determined by 5 of its 8 supersymmetry generators. A  $D = 4 \rightarrow D = 1$  dimensional reduction of these systems produces an  $N = 8$  Supersymmetric Quantum Mechanics (such that a vector multiplet  $(3, 4, 1)$  and a matter multiplet  $(2, 4, 2)$  are combined in a single  $(5, 8, 3)$   $N = 8$  multiplet, see [11]). The class of  $N = 5$  supersymmetric theories is more general than the ones obtained from  $D = 4$  dimensional reduction of SYM theories. It is quite important to investigate whether the [10] result is in consequence of some non-trivial deep property of the extended supersymmetry.

The computational method discussed in the next Section is a generalization of the one used in [2] to derive the  $(1, 8, 7)$   $N = 8$  off-shell action. The problem in [2] was considerably simplified by the symmetry properties of the  $(1, 8, 7)$  multiplet, which can

be covariantly expressed in terms of the octonionic structure constants. For more general multiplets, not exhibiting special symmetry properties, a brute-force technique is more suitable. We concretely applied it to the  $k = 2$  multiplets  $((2, 8, 6)_A$  and  $(2, 8, 6)_B$ , see Appendix). More cumbersome computations can be carried on for  $k > 2$  and  $N = 5, 6, 7$  length-4 multiplets (see [4]). In the next Section we describe the mathematical framework. The concrete application to the  $N = 5, 6, 7, 8$  one-dimensional  $\sigma$ -model with  $(2, 8, 6)$  field content will be discussed in Section 3. The resulting action is given in Section 4. In the Conclusions we discuss our results and point out possible applications and further lines of development. To save space we invite the reader to consult the references [4] and [5] for the information concerning the graphical interpretation of the supersymmetry transformations, the notion of multiplets of different length and/or inequivalent connectivity. The needed supersymmetry transformations are explicitly given in the Appendix.

## 2 $N$ -Extended off-shell actions.

We discuss a method to explicitly construct one-dimensional actions which are  $N = 5, 6, 7, 8$  off-shell invariant under the supersymmetry transformations closing the

$$\begin{aligned} \{Q_i, Q_j\} &= H, \\ [H, Q_i] &= 0 \end{aligned} \tag{1}$$

1D  $N$ -extended supersymmetry algebra (where  $i, j = 1, 2, \dots, N$ , while  $H$  coincides with a time-derivative).

As discussed in the Introduction we are dealing with multiplets with  $(k, 8, 8 - k)$  field-content, given by  $k$  fields of mass-dimension 0 (interpreted as coordinates of a  $k$ -dimensional target manifold), 8 fermions of mass-dimension  $\frac{1}{2}$  and  $8 - k$  auxiliary fields of mass-dimension 1.

A lagrangian  $\mathcal{L}$  generating no higher-derivatives equations of motion has mass-dimension 2 (the action is  $\mathcal{S} = \frac{1}{m} \int dt \mathcal{L}$ ).

For any  $k = 1, 2, \dots, 8$ , the most general homogeneous term  $\mathcal{T}_d$  of mass-dimension  $d$ , constructed with the fields entering the  $(k, 8, 8 - k)$  multiplet and their time-derivatives (a time-derivative counts as 1 in mass-dimension), involves the following number of independent functions of the  $k$  target coordinates:

$$\begin{aligned} \mathcal{T}_0 &: 1 \text{ function,} \\ \mathcal{T}_{\frac{1}{2}} &: 8 \text{ functions,} \\ \mathcal{T}_1 &: 36 \text{ function,} \\ \mathcal{T}_{\frac{3}{2}} &: 128 \text{ functions,} \\ \mathcal{T}_2 &: 402 \text{ functions,} \\ \mathcal{T}_{\frac{5}{2}} &: 1152 \text{ functions.} \end{aligned} \tag{2}$$

Collectively denoting with  $\vec{x}$  the  $k$  target coordinates and with  $\psi_j$  the 8 fermionic fields we have, e.g., that  $\mathcal{T}_0 \equiv F(\vec{x})$ ,  $\mathcal{T}_{\frac{1}{2}} \equiv \sum_j F_j(\vec{x})\psi_j$  and so on.

A manifestly  $\overline{N}$ -extended supersymmetric lagrangian  $\mathcal{L}_{\overline{N}}$  is produced through the position

$$\mathcal{L}_{\overline{N}} = Q_1 \cdots Q_{\overline{N}} F_{\overline{N}}, \quad (3)$$

with, in mass-dimension,  $[\mathcal{L}_{\overline{N}}] = 2$ ,  $[F_{\overline{N}}] = 2 - \frac{\overline{N}}{2}$ , provided that  $\mathcal{L}_{\overline{N}}$  is not expressed as a time-derivative of some  $d = 1$  function.

The supersymmetry operators  $Q_i$  have mass-dimension  $[Q_i] = \frac{1}{2}$ , making clear that we can have at most  $\overline{N} = 4$  manifest supersymmetries, with a lagrangian expressed in terms of unconstrained functions entering  $F_{\overline{N}}$  (their total numbers are 1, 8, 36, 128, 402 for  $\overline{N} = 0, 1, 2, 3, 4$ , respectively).

In order to have an  $N$ -extended supersymmetric action, with  $N > \overline{N}$ , we have to impose  $N - \overline{N}$  constraints, for  $j = \overline{N} + 1, \dots, N$ , expressed through

$$Q_j \mathcal{L}_{\overline{N}} = \partial_t R_{j,\overline{N}}, \quad (4)$$

where, in mass-dimension, we have

$$[R_{j,\overline{N}}] = \frac{3}{2}. \quad (5)$$

Each (4) constraint generates a system of 1152 constraining equations to be solved in terms of 128 functions (the coefficients entering  $R_{j,\overline{N}}$ ). Needless to say, the great majority of the 1152 equations are trivially satisfied, while most of the remaining ones are redundant, generating some constraint which is repeated over and over.

This is a general scheme that, with straightforward modifications, can be applied to  $N > 8$  extended supersymmetries and multiplets of length greater than 3. In the next Section we concretely apply it to analyze the off-shell invariant actions of the  $k = 2$  (2, 8, 6) multiplets.

### 3 The $N = 5, 6, 7, 8$ $\sigma$ -model with (2, 8, 6) field-content.

There are two inequivalent  $N = 5$  multiplets with (2, 8, 6) field-content [3, 4, 5]. The first one,  $(2, 8, 6)_A$ , is given [5] by the  $N = 4$  (the supersymmetric operators  $\widehat{Q}_1, \widehat{Q}_2, \widehat{Q}_3, \widehat{Q}_4$  in the Appendix) irreducible multiplets (2, 4, 2) and (0, 4, 4) linked together by a 5-th supersymmetry (the operator  $\widehat{Q}_5$ ). The second one,  $(2, 8, 6)_B$ , is given [5] by two  $N = 4$  (the supersymmetric operators  $\overline{Q}_1, \overline{Q}_2, \overline{Q}_3, \overline{Q}_4$  in the Appendix) irreducible multiplets (1, 4, 3) linked together by a 5-th supersymmetry (the operator  $\overline{Q}_5$ ).

In the latter case we can write a manifestly  $N = 4$  lagrangian  $\mathcal{L}$ , written as

$$\mathcal{L} = \overline{Q}_1 \overline{Q}_2 \overline{Q}_3 \overline{Q}_4 F(x, y), \quad (6)$$

with  $F$  an unconstrained function of the two target coordinates. Imposing, according to (4), the invariance of the action under the 5-th supersymmetry  $\overline{Q}_5$  has the effect of constraining  $F(x, y)$ , which must satisfy

$$\square F = \partial_x^2 F + \partial_y^2 F = 0. \quad (7)$$

No further constraint arises by imposing the 6-th, 7-th or 8-th supersymmetry (the operators  $\overline{Q}_6, \overline{Q}_7, \overline{Q}_8$ , respectively). Therefore, the invariance under  $N = 5B$  automatically induces the invariance under the full  $N = 8$  extended supersymmetry.

For what concerns the first case, it is a priori unclear whether a manifestly  $N = 4$  invariant lagrangian can even be produced. The  $\binom{8}{4} = 70$  inequivalent choices of 4 supersymmetry operators  $\widehat{Q}_i$  out of the total set of 8 operators produce the following results: in 16 cases a manifest  $N = 4$  lagrangian involving all fields entering the  $(2, 8, 6)_A$  multiplet can be encountered; in 6 cases the  $N = 4$  lagrangian involves only the fields entering a  $(2, 4, 2)$  submultiplet (due to the fact that there are 6 inequivalent ways of selecting  $N = 4$ -closed  $(2, 4, 2)$  multiplets inside  $(2, 8, 6)_A$ ); the remaining 48 cases give a total time-derivative.

We should not accept for granted that, starting from a manifestly  $N = 4$  lagrangian, we can get the most general off-shell invariant action for  $N > 4$  with the procedure discussed in the previous section. We explicitly verified the construction starting from a manifestly  $N = 3$  lagrangian which, in principle, can provide a more general result. After setting

$$\mathcal{L} = \overline{Q}_1 \overline{Q}_2 \overline{Q}_3 \left( \sum_j F_j(x, y) \psi_j \right) \quad (8)$$

we imposed the set of constraints generated by  $Q_4$  and, independently, the set of constraints generated by  $Q_5$ .

The  $Q_4$  constraints give the equations

$$\begin{aligned} \partial_x F_7 &= -\partial_y F_8, \\ \partial_x F_8 &= \partial_y F_7, \\ \partial_x F_6 &= \partial_y F_5, \\ \partial_x F_3 &= \partial_y F_4. \end{aligned} \quad (9)$$

The  $Q_5$  constraints give the equations

$$\begin{aligned} \partial_x F_7 &= -\partial_y F_8, \\ \partial_x F_8 &= \partial_y F_7, \\ \partial_x F_6 &= \partial_y F_5, \\ \partial_x F_1 &= -\partial_y F_2 \end{aligned} \quad (10)$$

(the only new equation is the last one).

The second, third and fourth equations in (9) and (10) can be solved in terms of the unconstrained fields  $H_1, H_2, H_3, H_4$  s.t.

$$\begin{aligned} F_7 &= \partial_x H_1, & F_8 &= \partial_y H_1, \\ F_5 &= \partial_x H_2, & F_6 &= \partial_y H_2, \\ F_4 &= \partial_x H_3, & F_3 &= \partial_y H_3, \\ F_2 &= \partial_x H_4, & F_1 &= -\partial_y H_4. \end{aligned} \quad (11)$$

The first equation in (9,10) produces the constraining equation  $\square H_1 = 0$ .

No further constraint arises by imposing invariance of the action under  $\widehat{Q}_6$ ,  $\widehat{Q}_7$  or  $\widehat{Q}_8$ .

The terms in the lagrangian depending on  $H_2, H_3, H_4$  correspond to a total derivative and can be eliminated. For instance, setting for simplicity  $K \equiv H_3$ , its corresponding term reads as  $\partial_t(K_x g_4 - K_y g_3 - K_{xx} \psi_5 \psi_2 - K_{xy} \psi_1 \psi_5 + K_{xy} \psi_2 \psi_6 - K_{yy} \psi_1 \psi_6)$ . The action only depends on the constrained field  $H_1$ . It coincides with the action produced from a manifestly  $N = 4$  construction and is automatically  $N = 8$  invariant. Since, contrary to the  $N = 5$  transformations, the  $N = 8$  transformations are uniquely defined [4], the action coincides with the one obtained by imposing the  $N = 5B$  invariance.

The overall result can be summarized in the following statements:

- the  $N = 5A$  invariance automatically induces an  $N = 8$  off-shell invariant action;
- the  $N = 5B$  invariance automatically induces an  $N = 8$  off-shell invariant action;
- the  $N = 8$  invariant action, uniquely determined in terms of a single, constrained function satisfying (7), is also invariant under the two inequivalent  $N = 5$  and the two inequivalent  $N = 6$  supersymmetry transformations.

## 4 The action

The most general off-shell invariant action is expressed in terms of a single prepotential  $\Phi(x, y)$  satisfying the constraint

$$\square \Phi = \partial_x^2 \Phi + \partial_y^2 \Phi = 0 \quad (12)$$

( $\Phi$  is explicitly recovered in terms of  $F$  entering (6) as  $\Phi = \partial_x^2 F$ ).

The lagrangian of the system is explicitly given by (the summation over repeated indexes is understood)

$$\begin{aligned} \mathcal{L} = & \Phi(\dot{x}^2 + \dot{y}^2 - \psi_0 \dot{\psi}_0 - \psi_i \dot{\psi}_i - \lambda_0 \dot{\lambda}_0 - \lambda_i \dot{\lambda}_i + g_i g_i + f_i f_i) + \\ & + \Phi_x [\dot{y}(\psi_0 \lambda_0 - \psi_i \lambda_i) - g_i(\psi_i \psi_0 + \lambda_i \lambda_0) + f_i(\psi_i \lambda_0 - \lambda_i \psi_0) + \\ & \epsilon_{ijk}(f_i \lambda_j \psi_k + \frac{1}{2} g_i(\lambda_j \lambda_k - \psi_j \psi_k))] + \\ & - \Phi_y [\dot{x}(\psi_0 \lambda_0 - \psi_i \lambda_i) + f_i(\psi_i \psi_0 + \lambda_i \lambda_0) + g_i(\psi_i \lambda_0 - \lambda_i \psi_0) + \\ & - \epsilon_{ijk}(g_i \psi_j \lambda_k - \frac{1}{2} f_i(\lambda_j \lambda_k - \psi_j \psi_k))] + \\ & + \Phi_{xx} [\frac{1}{6} \epsilon_{ijk}(\psi_i \psi_j \psi_k - 3 \lambda_i \lambda_j \psi_k) \psi_0] + \\ & + \Phi_{yy} [\frac{1}{6} \epsilon_{ijk}(\lambda_i \lambda_j \lambda_k - 3 \psi_i \psi_j \lambda_k) \lambda_0] + \\ & - \Phi_{xy} [\frac{1}{6} \epsilon_{ijk}(\psi_i \psi_j \psi_k \lambda_0 + \lambda_i \lambda_j \lambda_k \psi_0 + 3 \psi_i \lambda_j \lambda_k \lambda_0 + 3 \lambda_i \psi_j \psi_k \psi_0)]. \end{aligned} \quad (13)$$

The action associated to the lagrangian is invariant under  $N = 8$  off-shell supersymmetries. The presentation above makes use of the manifest, quaternionic-covariant,  $N = 4$  decomposition into  $(1, 4, 3) + (1, 4, 3)$  multiplets. It allows to present the results in terms

of the totally antisymmetric  $\epsilon_{ijk}$  tensor. By setting  $x_1 = x$  and  $x_2 = y$ , we can express the lagrangian as  $\mathcal{L} = g_{ij}\dot{x}_i\dot{x}_j + \dots$ , where  $g_{ij}(x_1, x_2)$  denotes the target manifold metric. In our case

$$g_{ij} = \delta_{ij}\Phi, \quad (14)$$

proving that the target is conformally flat. The constraint (7) implies that the target manifold corresponds to a special geometry of rigid type (see [12] for details).

Some comments are in order. The (13) lagrangian coincides with the result presented in [6] for the  $N = 8$  (2, 8, 6) action. Our analysis is however more complete and convenient for at least three reasons: we have imposed the invariance under  $N = 5$  generators and proven that the resulting action is automatically invariant under  $N = 8$  supersymmetry. We investigated the role of the multiplets with different connectivity and proved that the action obtained from (13) is invariant under both  $N = 5A$  and  $N = 5B$ . Finally, our computational scheme allows to present the supersymmetric action directly in terms of the component fields. It is quite a non-trivial and cumbersome task to extract the component fields from the constrained superfields employed in [6].

## 5 Conclusions

This work is the first one in an intended systematic program of constructing and analyzing the properties of off-shell invariant  $N$ -extended one-dimensional supersymmetric  $\sigma$ -models. This project is made possible due to two main reasons. The classification, given in [4], of the inequivalent irreducible multiplets of the  $N$ -extended supersymmetries (the [4] results also allow to explicitly construct a representative multiplet in each equivalence class) and the computational scheme, here discussed, to produce off-shell invariant actions.

We concretely investigated the simplest case (the (2, 8, 6) multiplets) admitting, for the same field content, inequivalent  $N = 5$  supersymmetries. The associated  $\sigma$ -model possesses a two-dimensional target. We proved that requiring an  $N = 5A$  invariance automatically guarantees an  $N = 8$  off-shell invariant action. Similarly, requiring an  $N = 5B$  invariance automatically guarantees a full  $N = 8$  invariance. The action, which is therefore invariant under both  $N = 5$  transformations, corresponds to a conformally flat  $2D$  target with a special metric of rigid type (satisfying the (12) equation).

There is a common wisdom, originated by scattered results produced by several groups using different methods, which deserves being tested. It states that for  $N > 4$  linear supersymmetries the target manifold should be conformally flat and the conformal factor should satisfy a Laplace equation. This is indeed what we obtained here. An explicit proof for all cases is however lacking. This would amount to investigate the remaining  $(k, 8, 8-k)$  multiplets, as well as the length-4 multiplets (those admitting fermionic auxiliary fields of mass-dimension  $\frac{3}{2}$ ). The length-4 multiplets, classified in [2] and [4], cannot be “oxidized” to  $N = 8$  (they only carry  $N = 5, 6$  or  $7$  supersymmetries, according to the cases). So far they have never been analyzed in the literature. They can now be investigated with the method here developed.

The issue of the  $N = 5$  invariance implying the  $N = 8$  invariance is also of utter importance. Our result corroborates (for a totally different and in principle unrelated system) the finding of [10] obtained by twisting the  $N = 2$  Super-Yang-Mills theory. It would be important to verify whether this is a general property, valid for any  $(k, 8, 8 - k)$  multiplet.

Our computational scheme is not limited to  $N = 8$ . It works in principle for  $N > 8$ . Some special cases are of particular importance. The dimensional reduction of the  $N = 4$  Super-Yang-Mills theory to 0+1 dimensions produces [11, 13] an  $N = 9$  off-shell invariance realized on a  $(9, 16, 7)$  multiplet which can be decomposed, under the  $N = 4$  subalgebra, into a single  $(3, 4, 1)$  vector multiplet and three  $(2, 4, 2)$  matter multiplets. In terms of the  $N = 8$  decomposition we end up with the  $(5, 8, 3)$  and  $(4, 8, 4)$  multiplets linked together by a 9-th supersymmetry [5].

The results on twist suggest [13] that 6 generators are sufficient to fully determine the full  $N = 9$  invariance. This would amount to use 5 out of the 8 generators in the  $N = 8$  decomposition. The remaining generator to be used would be the 9-th supersymmetry generator.

Our computational scheme can allow to attack this one as well as similarly related problems. It is worth mentioning that in the literature no systematic investigation of off-shell invariant actions for  $N > 8$  have been carried on.

## Appendix

The  $N = 5A, 5B, 6A, 6B, 7, 8$  extended supersymmetries represented on the multiplets with  $(2, 8, 6)$  field-content can be explicitly given as

$$\begin{aligned}
N = 5A & : \hat{Q}_1, \hat{Q}_2, \hat{Q}_3, \hat{Q}_4, \hat{Q}_5 \\
N = 5B & : \bar{Q}_1, \bar{Q}_2, \bar{Q}_3, \bar{Q}_4, \bar{Q}_5 \\
N = 6A & : \hat{Q}_1, \hat{Q}_2, \hat{Q}_3, \hat{Q}_4, \hat{Q}_5, \hat{Q}_6 \\
N = 6B & : \bar{Q}_1, \bar{Q}_2, \bar{Q}_3, \bar{Q}_4, \bar{Q}_5, \bar{Q}_6 \equiv \hat{Q}_1, \hat{Q}_2, \hat{Q}_3, \hat{Q}_4, \hat{Q}_5, \hat{Q}_7 \\
N = 7 & : \bar{Q}_1, \bar{Q}_2, \bar{Q}_3, \bar{Q}_4, \bar{Q}_5, \bar{Q}_6, \bar{Q}_7 \equiv \hat{Q}_1, \hat{Q}_2, \hat{Q}_3, \hat{Q}_4, \hat{Q}_5, \hat{Q}_6, \hat{Q}_7 \\
N = 8 & : \bar{Q}_1, \bar{Q}_2, \bar{Q}_3, \bar{Q}_4, \bar{Q}_5, \bar{Q}_6, \bar{Q}_7, \bar{Q}_8 \equiv \hat{Q}_1, \hat{Q}_2, \hat{Q}_3, \hat{Q}_4, \hat{Q}_5, \hat{Q}_6, \hat{Q}_7, \hat{Q}_8 \quad (15)
\end{aligned}$$

where the  $\hat{Q}_i$  and  $\bar{Q}_i$  generators are expressed in the tables below. The  $N = 5A$   $(2, 8, 6)_A$  multiplet admits  $2_5 + 2_4 + 4_3$  connectivity (see [4]) and  $N = 4$  decomposition (see [5]) given by  $(2, 4, 2) + (0, 4, 4)$ . Its component fields can be parametrized as  $(x, y; \psi_1, \dots, \psi_8; g_1, \dots, g_6)$ . The  $N = 5B$   $(2, 8, 6)_B$  multiplet admits  $6_4 + 4_3$  connectivity (see [4]) and  $N = 4$  decomposition given by  $(1, 4, 3) + (1, 4, 3)$  (see [5]). It is convenient to parametrize its component fields as  $(x, y; \psi_0, \psi_1, \psi_2, \psi_3, \lambda_0, \lambda_1, \lambda_2, \lambda_3; g_1, g_2, g_3, f_1, f_2, f_3)$  to make its quaternionic structure clear ( $g_i, f_i$  are the auxiliary fields). The  $N = 6A$  multiplet admits  $2_6 + 6_4$  connectivity, while  $N = 6B$  admits  $4_5 + 4_4$  connectivity. Note that, according to the ‘‘oxidation diagram’’ of [5],  $N = 5B$  can only be lifted to the  $B$ -type  $N = 6$  supersymmetry, while  $N = 5A$  can be lifted to both  $N = 6A$  and  $N = 6B$ .



The  $\widehat{Q}_i$  and  $\overline{Q}_i$  generators are explicitly given by

	$\widehat{Q}_1$	$\widehat{Q}_2$	$\widehat{Q}_3$	$\widehat{Q}_4$	$\widehat{Q}_5$	$\widehat{Q}_6$	$\widehat{Q}_7$	$\widehat{Q}_8$
$x$	$\psi_1$	$-\psi_2$	$-\psi_3$	$-\psi_4$	$\psi_5$	$\psi_6$	$\psi_7$	$\psi_8$
$y$	$\psi_2$	$\psi_1$	$\psi_4$	$-\psi_3$	$\psi_6$	$-\psi_5$	$\psi_8$	$-\psi_7$
$\psi_1$	$\dot{x}$	$\dot{y}$	$g_1$	$g_2$	$-g_3$	$-g_4$	$-g_5$	$-g_6$
$\psi_2$	$\dot{y}$	$-\dot{x}$	$-g_2$	$g_1$	$-g_4$	$g_3$	$-g_6$	$g_5$
$\psi_3$	$g_1$	$g_2$	$-\dot{x}$	$-\dot{y}$	$-g_5$	$g_6$	$g_3$	$-g_4$
$\psi_4$	$g_2$	$-g_1$	$\dot{y}$	$-\dot{x}$	$-g_6$	$-g_5$	$g_4$	$g_3$
$\psi_5$	$g_3$	$-g_4$	$-g_5$	$-g_6$	$\dot{x}$	$-\dot{y}$	$-g_1$	$-g_2$
$\psi_6$	$g_4$	$g_3$	$g_6$	$-g_5$	$\dot{y}$	$\dot{x}$	$-g_2$	$g_1$
$\psi_7$	$g_5$	$-g_6$	$g_3$	$g_4$	$g_1$	$g_2$	$\dot{x}$	$-\dot{y}$
$\psi_8$	$g_6$	$g_5$	$-g_4$	$g_3$	$g_2$	$-g_1$	$\dot{y}$	$\dot{x}$
$g_1$	$\dot{\psi}_3$	$-\dot{\psi}_4$	$\dot{\psi}_1$	$\dot{\psi}_2$	$\dot{\psi}_7$	$-\dot{\psi}_8$	$-\dot{\psi}_5$	$\dot{\psi}_6$
$g_2$	$\dot{\psi}_4$	$\dot{\psi}_3$	$-\dot{\psi}_2$	$\dot{\psi}_1$	$\dot{\psi}_8$	$\dot{\psi}_7$	$-\dot{\psi}_6$	$-\dot{\psi}_5$
$g_3$	$\dot{\psi}_5$	$\dot{\psi}_6$	$\dot{\psi}_7$	$\dot{\psi}_8$	$-\dot{\psi}_1$	$\dot{\psi}_2$	$\dot{\psi}_3$	$\dot{\psi}_4$
$g_4$	$\dot{\psi}_6$	$-\dot{\psi}_5$	$-\dot{\psi}_8$	$\dot{\psi}_7$	$-\dot{\psi}_2$	$-\dot{\psi}_1$	$\dot{\psi}_4$	$-\dot{\psi}_3$
$g_5$	$\dot{\psi}_7$	$\dot{\psi}_8$	$-\dot{\psi}_5$	$-\dot{\psi}_6$	$-\dot{\psi}_3$	$-\dot{\psi}_4$	$-\dot{\psi}_1$	$\dot{\psi}_2$
$g_6$	$\dot{\psi}_8$	$-\dot{\psi}_7$	$\dot{\psi}_6$	$-\dot{\psi}_5$	$-\dot{\psi}_4$	$\dot{\psi}_3$	$-\dot{\psi}_2$	$-\dot{\psi}_1$

(16)

and

	$\overline{Q}_1$	$\overline{Q}_2$	$\overline{Q}_3$	$\overline{Q}_4$	$\overline{Q}_5$	$\overline{Q}_6$	$\overline{Q}_7$	$\overline{Q}_8$
$x$	$-\psi_1$	$-\psi_2$	$-\psi_3$	$\psi_0$	$\lambda_0$	$\lambda_1$	$\lambda_2$	$\lambda_3$
$\psi_0$	$g_1$	$g_2$	$g_3$	$\dot{x}$	$-\dot{y}$	$-f_1$	$-f_2$	$-f_3$
$\psi_1$	$-\dot{x}$	$g_3$	$-g_2$	$g_1$	$-f_1$	$\dot{y}$	$f_3$	$-f_2$
$\psi_2$	$-g_3$	$-\dot{x}$	$g_1$	$g_2$	$-f_2$	$-f_3$	$\dot{y}$	$f_1$
$\psi_3$	$g_2$	$-g_1$	$-\dot{x}$	$g_3$	$-f_3$	$f_2$	$-f_1$	$\dot{y}$
$g_1$	$\dot{\psi}_0$	$-\dot{\psi}_3$	$\dot{\psi}_2$	$\dot{\psi}_1$	$\dot{\lambda}_1$	$-\dot{\lambda}_0$	$-\dot{\lambda}_3$	$\dot{\lambda}_2$
$g_2$	$\dot{\psi}_3$	$\dot{\psi}_0$	$-\dot{\psi}_1$	$\dot{\psi}_2$	$\dot{\lambda}_2$	$\dot{\lambda}_3$	$-\dot{\lambda}_0$	$-\dot{\lambda}_1$
$g_3$	$-\dot{\psi}_2$	$\dot{\psi}_1$	$\dot{\psi}_0$	$\dot{\psi}_3$	$\dot{\lambda}_3$	$-\dot{\lambda}_2$	$\dot{\lambda}_1$	$-\dot{\lambda}_0$
$y$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_0$	$-\psi_0$	$\psi_1$	$\psi_2$	$\psi_3$
$\lambda_0$	$-f_1$	$-f_2$	$-f_3$	$\dot{y}$	$\dot{x}$	$-g_1$	$-g_2$	$-g_3$
$\lambda_1$	$\dot{y}$	$-f_3$	$f_2$	$f_1$	$g_1$	$\dot{x}$	$g_3$	$-g_2$
$\lambda_2$	$f_3$	$\dot{y}$	$-f_1$	$f_2$	$g_2$	$-g_3$	$\dot{x}$	$g_1$
$\lambda_3$	$-f_2$	$f_1$	$\dot{y}$	$f_3$	$g_3$	$g_2$	$-g_1$	$\dot{x}$
$f_1$	$-\dot{\lambda}_0$	$\dot{\lambda}_3$	$-\dot{\lambda}_2$	$\dot{\lambda}_1$	$-\dot{\psi}_1$	$-\dot{\psi}_0$	$-\dot{\psi}_3$	$\dot{\psi}_2$
$f_2$	$-\dot{\lambda}_3$	$-\dot{\lambda}_0$	$\dot{\lambda}_1$	$\dot{\lambda}_2$	$-\dot{\psi}_2$	$\dot{\psi}_3$	$-\dot{\psi}_0$	$-\dot{\psi}_1$
$f_3$	$\dot{\lambda}_2$	$-\dot{\lambda}_1$	$-\dot{\lambda}_0$	$\dot{\lambda}_3$	$-\dot{\psi}_3$	$-\dot{\psi}_2$	$\dot{\psi}_1$	$-\dot{\psi}_0$

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## Acknowledgments

M.G. receives a CLAF grant. M.R. receives a FAPEMIG grant. The work has been supported by Edital Universal CNPq, Proc. 472903/2008-0.

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