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NONLINEAR ELECTRODYNAMICS AND THE SURFACE REDSHIFT OF PULSARS

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ABSTRACT

Currently is argued that the best method of determining the neutron star (NS) fundamental properties is by measuring the *gravitational redshift* (z) of spectral lines produced in the star photosphere. Measurement of z at the star surface provides a unique insight on the NS mass-to-radius relation and thus on its equation of state (EoS), which reflects the physics of the strong interaction between particles making up the star. Evidence for such a measurement has been provided quite recently by Cottam, Paerels & Mendez (2002), and also by Sanwal et *al.* (2002). Here we argue that although the quoted observations are undisputed for canonical pulsars, they could be misidentified if the NS is endowed with a super strong B as in the so-called magnetars (Duncan & Thompson 1992) and strange quark magnetars (Zhang 2002), as in the spectral line discovered by Ibrahim et *al.* (2002;2003). The source of this new "confusion" redshift is related to nonlinear electrodynamics (NLEDs) effects.

Subject headings: Gravitation: redshift — line: formation — line: identification — stars: Bs — nonlinear methods: electrodynamics — stars: pulsars: general

1. INTRODUCTION

Neutron stars (NSs), the death throes of massive stars, are among the most exotic objects in the universe. They are supposed to be composed of essentially neutrons, although some protons and electrons are also required in order to guarantee stability against Pauli's exclusion principle for fermions. In view of its density, a neutron star is also believed to trap in its core a substantial part of even more exotic states of matter (EoS). It is almost a concensus that these new states might exist inside and may dominate the star structural properties. Pions plus kaons Bose-Einstein condensates could appear, as well as "bags" of strange quark matter (Miller 2002). This last one believed to be the most stable state of nuclear matter (Glendenning 1997), which implies an extremely dense medium whose physics is currently under severe scrutiny. The major effect of these exotic constituents is manifested through the NS mass-radius ratio (M/R). Most researchers in the field think of the presence of such exotic components not only as to make the star more compact, i.e., smaller in radius, but also to lower the maximum mass it can retain. To get some insight into the neutron star most elusive properties: its mass (M) and radius (R), astronomers use several techniques at disposal, being the most prospective one the *gravitational redshift*. Since the redshift depends on the ratio M/R, then measuring NS spectral lines displacement leads to a direct insight into this dense matter equation of state.

In the late years strong evidence seems to have gathered around a new and exotic class of hyper magnetized neutron stars: the so-called "magnetars" (Duncan & Thompson 1992). These objects are supposed to be the final stage of newly-born neutron stars in which a classical alpha-omega dynamo mechanism has efficiently acted on during the early stages of its evolution, reaching field strentghs up to $B_{\rm Sup-Crit} \sim 10^{17}$ G. A peculiar class of gamma-ray sources known as "soft gamma-ray repeaters" (SGRs) (Kouveliotou et *al.* 1998), and a set of X-ray pulsars known as of "anomalous" (AXPs), have been claimed to be associated with these type of stars (Mereghetti 1999).

Although these magnetars are said to be the best model to explain the dynamics of SGRs and AXPs, accretion-driven models (Marsden et *al.* 2001), strange quark matter stars with normal Bs (Zhang, Xu & Qiao 2000; Xu & Busse 2001), or even the strange magnetar interpretation (Zhang 2002) have also been proposed as competing scenarios. Note in passing that in a recent paper Pérez Martínez et *al.* (2003) have provided arguments contending the formation itself of the so-called magnetars, in the context of the physics used by their mentors (Duncan & Thompson 1992) for proposing their occurrence in nature. Pérez Martínez et *al.* (2003) argue that a fully description of the physics taking place during the early evolution of NSs should not overlook fundamental issues, as for instance quantum electrodynamics effects, when discussing the rôle of super strong Bs on the stability of just-born NS pulsars. The *positive magnetization* of the neutron matter and the *appearance of a ferromagnetic configuration* in the star structure are examples of such effects. Thus the idea of magnetars is still contentious. Despite of that lively dispute, in this *Letter*, we present theoretical arguments which alert on the potential effects of NLEDs in the physics of strongly magnetized NSs.

A very interesting example of how this issue could be elusive is provided by the recent discovery by Ibrahim et *al.* (2002), and its subsequent confirmation by Ibrahim et *al.* (2003), of cyclotron resonance features from the SGR 1806-20. It is well-known, in accretion-powered models, that proton (p) and α-particles (He) produce, respectively, fundamental resonances of energy

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$$E_p =^{6.3|^p}_{3.2|_{He}} (1+z)^{-1} \left[\frac{B_{\rm Sup-Crit}}{10^{15}{\rm G}}\right] \ {\rm keV} \ . \qquad (1)$$

Ibrahim et *al.* (2002;2003) showed that the 5 keV absorption line in the spectrum of SGR 1806-20 is consistent with a proton-cyclotron fundamental resonance in a redshift-dependent super critical magnetic field (B) of strength: $B_{\rm Sup-Crit} \sim 7.9 \times 10^{14}(1+z)^{-1}$ G. This translates into a $B_{\rm Sup-Crit} \sim 1.0 \times 10^{15}$ G, for the mass and radius of a *canonical* NS ($\rho \sim 10^{14}$ g cm^{-3}, $R \sim 10$ km, $M \sim 1.4\ M_\odot$, $B \sim 10^{12}$ G). An estimate that agrees with the field strength inferred from the SGR 1806-20 spindown, i.e. from P and $\dot{P}$ (Kouveliotou et *al.* 1998).

2. GRAVITATIONAL VS. NLEDS REDSHIFT

In particular, we argue that for extremely supercritical magnetic fields NLEDs effects force photons to propagate along accelerated curves. In case the nonlinear Lagrangean density is a function only of the scalar $F = F_{\mu\nu}F^{\mu\nu}$, to say $L(F)$, the force accelerating the photons is given by

$$k_{\alpha||\nu}k^\nu = \frac{k^2}{2}L_F - 2(L_{FF}F^\mu_\beta F^{\beta\nu})_{|\alpha}k_\mu k_\nu \ , \qquad (2)$$

where k_ν is the wavevector, and L_F means partial derivative with respect to F (note that it does not depend on any intrinsic property of the photons). This feature allows for this force, acting along the photons path, to be geometrized (Novello et *al.* 2000; Novello & Salim 2001) in such a way that in an effective metric: $g^{eff}_{\mu\nu} = g_{\mu\nu} + g^{eletro}_{\mu\nu}$ the photons follows geodesic paths, as we shall show in section (III) in the particular case of the Lagrangean called for above. The standard geometric procedure used in general relativity (GR) to describe the photons can now be used upon substituting the usual metric by the effective metric. In particular, the outcoming redshifts prove to have now a couple of components, one due to the gravitational field and another stemming from the B.

A direct insight into the GR $z = z(M,R)$ at the surface of a compact star could be attained from the identification of absorption or emission lines from it. NS mass (M) can be estimated, in some cases, from the orbital dynamics of binary systems, while attempts to measure its radius (R) proceed via high-resolution spectroscopy (Sanwal et *al.* 2002; upon studying the star 1E1207.4-5209; Cottam, Paerels & Mendez 2002; by analysing type-I X-ray bursts from the star EXO0748-676). In these systems success was achieved in determining these parameters, or the relation in between, by looking at excited ions near the NS surface (arguments favoring a strange star in EXO0748-676 are given by Xu 2003). Gravity effects cause the observed energies of the spectral lines of excited atoms to be shifted to lower values by a factor

$$\frac{1}{(1+z)} \equiv \left(1 - \frac{2G}{c^2}\left[\frac{M}{R}\right]\right)^{1/2} \ . \qquad (3)$$

Measurements of such line properties: energy, width, and polarization; as here called for, would lead to an indirect, but highly accurate estimate of the NS mass-to-radius ratio (M/R); and a tight constraint on its EoS, and to strong limits on the B strength (but not on its configuration) at the star surface. The above analysis stands on whenever effects of NS Bs are negligible. However, if the NS is pervaded by a super strong B ($B_{\rm Sup-Crit}$), then NLEDs should be taken into account to describe the overall physics taking place on the pulsar surface. Our major result proves that for extremely high Bs the redshift induced by NLEDs can be as high as the produced by gravity alone, thus making hard to draw any conclusive claim on those NS fundamental properties.

As claimed here, the shift in energy, and width, produced by the effective metric "pull" of the star on laboratory known spectral lines, scales up directly with the strength of the effective potential associated to the effective metric. Thence, this shift has two contributions: one coming from gravitational and another from NLEDs. For hyper magnetized stars, e.g. magnetars, and if the near surface multipole field is much stronger than the dipole component (see Duncan 1998 for a possible toroidal configuration in SGR 0526-66 based on global seismic oscillations; Section IV discusses implications for the cyclotron line interpretation), the correction factor from NLEDs is substantial being both contributions of about the same order of magnitude. Thus, there is the possibility, for a given field strength, for gravity effects to be mimicked by electromagnetic (EM) ones, and for the phenomenon to entangle the fixing of constraints on the M/R ratio. This difficulty, we suggest, can be overcome by taking into account that the contribution of the B, that differs from that of the gravitational field which is isotropic, depends on the polarization b^α of the emitted photon, being different for the cases $B_\alpha b^\alpha = 0$ and $B_\alpha b^\alpha \neq 0$.

Our warning is, therefore, that the identification and analysis of spectral lines from high B NSs (in outbursts) must take into account the two possible different polarizations of the received photons, in order to discriminate between redshifts produced either gravitationally or electromagnetially. Putting this result in perspective, we claim that if the characteristic z, or M/R ratio, were to be inferred from this type of sources care should be taken since for this super strong B such z becomes of the order of the gravitational one expected from a canonical NS. It is, therefore, not clear whether one can cathegorically assert something about the, e.g. SGR 1806-20, M/R ratio under such dynamical conditions. We prove this claim next.

3. THE MODEL

The propagation of photons in NLEDs has been undertaken by several authors (Bialynicka-Birula & Bialynicki-Birula 1970; Garcia & Plebanski 1989; Dittrich & Gies 1998; De Lorenci, Klippert, Novello & Salim 2000). In the case of geometric optics, were the photons propagation can be identified with the propagation of discontinuities of the EM field in a nonlinear regime, a remarkable property appears: the discontinuities propagate along null geodesics of an effective geometry which depends on the EM field of the background (Novello et *al.* 2000; Novello & Salim 2001). According to quantum electrodynamics, vacuum has nonlinear properties and these novel properties of photons propagation in NLEDs can be showed up, in principle, in photons propagating in vacuum. In this specific case, the equations for the EM field in vacuum coincide in their form with the equations of continua where the electric and magnetic permittivity tensors $\epsilon_{\alpha\beta}$ and $\mu_{\alpha\beta}$ are functions of the electric and magnetic fields determined by some observer represented by its velocity 4-vector V^{μ}. In curved space-time these equations are written as (symbol "||" stands for covariant derivative)

$$D^{\alpha}_{||\alpha} = 0, \quad B^{\alpha}_{||\alpha} = 0 \; , \tag{4}$$

$$D^{\alpha}_{||\beta}\frac{V^{\beta}}{c} + \eta^{\alpha\beta\rho\sigma}V_{\rho}H_{\sigma||\beta} = 0, \tag{5}$$

$$B^{\alpha}_{||\beta}\frac{V^{\beta}}{c} - \eta^{\alpha\beta\rho\sigma}V_{\rho}E_{\sigma||\beta} = 0, \tag{6}$$

where $\eta^{\alpha\beta\rho\sigma}$ is the completely antisymmetric Levi-Civita tensor. The 4-vectors representing the EM field are defined as usual in terms of the EM field tensor $F_{\mu\nu}$ and polarization tensor $P_{\mu\nu}$

$$E_{\mu} = F_{\mu\nu}\frac{V^{\nu}}{c}, \quad B_{\mu} = F^{*}_{\mu\nu}\frac{V^{\nu}}{c} \; , \tag{7}$$

$$D_{\mu} = P_{\mu\nu}\frac{V^{\nu}}{c}, \quad H_{\mu} = P^{*}_{\mu\nu}\frac{V^{\nu}}{c} \; , \tag{8}$$

where the dual tensor $X^{*}_{\mu\nu}$ is defined as $X^{*}_{\mu\nu} = \frac{1}{2}\eta_{\mu\nu\alpha\beta}X^{\alpha\beta}$, for any antisymmetric second order tensor $X_{\alpha\beta}$. The meaning of vectors D^{μ} and H^{μ} comes from the Lagrangean of the EM field, and in the case of vacuum they read: $H_{\mu} = \mu_{\mu\nu}B^{\nu}$, $\quad D_{\mu} = \epsilon_{\mu\nu}E^{\nu}$, where the permissivity tensors are given as

$$\mu_{\mu\nu} = \left[1 + \frac{2\alpha}{45\pi B_q^2}\left(B^2 - E^2\right)\right]h_{\mu\nu} - \frac{7\alpha}{45\pi B_q^2}B_{\mu}B_{\nu} \; , \tag{9}$$

$$\epsilon_{\mu\nu} = \left[1 + \frac{2\alpha}{45\pi B_q^2}\left(B^2 - E^2\right)\right]h_{\mu\nu} + \frac{7\alpha}{45\pi B_q^2}E_{\mu}E_{\nu} \; . \tag{10}$$

In these expressions α is the EM coupling constant ($\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$) and B_q is a quantum electrodynamic parameter: $B_q = \frac{m^2c^3}{e\hbar} = 4.41 \times 10^{13}$ G, also known as the Schwinger critical B, i.e., $B_q \equiv B_{Crit}$. The tensor $h_{\mu\nu}$ is the metric induced in the reference frame perpendicular to the observers determined by the vector field V^{μ}. Our main concern in this paper is the behavior of NLEDs in either a pulsar or magnetar, so in this particular case $E^{\alpha} = 0$ and $\epsilon^{\alpha}_{\beta} = h^{\alpha}_{\beta}$. We will deal with light propagation in NLEDs in optical aproximation. The EM wave will be represented by 3-surfaces of discontinuities of the EM field that propagate in the nonlinear background. As we will show the EM wave propagation can be described as if the metric of the background were changed from its Minkowskian value into another effective metric which depends on the dynamics of the background EM field. This formalism allows us to use the well-known results from Riemann geometry largely applied in GR.

Following Hadamard (1903), the surface of discontinuity of the EM field is denoted by Σ. The field is continuous when crossing Σ, while its first derivative presents a finite discontinuity specified as follows

$$[B^{\mu}]_{\Sigma} = 0, \quad [\partial_{\alpha}B^{\mu}]_{\Sigma} = b^{\mu}k_{\alpha}, \quad [\partial_{\alpha}E^{\mu}]_{\Sigma} = e^{\mu}k_{\alpha} \; , \tag{11}$$

where the symbol

$$[J]_{\Sigma} = \lim(J_{\Sigma+\delta} - J_{\Sigma-\delta}) \tag{12}$$

represents the discontinuity of the arbitrary function J through the surface Σ. The tensor $f_{\mu\nu}$ is called the discontinuity of the field and $k_{\lambda} = \partial_{\lambda}\Sigma$ is the propagation vector. Applying the conditions (11) and (12) to the field equations in the particular case of $E^{\alpha} = 0$, we obtain the constrains $e^{\mu}k_{\mu} = 0, b^{\mu}k_{\mu} = 0$ and the following equations for the discontinuity fields e_{α} and b_{α}:

$$e^{\lambda}k_{\alpha}\frac{V^{\alpha}}{c} + \eta^{\lambda\mu\rho\nu}\frac{V^{\rho}}{c}\left(\mu b_{\nu}k_{\mu} - \mu'\lambda_{\alpha}B_{\nu}k_{\mu}\right) = 0 \; , \tag{13}$$

$$b^{\lambda}k_{\alpha}\frac{V^{\alpha}}{c} - \eta^{\lambda\mu\rho\nu}\frac{V^{\rho}}{c}\left(e_{\nu}k_{\mu}\right) = 0 \; , \tag{14}$$

where μ defines the magnetic permissivity, and by ($'$) we mean $\frac{d}{dB}$. A lengthy algebraic manipulation then comes. It starts by writing e^{α} in Eq.(13) as a function of μ, b_{ν}, B_{α}, to obtain

$$e^{\lambda}k_{\alpha}\frac{V^{\alpha}}{c} = -\eta^{\lambda\mu\rho\nu}\frac{V^{\rho}}{c}\left(\mu b_{\nu}k_{\mu} - \mu'\lambda_{\alpha}B_{\nu}k_{\mu}\right) \; . \tag{15}$$

Upon substituting the term e^{ν} in Eq.(14) by the expression (15), what comes out is a long polynomial type expression involving $b_{\alpha}, k^{\beta}, V^{\mu}, \mu, B_{\gamma}$, which reads

$$b^{\lambda}[1 + \frac{\mu k^2V^2}{(k^{\alpha}V_{\alpha})^2} - \mu] + k^{\lambda}\frac{\mu' l^{\alpha}b_{\alpha}k^{\beta}B_{\beta}}{(k_{\mu}V^{\mu})^2} - \frac{V^{\lambda}}{(k_{\beta}V^{\beta})^2}\mu' l_{\alpha}$$
$$\times b^{\alpha}k_{\mu}B^{\mu}k_{\nu}V^{\nu} = -B^{\lambda}\mu' l_{\alpha}b^{\alpha}[1 - \frac{k^2V^2}{(k_{\nu}V^{\nu})}]^2. \tag{16}$$

This expression is already squared in k_{ν} but still has an unknown b_{α} term. To get rid of it one multiplies by B, to take advantage of the EM wave polarization-dependence. By noting that if $B \cdot b = 0$ one obtains the *dispersion relation* by separating out the $k^{\mu}k^{\nu}$ term, what remains is the (-) effective metric. Similarly, if $B \cdot b \neq 0$ one simply divides by $B \cdot b$ so that by factoring $k^{\mu}k^{\nu}$ what results is the (+) effective metric. For the case $B_{\alpha}b^{\alpha} = 0$ one obtains

$$\left[g^{\alpha\beta} + \left(\frac{1}{\mu} - 1\right)\frac{V^{\alpha}V^{\beta}}{c^2}\right]k_{\alpha}k_{\beta} = 0 \; , \tag{17}$$

whereas for the case $B_{\alpha}b^{\alpha} \neq 0$ it results

$$\left[(1-\xi)g^{\alpha\beta} + \left(\frac{1}{\mu} + \xi - 1\right)\frac{V^{\alpha}V^{\beta}}{c^2} + \xi l^{\alpha}l^{\beta}\right]k_{\alpha}k_{\beta} = 0 \; , \tag{18}$$

where $\xi = \frac{\mu' B}{\mu}$ and $l^{\mu} \equiv \frac{B^{\mu}}{|\vec{B}\cdot\vec{B}|^{1/2}}$. From the above expressions we can read the effective metric $g^{\alpha\beta}_{+}$ and $g^{\alpha\beta}_{-}$, where the labels $(+)$ and $(-)$ refers to extraordinary and ordinary polarized rays, respectively. To determine the redshift we need the covariant form of the metric tensor, obtained from the expression: $g_{\mu\nu}g^{\nu\alpha} = \delta^{\alpha_{\mu}}$. It reads

$$g^{+}_{\mu\nu} = \frac{g_{\mu\nu}}{(1-\xi)} + \left[\mu + \frac{1}{\xi - 1}\right]\frac{V_\mu V_\nu}{c^2} + \frac{\xi}{(\xi-1)(1-2\xi)} l_\mu l_\nu \;,$$

$$g^{-}_{\mu\nu} = g_{\mu\nu} + (\mu - 1)\frac{V_\mu V_\nu}{c^2} \;. \quad (19)$$

The function ξ can be expressed in terms of the magnetic permissivity of the vacuum, and is given as: $\xi = 2(1-\mu^{-1})$. In the particular case we are focusing on, both the emitter and server are inertial frames, that is: $V^\mu = \delta^\mu_0/\sqrt{g_{00}}$, therefore the components of both effective metrics above get coincident and given a $g^{eff}_{00} = \mu \; g_{00}$. The general expression for the redshift is then given as

$$\frac{\nu_B}{\nu_A} = \frac{\lambda_A}{\lambda_B} = \left[\frac{g_{00}(e)}{g_{00}(o)}\right]^{1/2} \;, \quad (20)$$

or

$$z = \frac{\lambda_B - \lambda_A}{\lambda_A} = \left[\frac{g_{00}(e)}{g_{00}(o)}\right]^{-1/2} - 1. \quad (21)$$

where $g_{00}(e), g_{00}(o)$ stands for the (00) effective metric components at emission and observation, respectively. Hence, for a collapsed star endowed with a very high B, the magnetic permissivity approximates as: $\mu = 1 + \frac{2\alpha}{45\pi B_q^2} B^2$, so that very far from the star the redshift can be approximated as (B_{15} as B in units of 1el5)

$$z + 1 = \left[\frac{g_{00}(o)}{g_{00}(e)}\right]^{1/2} = \frac{\left(1 - \frac{2GM}{c^2 R}\right)^{-1/2}}{\left[1 + 0.0543 B^2_{15}\right]^{1/2}} \;. \quad (22)$$

Note, however, that in the present case the correction brought to z by the nonlinear contribution of B does not depend on the polarization b^μ of the emitted photons, as one would expect from the expressions for ordinary $(-)$ and extraordinary $(+)$ rays discussed above. This is simply because in such a case, when computing the g_{00} metric component, or source *redshift*, there appears no terms coming, in Eq.(19), from the $l_0 l_0$ unitary vector along the $\vec{B}$ direction, because the source is at rest. On the other hand, if the hyper magnetized NS were either spinning or part of a binary system (seemingly not the case for magnetars), then one could take advantage from the star rotation ($P \sim 5 - 10$ s for magnetars) or orbital motion, which would bring with a velocity ($V = \Omega R$)-dependence component: $\propto |V/c|^2$, to Eq.(19). This effect, small for magnetars but rather large for fast rotators (millisecond type) NSs, may in principle allow to disentangle both components of the total pulsar surface redshift.

4. DISCUSSION AND CONCLUSION

The 5.0 keV feature discovered with *Rossi XTE* is strong, with an equivalent width of ~ 500 eV and a narrow width of less than 0.4 eV (Ibrahim et *al.* 2002;2003). When these features are viewed in the context of accretion models one arrives to a $M/R > 0.3$ M$_\odot$ km^{-1}, which is inconsistent with NSs, or that requires a low $B \sim (5-7)\times 10^{11}$ G, which is said not to correspond to any SGRs (Ibrahim et *al.* 2003). In the magnetar scenario, meanwhile, the features are plausibly explained as being ion-cyclotron resonances in an ultra strong B $B_{\rm Sup-Crit} \sim 10^{15}$ G, whose energy and width are close to model predictions (Ibrahim et *al.* 2003). According to Ibrahim et *al.* (2003), the confirmation of this findings would allow to estimate the gravitational redshift, mass and radius of the quoted “magnetar”: SGR 1806-20.

Here we alert on the possibility that it could also be due to NLEDs in the same super strong B of SGR 1806-20, as suggested by Eq.(22). To get our conclusion we used $B \sim 5 \times 10^{15}$ G, which is within the uncertainty in the B strength estimate from P and $\dot{P}$, and the likely B near-surface multipole structure, as suggested by various workers in the field. In particular, Duncan (1998) interpreted the 23 ms *global* oscillations observed in the “magnetar-like” object SGR 0526-66 as being a fundamental *toroidal mode* if a field $B \sim 4 \times 10^{15}$ G underneaths the star crust. Other authors hint at the coexistence of *poloidal* configurations, as well. For such fields, the cyclotron viewpoint could be sustained only whenever the dipole component is the dominant emission mechanism. If this were the case, no conclusive assertion on the M/R ratio of the compact star glowing in SGR 1806-20 could be consistently made, since NLEDs might well be mimicking the standard gravitational one associated with the pulsar surface redshift. More fundamentally yet, if new spectral lines were measured with high precision (as in Cottam, Paerels & Mendez 2002) from heavy elements on compact object with fields $B \gtrsim 10^{15}$ G, then the $\Delta z \gtrsim 10\% z$ correction brought to by NLEDs would prove critical regarding both its M/R ratio and EoS.

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