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CRITICALITY OF THE D=2 ANISOTROPIC QUANTUM HEISENBERG MODEL

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ABSTRACT

Within a real space renormalization group framework, we discuss the square-lattice spin - $\frac{1}{2}$ Heisenberg ferromagnet in the presence of an Ising-like anisotropy. The controversial point on how T_c vanishes in the isotropic Heisenberg limit is analyzed: quite strong evidence is presented favoring a *continuous* function of anisotropy. The crossover from the isotropic Heisenberg model to the pure Ising one is exhibited.

INTRODUCTION

Continuous symmetries cannot be spontaneously broken in short-range-interaction two-dimensional systems[1]. Consequently the order parameter associated with the Heisenberg (SO(3)) and XY (SO(2)) models vanishes for all finite temperatures. Nevertheless[2] such models are not prevented from having a phase transition at a finite temperature T_c where quantities like the susceptibility diverge (essentially associated with the fact that the two-body correlation function presents, for a finite interval of temperatures below T_c, a power-law behavior). In the case of the XY model in the presence of an Ising-like anisotropy, it is now well established[3] that, in the limit of the isotropic XY model, T_c remains *finite*; furthermore T_c most likely *continuously* varies as a function of the anisotropy. In the case of the Heisenberg model in the presence of the same type of anisotropy, the situation is less clear. Although it is a common belief that $T_c = 0$ for the *isotropic* Heisenberg model, controversy exists[4] concerning the *continuous or discontinuous* behavior of T_c as a function of the anisotropy. The calculation of this function is the main scope of the present paper. This is done within a real space renormalization group (RG) framework. The RG procedures have been applied with success for the isotropic Heisenberg model[5] as well for *discrete* group of symmetries (e.g. the q-state Potts model[6] of which the Ising model is the $q = 2$ particular case). The spin - $\frac{1}{2}$ anisotropic Heisenberg model has been treated [7] within a Migdal-Kadanoff framework. We present herein a simple single-shot treatment of this ferromagnet, whose *dimensionless*

Hamiltonian is given by

$$\mathcal{H} = 4K \sum_{<i,j>} [(1-\Delta)(S_i^x S_j^x + S_i^y S_j^y) + S_i^y S_j^z] \quad (K>0;\ 0 \leq \Delta \leq 1) \qquad (1)$$

where $K \equiv J/k_B T$ (J being the exchange integral) and where $<i,j>$ run over first-neighboring sites of a square lattice.

We shall exhibit that this Hamiltonian can be renormalized into itself (*no proliferation of coupling constants*) if convenient two-terminal graphs are used (see Fig.1; notice that both graphs share topological self-duality with the square lattice). We impose that the cluster partition function is preserved through renormalization, i.e.

$$e^{\mathcal{H}'_{12}} = \mathrm{Tr}_{3,4}\, e^{\mathcal{H}_{1234}} \qquad (2)$$

where

$$\mathcal{H}'_{12} \equiv K'_0 + 4K'[(1-\Delta')(S_1^x S_2^x + S_1^y S_2^y) + S_1^z S_2^z] \qquad (3)$$

and

$$\mathcal{H}_{1234} \equiv 4K \sum_{i<j} [(1-\Delta)(S_i^x S_j^x + S_i^y S_j^y) + S_i^z S_j^z] \qquad (4)$$

where the sum runs over the 5 bonds of the graph of Fig. 1b.

The non-commutative aspects of the present quantum problem makes Eq.(2) an operationally complex one to handle, in spite of its apparent simplicity. A similar problem has been solved[8] for the isotropic case (Δ=0). In the present case we have proceeded as follows. First we expand $e^{\mathcal{H}'_{12}}$ and obtain

$$e^{\mathcal{H}'_{12}} = a' + b'_{12}\,(S_1^x S_2^x + S_1^y S_2^y) + c'_{12}\; S_1^z S_2^z \qquad (5)$$

where a', b'_{12} and c'_{12} are functions of K'_o, K' and Δ' that we have determined. Similarly we expand $e^{\mathcal{H}_{1234}}$ and obtain

$$\begin{aligned} e^{\mathcal{H}_{1234}} = a + \sum_{i<j} [& b_{ij}(S_i^x S_j^x + S_i^y S_j^y) + c_{ij}\; S_i^z S_j^z \\ & + d_{ij}(S_i^x S_j^x + S_i^y S_j^y) S_k^z S_\ell^z \\ & + e_{ij}(S_i^x S_j^x + S_i^y S_j^y)(S_k^x S_\ell^x + S_k^y S_\ell^y)] \\ & + f\; S_1^z S_2^z S_3^z S_4^z \end{aligned} \qquad (6)$$

where a, b_{ij}, c_{ij}, d_{ij}, e_{ij} and f are functions of K and Δ and $(k,\ell) \neq (i,j)$. The use of Eqs. (2), (5) and (6) implies $a' = 4a$, $b'_{12} = 4b_{12}$ and $c'_{12} = 4c_{12}$, and therefore only the calculation of a, b_{12} and c_{12} is needed. To perform this calculation it is useful to notice that $S^z \equiv \sum_{i=1}^{4} S_i^z$ commutes with $\mathcal{H}_{1234}$, and consequently the 16×16 matrices associated with $\mathcal{H}_{1234}$ and $e^{\mathcal{H}_{1234}}$ can be presented in two $1 \times 1\,(M = \pm 2)$, two $4 \times 4\,(M = \pm 1)$ and one $6 \times 6\,(M=0)$ blocks where M is the quantum number corresponding to S^z. We finally obtain

$$e^{4K'} = H^2/4FG \qquad (7)$$

$$e^{4K'\Delta'} = H^2/4F^2 \qquad (8)$$

$$F \equiv AB\, e^{3(\Delta-1)K} + 2B\, e^{K}[A \cosh AK + \Delta \sinh AK] + A\, e^{-K}[B \cosh BK + \Delta \sinh BK] \quad (9)$$

$$G \equiv 2AB\, e^{\Delta K}[e^{K} + e^{-K}\cosh 2K(1-\Delta)] \quad (10)$$

$$H \equiv AB\, e^{(1+\Delta)K}[2e^{4K} + e^{-4K}(1+2e^{2\Delta K})] + 2B\, e^{K}[A \cosh AK - \Delta \sinh AK] + A\, e^{-K}[B \cosh BK - (2-\Delta)\sinh BK] \quad (11)$$

$$A \equiv [\Delta^2 + 16(1-\Delta)^2]^{1/2} \quad (12)$$

$$B \equiv [(2-\Delta)^2 + 32(1-\Delta)^2]^{1/2} \quad (13)$$

The $\Delta = 1$ particular case recovers the $q = b = 2$ one of Ref.[6]. The flow diagram in the $(1/K,\Delta)$ space is presented in Fig. 2. The $\Delta = 0$ axis (isotropic Heisenberg model) renormalizes into itself and contains a fixed point at $1/K = 0$ which reproduces the *exact* answer. The $\Delta = 1$ axis (Ising model) renormalizes into itself as well, and contains a fixed point at $1/K = 1/K^* \equiv 2/\ln(\sqrt{2}+1)$ which reproduces the *exact* answer. Furthermore the ferro-paramagnetic critical line presents the expected Ising criticality. At the Ising fixed point we obtain $\nu = \ln 2 / \ln(\partial K'/\partial K)\big|_{\Delta = 1, K = K^*} \simeq 1.149$ (the results corresponding to b=3,4 and 5 are 1.109, 1.095 and 1.088 respectively, to be compared with the exact result $\nu = 1$). At the isotropic Heisenberg fixed point we obtain $\nu = \infty$ which is the *exact* answer[9]. Finally Eqs. (7-13) provide the following asymptotic behaviors:

$$T_c(\Delta)/T_c(1) \sim 1 - \frac{2}{3} (1-\Delta)^2 \qquad (\Delta \to 1 \text{ limit}) \qquad (14)$$

$$e^{-4J/k_B T_c(\Delta)} \sim \Delta \qquad (\Delta \to 0 \text{ limit}) \qquad (15)$$

Let us conclude by a synthesis. The present real-space renormalization-group approach of the square-lattice spin-$\frac{1}{2}$ anisotropic Heisenberg ferromagnet reproduces: (i) the *exact*[9] $T_c=0$ and $\nu=\infty$ for the isotropic Heisenberg model ($\Delta = 0$); (ii) the *exact* T_c and a satisfactory ν for the Ising model ($\Delta=1$); (iii) the *correct* Ising criticality for $0 < \Delta \leq 1$. Consequently we are tempted to consider the critical line of Fig. 2 as a very good approximation, and the asymptotic behaviors indicated in Eqs. (14) and (15) as exact or almost exact.

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CAPTIONS FOR FIGURES

Fig. 1 - The self-dual two-terminal graphs on which the present RG is constructed. o(•) are terminal (internal) nodes on which the spins are located. b and b' are the distances between terminals (b/b' = 2 is the linear scaling factor).

Fig. 2 - RG flow diagram. The solid line is the ferro (F)-para (P)-magnetic critical frontier; the dashed lines are indicative. ■(•) denotes the Ising (isotropic Heisenberg) fixed points.

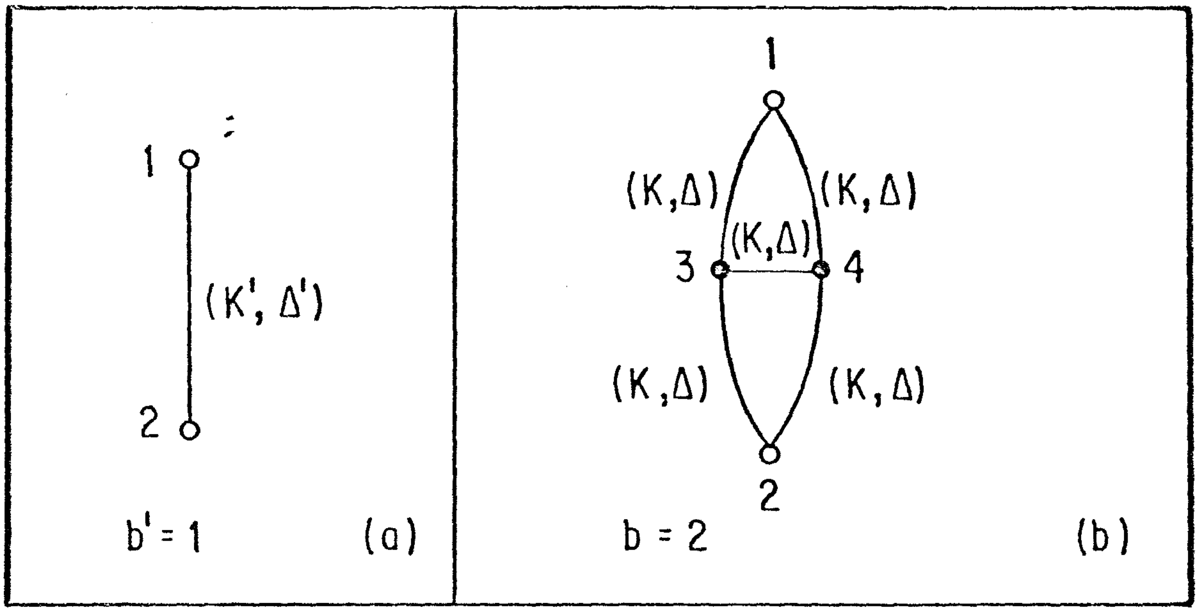

FIG.1

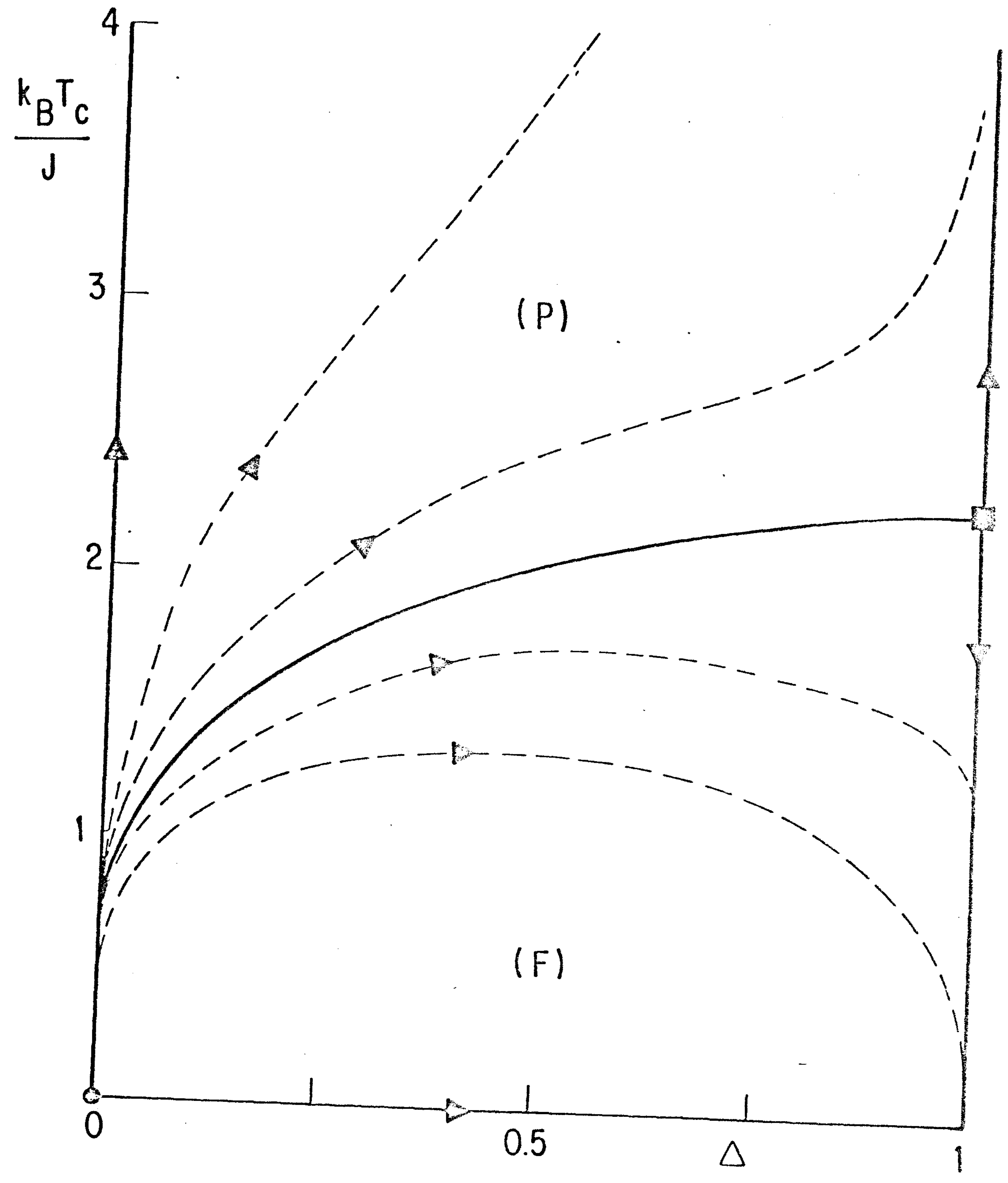

FIG.2