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Abstract

Landau's theory of Fermi liquids is applied to normal neutron matter . The quasiparticle interaction function $f_{pp'}$ is calculated in the Hartree-Fock approximation for the energy of the system . Hereby the semi-realistic hard-core potential of Gammel-Christian-Thaler is used . It is found that the resulting Landau parameters fulfill the sufficient condition for the existence of spin zero sound . The velocities of the spin waves are calculated from the linearized Landau-Boltzmann equation, when no collisions between the quasiparticles are present . In addition the spin diffusion coefficient is determined .

I. INTRODUCTION

We apply Landau's theory [1-3] for normal Fermi liquids to neutron matter. The notation of a quasiparticle was first used by Landau and later made precise in terms of formal many-body perturbation theory in order to construct a theory of normal Fermi liquids valid for long wavelength excitations.

The dynamics of the quasiparticles is described in terms of a Boltzmann type transport equation.

The energy E of the system is a complicated functional $E[n_p]$ of the quasiparticle distribution function n_p. The change in the ground state energy caused by a small number of elementary excitations is given by

$$\delta E = \sum_p \varepsilon^o(p)\delta n(p) + \frac{1}{2}\sum_{p,p'} f_{pp'}\, \delta n(p)\delta n(p') \qquad (1,1)$$

where $\varepsilon^o(p)$ is the excitation energy of the system if only one quasiparticle is present, and $\delta n(p)$ represents the deviation of $n(p)$ from the ground state distribution $n^o(p)$ of the quasiparticles.

In the presence of other quasiparticles we have to take into account their interaction energy $f_{pp'}$ which can be equivalently redefined by

$$f_{pp'} = \frac{\delta^2 E}{\delta n_p \delta n_{p'}} \quad . \qquad (1,2)$$

Since the quasiparticle interaction $f_{pp'}$ is defined only in the region of the Fermi surface and is assumed to be a smooth function of its arguments, we consider $f_{pp'}$ for momenta on the Fermi surface (i.e. $|\vec{p}| = |\vec{p}'| = k_F$).
Hence $f_{pp'}$ only depends on both the angle between $\vec{p}$ and $\vec{p}'$ and on the spin quantum numbers of the quasiparticles.

In the absence of an external magnetic field $f_{pp'}$ can be decomposed into two independent components, corresponding to parallel and antiparallel spins. We write these components in the following form

$$f^{\uparrow\uparrow}_{\vec{p}\vec{p}'} = f^{s}_{\vec{p}\vec{p}'} + f^{a}_{\vec{p}\vec{p}'}$$
$$f^{\uparrow\downarrow}_{\vec{p}\vec{p}'} = f^{s}_{\vec{p}\vec{p}'} - f^{a}_{\vec{p}\vec{p}'} \qquad (1,3)$$

where $f^{s}_{\vec{p}\vec{p}'}$ and $f^{a}_{\vec{p}\vec{p}'}$ are the spin symmetric and spin antisymmetric parts of the quasiparticle interaction .

We expand $f^{s(a)}_{\vec{p}\vec{p}'}$ in a series of Legendre polynominals, where $f^{s(a)}_{l}$ are constants depending on the Fermi momentum p_F . One usually defines dimensionless parameters representing the strength of the interaction

$$F^{s(a)}_{l} = \frac{m^{*} p_F}{\pi^2 \hbar^3} f^{s(a)}_{l} \quad . \qquad (1,4)$$

Here m^{*} means the effective mass of the quasiparticles at the Fermi surface and $m^{*} p_F / \pi^2 \hbar^3$ is the density of states per unit volume at the Fermi surface . The quantities F_l are referred to as Landau parameters .

A sum rule [3]

$$A = \sum_{l=0}^{\infty} \left(\frac{F^{s}_{l}}{1 + \frac{F^{s}_{l}}{2l+1}} + \frac{F^{a}_{l}}{1 + \frac{F^{a}_{l}}{2l+1}} \right) \qquad (1,5)$$

for the Landau parameters is obtained from the exclusion principle condition that the forward particle-particle scattering amplitude A (p,p′) for two like particles with parallel spins tends to zero when $\vec{p} \rightarrow \vec{p}'$.

We calculate the Landau parameters in a simple approximation by means of a microscopic theory outlined in a recent paper by Nitsch[4] . Based on his treatment we are able to calculate transport coefficients, the velocity of spin waves, etc. in the same approximation .

In Section II we derive the velocity of spin waves in neutron matter from a Boltzmann equation, the sufficient condition for their existence being satisfied [5]. In Section III the spin diffusion coefficient D and a corresponding relaxation time τ_D of the quasiparticles are calculated. A summary and a brief discussion, in particular dealing with the microscopic method of calculating the Landau parameters, are given in Section IV.

II. SPIN WAVES IN NEUTRON MATTER

At low temperature collisions between quasiparticles are so infrequent that the system can no longer support low frequency distortion (e.g. first sound) . A higher frequency collective mode (spin density fluctuations or zero sound) can exist, however . For this collective mode the average field of the other particles is responsible for the restoring force . The perturbation of the system is described by the Boltzmann equation

$$\frac{\partial n_p}{\partial t} + \{ n, \varepsilon \} = I \tag{II,1}$$

$$n_p(\vec{r},t) = n_p^0 + \delta n_p(\vec{r},t) \tag{II,2}$$

$$\varepsilon_p(\vec{r},t) = \varepsilon_p^0 + \sum_{p'} f_{pp'} \, \delta n_{p'}(\vec{r},t) \quad . \tag{II,3}$$

I is the collision integral , ε_p (r,t) means the true energy of the quasiparticle and the curly bracket is the Poisson-bracket .

Combining equations (II,1) to (II,3) and considering a periodic perturbation

$$\delta n_p(\vec{r},t) = \delta n_p(\vec{q},\omega) \; e^{i(\vec{q}\vec{r}-\omega t)} \tag{II,4}$$

where $\hbar q \ll p_F$ and $\hbar\omega \ll \mu$

we get an equivalent equation for δn_p , noting that $\nabla_{\vec{p}} \varepsilon_p = \vec{v}_p$ and $\nabla_{\vec{p}} n_p^0 = -\vec{v}_p \, \delta(\varepsilon_p - \mu)$:

$$(\vec{q}\vec{v}_p - \omega)\,\delta n_p(\vec{q},\omega) - \vec{q}\vec{v}_p \frac{\partial n_p^0}{\partial \varepsilon_p} \sum_{p'} f_{pp'} \, \delta n_{p'} = 0 \quad . \tag{II,5}$$

In equation (II,5) we have dropped I since we consider the collisionless regime , i.e., we assume the collision frequency $\nu \gg \omega$.

The Fermi surface is no longer isotropic ,

$$\delta n_p = \frac{\partial n^0}{\partial \varepsilon_p} \; u_\sigma(\theta, \varphi) \tag{II,6}$$

where $u_\sigma(\theta,\varphi)$ is the normal distortion of the Fermi surface for the spin orientation σ, or more conveniently

$$u^{\pm}(\theta,\varphi) = u^{s}(\theta,\varphi) \pm u^{a}(\theta,\varphi) . \qquad (II,7)$$

From equations (II,5) to (II,7) we obtain

$$(\cos\theta - \lambda)\, u^{s,a}(\theta,\varphi) + \frac{\cos\theta}{4\pi} \int F^{s,a}(\xi)\, u^{s,a}(\theta',\varphi')\, d\Omega' = 0 . \qquad (II,8)$$

Here θ denotes the angle between the vectors $\vec{q}$ and $\vec{v}_p$, ξ the angle between (θ,φ) and (θ',φ'), $\lambda = \omega/(q v_F)$ and $d\Omega' = \sin\theta' d\theta' d\varphi'$.

For $|\vec{p}| \to p_F$ and $|\vec{p}\,'| \to p_F$, the interaction function $F^{s,a}_{\vec{p}\vec{p}'}$ is expanded in terms of Legendre polynominals

$$F^{s,a}(\theta) = \sum_{l=0}^{\infty} F_l^{s,a} P_l(\cos\theta) . \qquad (II,9)$$

Equations (II,8), (II,9) and the addition theorems for spherical harmonics yield

$$\sum_{l'} \left[\delta_{ll'} + F_l^{s,a} \frac{(l-m)!}{(l+m)!} \int P_l^m(\theta') \frac{\cos\theta'}{\cos\theta' - \lambda} P_{l'}^m(\theta') \frac{d\Omega'}{4\pi} \right] \Phi_{l'm}^{s,a} = 0 \qquad (II,10)$$

where

$$\Phi_{lm}^{s,a} = \frac{F_l^{s,a}}{4\pi} \frac{(l-m)!}{(l+m)!} \int P_l^m(\theta')\, e^{im\varphi'}\, u^{s,a}(\theta',\varphi')\, d\Omega' . \qquad (II,11)$$

Equation (II,10) has a nontrivial solution when

$$\left| \delta_{ll'} + F_l^{s,a} \Omega_{ll'}^m \right| = 0 \qquad (II,12)$$

with

$$\Omega_{ll'}^m = \frac{(l-m)!}{(l+m)!} \int P_l^m(\theta') \frac{\cos\theta'}{\cos\theta' - \lambda} P_{l'}^m(\theta') \frac{d\Omega'}{4\pi} \qquad (II,13)$$

and $|m| \le l, l'$.

Equation (II, 10) represents a system of homogeneous equations for the quantities $\Phi^{s,a}_{lm}$. This system separates into independent subsystems corresponding to different values of m. But the modes of different l are not decoupled .

Spin waves correspond to the longitudinal (m = 0) antisymmetric (F^a) mode .

We obtain explicitly :

$$\begin{vmatrix} 1+F_0^a\Omega^0_{00}, & F_0^a\Omega^0_{10}, & F_0^a\Omega^0_{20}, & F_0^a\Omega^0_{30} \\ F_1^a\Omega^0_{01}, & 1+F_1^a\Omega^0_{11}, & F_1^a\Omega^0_{21}, & F_1^a\Omega^0_{31} \\ F_2^a\Omega^0_{02}, & F_2^a\Omega^0_{12}, & 1+F_2^a\Omega^0_{22}, & F_2^a\Omega^0_{32} \\ F_3^a\Omega^0_{03}, & F_3^a\Omega^0_{13}, & F_3^a\Omega^0_{23}, & 1+F_3^a\Omega^0_{33} \end{vmatrix} = 0 \, . \qquad \text{(II, 14)}$$

This equation is a nonlinear equation in λ , the solution λ_s provides the velocity of the spin waves $V_\sigma = \lambda_s V_F$. We have included only the first four Landau parameters F_l^a . These coefficients are all positive for the densities considered ($0 \leq F_l^a \leq 0.6$) . We find real values for λ_s which do not differ remarkably from unity, i.e. $V_\sigma \approx V_F$, due to the "weak" exchange interaction between the quasiparticles .

Numerical results for λ_s and v_F are given in Table 1 .

III SPIN DIFFUSION COEFFICIENT

We consider a system of neutrons in which there exists a magnetization gradient without an external field being present . This gradient is maintained by not specified sources of "up" and "down" spins and gives a steady-state diffuse flow .

Investigating spin diffusion we have to take those terms at the left hand side of (II,1) which contain a magnetization gradient . We get (cf. Sykes and Brooker [6]))

$$- \frac{\partial n^0}{\partial \varepsilon^0} \frac{\partial \varepsilon^0}{\partial \vec{p}} \nabla \mu\uparrow = I(p,\uparrow) \tag{III,1}$$

with n^0 meaning the equilibrium distribution function and ε^0 the equilibrium energy of a quasiparticle . $\mu\uparrow$ denotes the Fermi energy of the quasiparticles with spin $\uparrow$. Following Hone [7]) we calculate the spin diffusion coefficient in a volume with vanishing net magnetization . Thereby the calculation simplifies because of the symmetries

$$\nabla \mu\uparrow = - \nabla \mu\downarrow \ ; \quad \nabla n\uparrow = - \nabla n\downarrow \ ; \quad \frac{\partial n\uparrow}{\partial t} = - \frac{\partial n\downarrow}{\partial t}$$

where $n\uparrow$, $n\downarrow$ are the quasiparticle numbers per unit volume with spin up and spin down, respectively .

The connection between $\nabla\mu\uparrow$ and $\nabla n\uparrow$ is given by

$$\nabla \mu\uparrow = 2 \frac{m^* \chi_F}{m \, \chi} \nu_f \nabla n\uparrow \ , \quad \nu_f = \frac{\Omega \, m^* k_F}{\hbar^2 \pi^2} . \tag{III,2}$$

χ_F / χ represents the ratio of the spinsusceptibilities of the noninteracting and the interacting system (cf. Pfarr [8]) . Torrey [9]) has shown by a semiclassical argument that the magnetization M satisfies the continuity equation

$$\frac{\partial \vec{M}}{\partial t} = \nabla D \nabla \vec{M} \tag{III,3}$$

and Hart [10]) proved the validity of this relation for quantum fluids . In a series of papers Leggett and Rice [11]) and Leggett [12]) showed that equation (III,3) is incorrect when $\frac{\partial}{\partial x_i} \vec{M}$ is not parallel to $\vec{M}$ (in spin space) . Since we are interested in the magnitude of the spin diffusion coefficient D rather than in the deviations of the transport equation when ∇M is not parallel to $\vec{M}$ we may omit the corrections given by Leggett [12)].

Taking into account the relation

$$M = \mu_n \left(n\uparrow - n\downarrow \right)$$

(μ_n magnetic moment of the neutron) we immediately get from (III,3) a similar relation for the quasiparticle density :

$$\frac{\partial n\uparrow}{\partial t} - \nabla \vec{j}_\uparrow = 0 \qquad (III,4)$$

where

$$\vec{j}_\uparrow = - D \nabla n\uparrow \qquad (III,5)$$

and D is the spin diffusion coefficient .

On the other hand

$$\vec{j}_\uparrow = \frac{1}{2} \int d\tau \left(\frac{\partial \varepsilon^0}{\partial \vec{p}} \right) \left(\frac{\partial n^0}{\partial \varepsilon} \right) \psi(p,\uparrow) \, . \qquad (III,6)$$

Using (III,2), (III,5) and (III,6) we get for D (cf. ref. 6)

$$D = \frac{1}{3} \left(1 + F_0^a \right) \tau_D V_F^2 \, c(\lambda_D) \qquad (III,7)$$

where

$$\tau_D T^{2} = 8 \frac{\hbar^3 k_F^2}{\pi^3 m^* k_B^2} V_D^{-1}$$

means the relaxation time appropriate to our problem and

$$V_D = \frac{1}{\pi} \int_0^\pi \int_0^\pi d\theta \, d\varphi \, \sin^3 \frac{\theta}{2} \left(1 - \cos\varphi \right) \left| A^s - A^a \right|^2 .$$

The numerical values for DT^2, $\tau_D T^2$ and V_D may be taken from Table 2 . The numerical value of $c(\lambda_D)$ turns out to be about 0,8 for all densities .

IV. SUMMARY AND DISCUSSION

We have found that the spin density fluctuations propagate essentially at the velocity of the quasiparticles . Because of the "weak" exchange interaction only a few of them are included in the collective antisymmetric mode .

Furthermore we note the dependence of the spin diffusion coefficient on the nuclear forces which cause a change of D in the considered region of density of two orders of magnitude .

The interaction energy $f_{pp'}$ of the quasiparticles was calculated by functional differentiation of the Hartree-Fock energy of the system . The expansion coefficients of the quasiparticle interaction $F_l^{s,a}$ are connected with those of the forward scattering amplitudes $A_l^{s,a}$ [3] (cf. eq. (I,5)) . The Pauli principle provides us with a sum rule for the expansion coefficients of the forward scattering amplitude [3,13] .
In our simple approximation for the quasiparticle-interaction the Pauli principle sum rule is violated which might be due to the low order estimation for the energy . In order to get more accurate results many-body ($n \geq 3$) contributions to the energy expectation value have to be included . Moreover we expect that incorporation of a second-order contribution of normal perturbation theory involving non-diagonal matrix elements will improve our results . A more elaborate discussion related with the above problems is provided in References 14 - 17) and in the literature cited there .

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Table 1

k_F [fm^{-1}]	$\lambda = \frac{v_\sigma}{v_F}$	$v_F \times 10^9$ [cm/sec]
0.7	1.021	4.6
1.0	1.065	7.0
1.4	1.057	11.3
2.0	1.040	21.2
2.5	1.016	34.6

Table 2

k_F [fm^{-1}]	v_D	$\tau_D T_8^2$ [10^{-16} sec]	$c^{-1}(\lambda_D) D T_8^2$ [$10^2 cm^2 sec^{-1}$]
0.7	5.65	0.09	0.92
1.0	7.98	0.13	3.29
1.4	1.18	2.02	121.73
1.5	0.56	5.12	378.44
1.6	0.41	8.16	738.05
2.0	2.69	2.37	473.23

Table captions :

Table 2 : T_8 is the temperature in units of 10^8 K .