NOTAS DE FÍSICA
VOLUME XVIII
Nº 25

WEAK AND ELECTROMAGNETIC FORCES AS A CONSEQUENCE OF THE SELF-INTERACTION OF THE γ - FIELDS*

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RIO DE JANEIRO, BRASIL
1972

Notas de Física - Volume XVIII - Nº 25

WEAK AND ELECTROMAGNETIC FORCES AS A CONSEQUENCE OF THE SELF-INTERACTION OF THE γ - FIELDS=

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(Received october 31th, 1972)

ABSTRACT

A model is proposed which geometrizes the electromagnetic and the weak forces. The absence of electromagnetic properties of neutrino is understood in terms of a symmetry principle.

1. INTRODUCTION

The idea that considers all process in nature linked to geometry is as old as Einstein's theory, at least(1). However, the Eötvos experiment that gave a good reason to try the geometrical way to describe gravitational (G) interaction rests without similar in the rest of physics. Nevertheless, the fact that all known elementary particles have the same coupling constant with the electromagnetic (EM) field could also suggest that EM properties can be described by means of a modification of the structure of space time. There is an important distinction, among others, between EM and G forces: the universality of gravitational process and the lack of electromagnetic properties in some particles, the neutrino for instance.

So, if one considers the geometric approach, one should be faced with the first crucial problem: why neutrino does not have an electric charge? The second point is: what could be the modification on the riemannian nature of space time that electromagnetic or even weak forces could make? It is our purpose in this paper to show that these two questions can be answered. We will prove it, in a naif model, in the following.

2. THE FUNDAMENTAL OBJECTS OF THE SPACE TIME -

As in (2) we will assume that the fundamental objects of space time are the generalized γ's that are linked tc the metric tensor by the anticommutation relation

$$\{\gamma_\mu(x), \gamma_\nu(x)\} = 2g_{\mu\nu}(x) \qquad (1)$$

The choice of a set of elements of a Clifford (C) algebra as fundamental has been considered in the literature many times. The real motivation for doing this rests on the fact that going from the $g_{\alpha\beta}(x)$ metric to the $\gamma_\alpha(x)$ sub-metric the additional degrees of freedom could be used to introduce new features in the theory. It seems to us, however, that this program was not fully realized and that is the reason to come back to it here.

Expression (1) shows that besides the manifold mapping group, the γ's admit a more general transformation

$$\gamma_\mu(x) \rightarrow \gamma'_\mu(x) = M(x)\, \gamma_\mu(x)\, M^{-1}(x) \tag{2}$$

Expression (2) is generally assumed to represent a Lorentz rotation of a tetrad frame that depends on the space time coordinates. This space time dependent transformation generates, in the usual manner, a modification of the derivative operation into a covariant derivative. We write

$$D_\mu\, \gamma_\nu(x) = \partial_\mu\, \gamma_\nu(x) - \Gamma^{\varepsilon}_{\mu\nu}(x)\, \gamma_\varepsilon(x) + [\tau_\mu(x), \gamma_\nu(x)] \tag{3}$$

Where $\Gamma^{\varepsilon}_{\mu\nu}(x)$ are the Christoffel symbols and $\tau_\mu(x)$ is an internal connection. It is usually assumed that the covariant derivative of the $\gamma_\mu(x)$ is null, as this imply that the covariant derivative of the metric tensor is null. However, it is a simple matter to show that this condition is only sufficient but not necessary. Indeed, if we put

$$D_\mu\, \gamma_\nu(x) = [\, V_\mu(x), \; \gamma_\nu(x)\,] \tag{4}$$

for an arbitrary element $V_\mu(x)$ of the C- algebra, the riemannian nature of the space time will be preserved. Throughout this paper we will assume that equation (4) holds.

The most general form of $V_\mu(x)$ may be written as

$$(5)\quad V_\mu(x) = q_1 A_\mu(x)\, 1 + q_2 F^\lambda(x)\, \gamma_\mu(x)\, \gamma_\lambda(x)\, \gamma_5(x) + q_3\, \phi(x)\gamma_\mu(x)\gamma_5(x) + q_4 B_\mu(x)\, \gamma_5(x) + q_5\, \xi(x)\, \gamma_\mu(x)$$

Where $A_\mu(x)$, $F_\mu(x)$, $B_\mu(x)$ are vector fields; $\phi(x)$ and $\xi(x)$ are scalar fields; and $q_1,\ldots, q_5$ are a set of arbitrary constants. This set of q's is introduced only as a device to permit the fields to be adimensional. So, the dimension of the q 's is $(\text{length})^{-1}$.

3. ELECTROMAGNETIC AND WEAK FORCES -

The step that takes from Dirac's equation in a minkowskian (flat) space to a general (riemannian) one consists in the usual way of replacing the derivative ∂_μ by a covariant derivative $D_\mu \equiv \partial_\mu + \tau_\mu$ by means of the minimal coupling principle. Let us do this generalization by writing the lagrangian of a spinor $\chi(x)$ under the form

$$(6)\quad L = i\, \bar{\chi}(x)\, \gamma^\mu(x) D_\mu \chi(x) - i(D_\mu \bar{\chi}(x))\, \gamma^\mu(x)\, \chi(x) - 2m\, \bar{\chi}(x)\, \chi(x)$$

Then, by means of a variational principle and taking (4)- (5) into account we arrive at the modified form of the equation of χ

$$(7)\quad i\, \gamma^\mu(x)\, D_\mu \chi(x) + i q_1\, A_\mu(x)\, \gamma^\mu(x)\, \chi(x) + 3i\, q_2\, F^\mu(x)\, \gamma_\mu(x)\, \gamma_5(x)\chi(x) + 4i\, q_5\, \xi(x)\, \chi(x) - m\, \chi(x) = 0$$

In (7) use has been made of the relation $D_\mu \chi(x) = \Delta_\mu \chi(x) + V_\mu(x)\chi(x)$ where Δ_μ is the usual covariant derivative defined by means of the Fock--Ivanenko coefficients:

$$\Delta_\mu \chi(x) = \partial_\mu \chi(x) + \tau_\mu^{(FI)}(x)\, \chi(x)$$

with

(8) $\tau_\mu^{(FI)} = 1/8 \{ \gamma^\lambda(x)\, \partial_\mu \gamma_\lambda(x) - \partial_\mu \gamma_\lambda(x) \gamma^\mu(x) + \Gamma^\varepsilon_{\lambda\mu}(x) .$

$. (\gamma_\varepsilon(x)\, \gamma^\lambda(x) - \gamma^\lambda(x)\, \gamma_\varepsilon(x)) \}$

We remark here that the vector field $B_\mu(x)$ and the scalar field $\phi(x)$ have no influence on the evolution of the χ-field.

Let us define a new combination of the dynamical vector fields of the form

(9) $e\, \phi_\mu(x) = q_1\, A_\mu(x) - 3q_2\, F_\mu(x)$

$g_F\, W_\mu(x) = 3q_2\, F_\mu(x)$

Then, equation (7) can be identified with the equation of a spinor χ in a gravitational field plus an interaction term given by the lagrangian

$$L_{int} = i\, e\, \phi^\mu(x)\, \bar{\chi}(x)\, \gamma_\mu(x)\chi(x) + i g_F\, W^\mu(x)\, \bar{\chi}(x)\, \gamma_\mu(x)\, (1+\gamma_5(x))\, \chi(x)$$
$$+\ 4\, q_5\, \xi(x)\, \bar{\chi}(x)\, \chi(x) \qquad (10)$$

The first term may be interpreted as an electromagnetic-type interaction; the second term as a weak interaction with a vectorial boson;

and the third term may be interpreted as a correction of mass.

So, we could consider EM and W forces as a consequence of the self interaction of the fundamental γ- field. However, if this is true, and all these processes are linked with geometry then, why neutrino, for instance, is blind to the EM modification of the space time? To answer this question we have to study the symmetries of equation (7). Let us put $q_5 = 0$, as we are interested only in the EM and W effects. We will require, as a natural generalization of the flat case, that equation (7) with null mass be invariant under the mapping

(11) $$\begin{aligned} \chi(x) &\to \chi'(x) = \gamma_5(x)\ \chi(x) \\ \Delta_\mu &\to \Delta'_\mu = \gamma_5(x)\ \Delta_\mu \end{aligned}$$

Then, the condition of invariance imposes

(12) $$q_1\ A_\mu(x) = 3q_2\ F_\mu(x)$$

In terms of the electromagnetic and weak forces, $\phi^\mu(x)$ and $W_\mu(x)$, this symmetry implies that there is not an EM-type interaction for the massless particle described by $\chi(x)$. This is not the case for a massive particle because the mass term breaks the symmetry of the theory. So, the absence of EM-forces for a massless particle is a consequence of the invariance of the theory under the mapping (11).

4. CONCLUSION

We have succeeded, in a very simple model, to show that electromag netic and weak - type forces appear as a consequence of the self interac tion of the γ - field as given by equations (4) and (5). This result is independent of the particular representation chosen for describing the electrons and neutrinos. Indeed, the same result is obtained by means of the Weyl's two - component representation. Furthermore, the symmetries of the equation of the spinor field gives a natural explanation of the absence of the electric charge of the neutrino. These results suggest to us the question: is it possible to assign values on the fundamental set $\{q_1, \ldots q_5\}$ of the coupling constants? We intend to consider this problem elsewhere.

ACKNOWLEDGEMENTS

I would like to thank Dr. C.G. de Oliveira for his coments on this paper.

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