UNCOMMUNICATING VACUA GENERATED BY SPONTANEOUS BREAKING OF SYMMETRY IN A CURVED SPACE-TIME.

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Abstract: We show that the existence of a cosmological constant induces a spontaneous breaking of symmetry of a scalar field coupled non-minimally to gravity. We give an example of a case in which the ground state is degenerated generating three disconnected vacua.

One of the main recent interest on models which admits spontaneous breaking of gauge symmetries is intimately related to the old problem of the origin of the mass. After Mach's investigations on this problem, in the modern era, the connection of the global properties of the Universe with the localized quantity of matter has been questioned in many different ways.

The modern theory of gravitation[1], was at its origins, believed to be a consequence of the proposal of A. Einstein to include Mach's ideas in a coherent context which should relate the dynamics od space-time with the energy-matter content on it.

The popular Higgs[2,3] mechanism of generating mass to a gauge field through a spontaneaous breaking of symmetry of a scalar field seems to be very far from Mach's project. Even in some recent tentatives [4,5,6,7] to include gravitation into the Goldstone-Higgs model have not completely succeeded in creating a model which should relate the creation of mass mechanism to a global context of the Universe.

The proposal of this letter is to suggest such model and thus restoring the importance, in modern theories, of Mach's idea on the relation between local and global properties, e.g., the cosmical dependence of the mass.

Let me start with a theory which consists of a scalar field ϕ coupled non-minimally to gravity, the dynamics of which is governed by the Lagragian

$$L = \sqrt{-g}\,\{\partial_\mu\phi^*\partial_\nu\phi g^{\mu\nu} - m^2\phi^2 + U(\phi) - \alpha R\phi^2 + \frac{1}{k} R + 2\Lambda\} \qquad (1)$$

We restrict ouselves in the present paper to the analysis of the case in which the constant $\alpha = \frac{1}{6}$ and the self-interacting term $U(\phi)=\sigma\phi^4$. The reason for the choice 1/6 for α is related to the conformal invariance of the equation of ϕ in the absence of mass, although as is easily recognizable, the full theory of the coupled system $(\phi, g_{\mu\nu})$, even for massless ϕ-field, is not conformally invariant.

The factor 2Λ gives the contribution of the whole Universe in the form of a cosmological constant. We call the attention of the reader to the fact that even if we do not introduce explicitly at the beginning such Λ term, it will appears as a consequence of spontaneous breaking of symmetry of the ϕ-field in the presence of gravitation [8].

Let me remark that the factor $U(\phi)=\sigma\phi^4$, which in the flat space-time, is essential for the existence of a spontaneous breaking of symmetry, has not in the general curved space-time a fundamental importance, but it only acts as a renormalization of the vacuum value of the ϕ-field. The symmetry is broken by the cosmological constant.

This can be easily recognized by a simple inspection on the equation of the system $(\phi, g_{\mu\nu})$ which we get from varying Langrangian(1). However, we will not consider the case $U(\phi)=0$ here. We postpone such discussion for another place [9].

The equations of motion for the system $(\phi, g_{\mu\nu})$

are given by

$$\Box\phi + (m^2 + \frac{1}{6} R)\phi - \frac{1}{2} \frac{\delta U}{\delta\phi} = 0 \tag{2a}$$

$$(\frac{1}{k} - \frac{1}{6}\phi^2)(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) = -\phi_{|\mu}\phi_{|\nu} + \frac{1}{2} g_{\mu\nu}(\phi_{|\lambda}\phi_{|\varepsilon}g^{\lambda\varepsilon} -$$

$$- m^2\phi^2 + U) - \frac{1}{6}\Box\phi^2 g_{\mu\nu} + \frac{1}{6}(\phi^2)_{|\mu||\nu} + \Lambda g_{\mu\nu} \tag{2b}$$

From now on we set Einstein's constant $k = 1$, which causes no limitation at all in the generality of our discussion.

Taking the trace of equation (2b) we obtain for the scalar of curvature R the relation

$$R = m^2\phi^2 - 4\Lambda \tag{3}$$

Using this result we obtain that, besides the trivial solution $\phi=0$, there are two constant solutions of the equation of evolution of ϕ, which are given by:

$$\phi_0^2 = \frac{6m^2 - 4\Lambda}{12\sigma - m^2} \tag{4}$$

We note, once more, that even in the absence of the quartic self-interacting term $\sigma\phi^4$ the existence of a cosmological constant induces the mechanism of the spontaneous breaking of symmetry. In this case, we should have a positive value for Λ limited by the inequality $2\Lambda > 3m^2$.

We remark that the existence of a non-null cosmological constant makes unnecessary the usual assumption of starting the theory with tachyons (imaginary mass for the ϕ-field) and the jump from the tachionic character(outside the light cone classical trajectories) to the positive real mass field (inside the light cone) by the spontaneous breaking of symmetry mechanism.

The reason for this is easy to understand if we realize that the presence of a positive cosmological constant, which satisfies the condition $2\Lambda>3m^2>0$, has the effect of renormalizing the mass of the ϕ-field, to an effective imaginary mass given by the value $(M_{eff})^2= m^2-\frac{2}{3}\Lambda$, which under our hypothesis, is negative. This can be seen using equation (3) into (2a).

Thus, we have so far obtained three values for constant solutions of equation (2a): $\phi=0$ and $\phi=\pm\left(\frac{6m^2-4\Lambda}{12\sigma-m^2}\right)^{1/2}$. What about the stability of these solutions?

In order to answer this question we have to analyse the behavior of the energy of the ϕ-field in the presence of non-minimal coupling with gravity. Following Callan et al |10| we define the energy for the constant ϕ-field by the expression

$$E(\phi) = \frac{3m^2\phi^2-3\sigma\phi^4-6\Lambda}{6-\phi^2} \tag{5}$$

There are many cases worth of investigation depending on the relative values of the constants envolved in the expression of the energy (5).

We will analyse a particular case, defined by a special set of values of the constants which seems to me to exemplify very clearly the new features which appear due to the presence of the Λ-factor.

The case we wish to examine imposes the constraint $\sigma = \frac{m^4}{8\Lambda} > 0$ and $2\Lambda > 3m^2 > \Lambda > \frac{3}{2} m^2 > 0$. The fact that the self coupling constant σ depends on the cosmological constant, which is a straight consequence of Mach's ideas, is set in order to the constant solution ϕ_0 to be a extremum of the energy. Its positive value ($\sigma > 0$) may appear at first sight , a wrong assumption once we could be creating conditions for generating unbounded negative energy states. However, this is not the case. The simplest way to see this is to look to the figure (1) which represents the configuration of the energy $E(\phi)$, given by (5), against ϕ.

We remark that the three constant solutions minimized the energy and thus can be associated to stable vacua.

Further, once there is an infinite barrier at $\phi_0^2 = 6$, these three vacua are disconnected and defines three uncoupled sectors of the ϕ-field which should be interpreted as a kind of uncommunicated world. [see figure 1]

Thus, we have shown that the existence of a cosmical constant Λ has a similar effect of a quartic self-coupling term $\sigma\phi^4$ and thus induces spontaneous breaking of symmetry irrespectly of the properties of the gravitational field [11]. Once such breaking symmetry is achieved we can set the Higgs mechanism to operate and by introducing a massless gauge field, minimally coupled both to gravity and to the scalar field, we generate mass for this gauge field , thus making such mass to depend on the cosmological constant. This seems to me a triumph of Mach's ideas.

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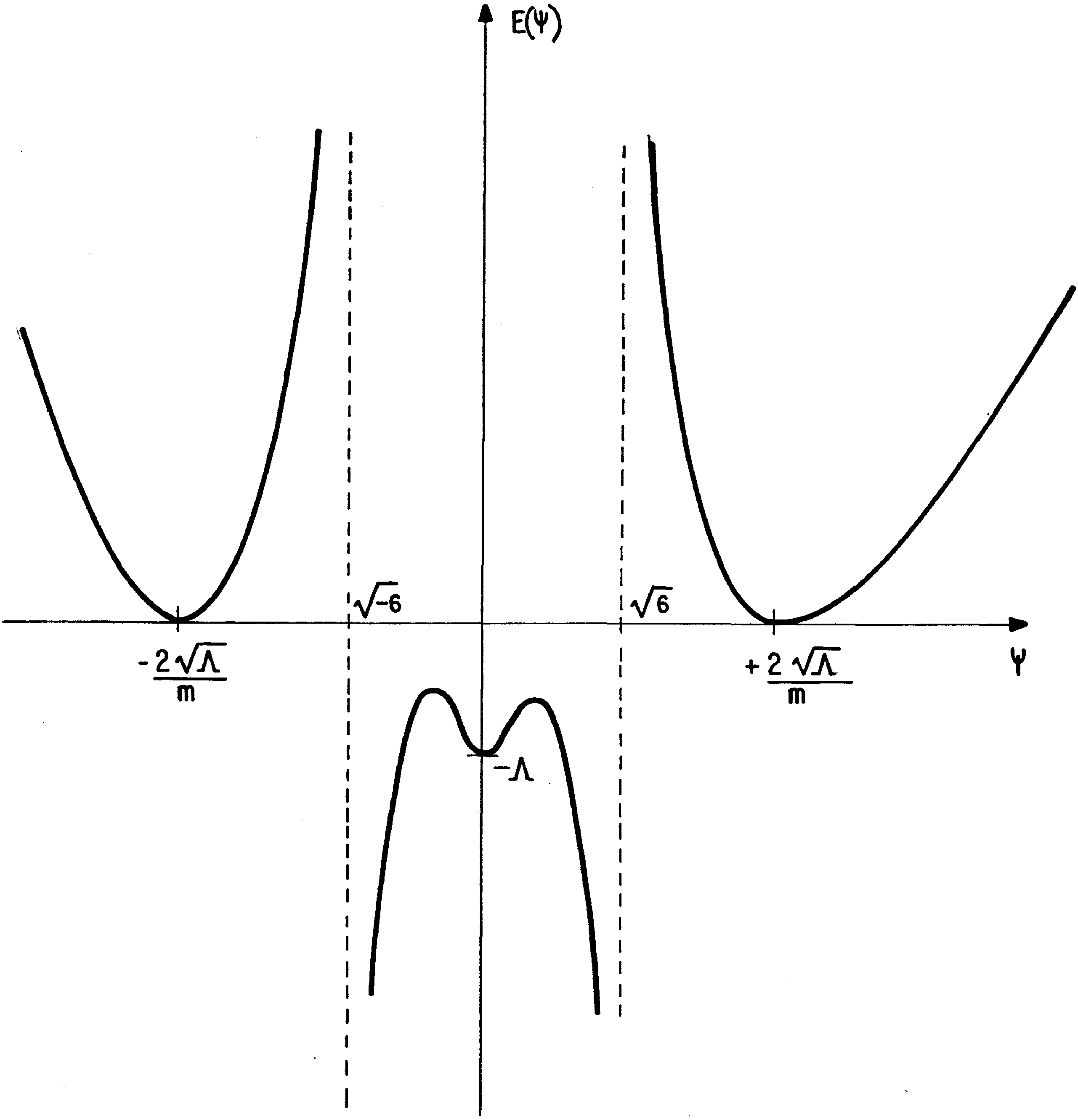

Fig.1 - Configuration of the dependence of the energy of the ϕ-field in a curved space time with positive cosmological constant Λ. The figure is drawn for constant ϕ and for the case $2\Lambda > 3m^2 > \Lambda > \frac{3}{2}\, m^2$.

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[11] See, for some different arguments, reference |4|for instance.