CBPF
CENTRO BRASILEIRO
DE PESQUISAS FÍSICAS

A0024/76

AGO,1976

DIFFERENTIAL ROTATION OF VISCOUS NEUTRON MATTER

Jürgen Nitsch and Joachim Pfarr

Institut für Theoretische Physik

Universität zu Köln, Köln, Zülpicher Str. 11, Germany

H. Heintzmann

Centro Brasileiro de Pesquisas Físicas

Av. Venceslau Brás, 71 - 20000 Rio de Janeiro

RIO DE JANEIRO

BRASIL

DIFFERENTIAL ROTATION OF VISCOUS NEUTRON MATTER

Jürgen Nitsch and Joachim Pfarr

Institut für Theoretische Physik
Universität zu Köln, Köln, Zülpicher Str. 77, Germany

H. Heintzmann

Centro Brasileiro de Pesquisas Físicas
Av. Venceslau Braz, 71 - 20000 Rio de Janeiro

Abstract :

We have investigated the reaction of a homogeneous sphere of neutron matter set in rotational motion under the influence of an external torque acting on its surface . For neutron matter with a typical neutron star density of 10^{15} g cm^{-3} and a temperature varying between 10^{6} and 10^{9} K originally in uniform rotation a time dependent differential motion sets in which lasts a time scale of hours to some decades resulting finally in corotation . During these times the braking index of a magnetic neutron sphere very sensitively depends on time .

Introduction

It is generally accepted that pulsars are rotating neutron stars . As a rule the interior of such a neutron star contains a liquid phase which mainly consists of neutrons and also contains protons, electrons and negative muons . In more massive neutron stars with a central density of $\varrho_c > 10^{15}\ \mathrm{gcm}^{-3}$ also hyperons become important constituents (Ruderman [1] , Baym and Pethick [2]) . However, we are interested in that part of a neutron star which contains normal (i.e. nonsuperfluid) neutron matter . Between densities of about $5 \cdot 10^{11}\ \mathrm{gcm}^{-3}$ and $2 \cdot 10^{14}\ \mathrm{gcm}^{-3}$ the neutrons are superfluid because of the ^1S_o - attraction (Krotscheck [3] , Chao et al. [4]) . The neutrons are supposed to be superfluid again at densities which exceed the nuclear densities because of the $^3P_2$ - neutron pairing (Tamagaki [5] , Tamagaki and Takatsuka [6]). It should be pointed out, however, that all these calculations concerning the ^1S_o - pairing as well as the anisotropic $^3P_2$ - superfluidity should be regarded as first estimates ; since the energy gaps very sensitively depend on the nuclear potential used .. Detailed calculations performed by Takatsuka [6] confirm that the onset of anisotropic superfluidity is a delicate function of the neutron effective mass . Moreover it has been investigated by Weyer [7,8] that in the regime of high densities in neutron matter ($\varrho \gtrsim 5 \cdot 10^{14}\ \mathrm{gcm}^{-3}$) a certain pairing correlation of neutrons in relative singlet states may occur . This pairing is due to the asymmetry of nuclear forces between even and odd states which suggests the preference of singlet states for neutrons with equal momentum quantum numbers . These correlations build up so called dineutron clusters, analogously to the α - particle picture . The decision whether the dineutron correlation or the anisotropic superfluidity prevails in neutron matter still remains unsettled. Therefore we assume [9] - even in the presence of the $^3P_2$ - superfluidity - at least a normal component between $\sim 2 \cdot 10^{14}$ and $4 \cdot 10^{14}\ \mathrm{gcm}^{-3}$.

Our following model calculation which refers to a normal neutron fluid may be applied to the normal component in the interior of a neutron star .

The question we would like to answer is how the surface angular velocity $\Omega_S(t)$ and the braking index n defined by

$$n(t) := \Omega_S(t) \cdot \ddot{\Omega}_S(t) \cdot \dot{\Omega}_S^{-2}(t) \qquad (1)$$

are modified by the existence of viscous normal matter in the interior of a neutron star . This analysis is useful, since the experimentally found values of n are about 2.5 and special and general relativistic effects have been proven to be too small to give a correction of the desired order (Pfarr [11]) .

In the first section we give a short epitome as to the calculation of the first viscosity which we need in the second section for an approximate solution of the Navier-Stokes equation .

I Viscosity of Normal Neutron Matter

In this section we briefly review the essential steps for the evaluation of the first viscosity in neutron matter in the framework of the Landau theory (Nitsch [12] , Heintzmann and Nitsch [9] and the literature cited there) . In doing so we derive in a first step a simple representation of Landau's interaction function of the quasiparticles which we need as the most important ingredient in the Boltzmann-transport equation to approximate the collision integral . There this function is related to the forward scattering amplitude of two quasiparticles [13] . The second step only gives a rough sketch of the general assumptions for the evaluation of the first viscosity .

The total energy E of an interacting system is a functional of the distribution function $n_\sigma(p)$ of the quasiparticles . If the function $n_\sigma(p)$ is sufficiently close to the ground state distribution function $n^0_\sigma(p)$ we carry out an expansion of E [n] (Pines and Nozières [13])

$$E[n]-E_0 = \sum_{\sigma;p} \epsilon_\sigma(p)\delta n_\sigma(p) + (2!)^{-1}\sum_{\sigma,\sigma';p,p'} f_{\sigma\sigma'}(p,p')\delta n_\sigma(p)\delta n_{\sigma'}(p') \quad (2)$$

where the quasiparticle energy $\epsilon_\sigma(p)$ is the first and the interaction function between the quasiparticles $f_{\sigma\sigma'}(p,p')$ is the second variational derivative of the total energy E [n] , i.e.

$$\epsilon_\sigma(p) := \delta E[n] / \delta n_\sigma(p) \tag{3}$$

and

$$f_{\sigma\sigma'}(p,p') := \delta^2 E[n] / \delta n_\sigma(p) \delta n_{\sigma'}(p') \quad . \tag{4}$$

The deviation $\delta n_\sigma(p)$ from $n^0_\sigma(p)$ is defined by

$$\delta n_\sigma(p) := n_\sigma(p) - n^0_\sigma(p) \quad . \tag{5}$$

A simple approximation of the quantities $\epsilon_\sigma(p)$ and $f_{\sigma\sigma'}(p,p')$ we get by means of the Hartree-Fock theory :

$$E_{HF}[n] = \sum_{\sigma;p} p^2 (2m)^{-1} n_\sigma(p) + (2!)^{-1} \sum_{\sigma,\sigma';p,p'} \langle p\sigma p'\sigma' | V | p\sigma p'\sigma' \rangle_a n_\sigma(p) n_{\sigma'}(p') \tag{6}$$

with

$$\epsilon_\sigma(p) = p^2 (2m)^{-1} + \sum_{\sigma';p'} \langle p\sigma p'\sigma' | V | p\sigma p'\sigma' \rangle_a n_{\sigma'}(p') \tag{7}$$

and

$$f_{\sigma\sigma'}(p,p') = \langle p\sigma p'\sigma' | V | p\sigma p'\sigma' \rangle_a \quad . \tag{8}$$

We describe the interacting forces between the neutrons using the unitarily transformed (Mittelstaedt et al. [14]) Gammel-Christian-Thaler potential (Gammel et al. [15]) .

In the considerations above we dealt with stable, homogeneous distributions for which the function $n_\sigma(p)$ neither depended on time nor on the relative position of the quasi-particles . In a more general case, however, we consider a weak time dependent inhomogeneous perturbation of the ground state of our system . As a consequence the distribution function of the quasiparticles (in the classical limit) explicitly becomes time and position dependent : $n_\sigma = n_\sigma(p,r,t)$.

We determine $n_\sigma(p,r,t)$ by solving the Boltzmann equation

$$\partial n/\partial t + \{n,\tilde{\epsilon}\} = I(n) \tag{9}$$

where $I(n)$ is the collision integral of the quasiparticles which we approximate in the case of binary collisions , specified by

$$P_1 + P_2 \longrightarrow P_1' + P_2' \tag{10}$$

in terms of Born collision cross sections using the interaction function of Equation (8) .. The local excitation energy of a quasiparticle is equal to

$$\tilde{\epsilon}_\sigma(p,r) = \epsilon_\sigma(p) + \sum_{\sigma';p'} f_{\sigma\sigma'}(p,p')\,\delta n_{\sigma'}(p',r) \quad . \tag{11}$$

Once the collision integral is known we can study the transport properties of the system such as the viscosity, thermal conductivity or the spin diffusion . If we impose an inhomogeneous static perturbation containing a velocity gradient to the system, this gradient induces a flow of momentum which is only limited by the collisions between quasiparticles , and which is – in the comoving coordinate system – proportional to the imposed velocity gradient . Thus the first viscosity η is defined as the proportionality coefficient between the momentum flux density tensor Π_{ik} and the expression

$$\{ \partial v_i/\partial x_k + \partial v_k/\partial x_i - 2\cdot(3)^{-1}\partial v_l/\partial x_l\,\delta_{ik} \} \quad . \tag{12}$$

Since the calculation of η is somewhat lengthy and cumbersome we here only quote the final result [16] :

$$\eta = \omega(\rho)\cdot T^{-2} \quad . \tag{13}$$

The density dependent function $\omega(\varrho)$ contains mainly all those quantities which arise from the interaction of the neutrons such as forward scattering amplitudes and the effective mass m^* of the quasiparticles. The explicit expression for $\omega(\varrho)$ as well as a table for some η - values are given in the work by Heintzmann and Nitsch [9]. However, the result (13) can easily be eludicated by some qualitative arguments : since the neutron-neutron scattering is restricted to lie within a layer of width $(k_B T)$ around the Fermi surface (i.e. all elementary excitations of interest are to be found in this layer) the transition probability for the process (10) in the "thermal limit" [13] is of order $(k_B T)^2$. This leads to a quasiparticle lifetime τ_p proportional to T^{-2}. Moreover τ_p represents a qualitative measure for the collision time of quasiparticles which is proportional to the first viscosity η according to the elementary kinetic theory of gases. We shall see in the next section that the temperature dependence of η very sensitively effects the duration of differential rotation in neutron matter.

II. Differentially Rotating Neutron Fluid

First we want to describe our model : We consider a uniformly rotating sphere of neutron fluid with a solid outer layer. At a certain time t_o we apply a torque from outside to the surface of the sphere by switching on a homogeneous magnetic field in its interior. This torque causes a braking at the surface and thereby produces - because of the viscosity η - a velocity gradient within the star's matter (cf. Fig. 1). We determine this velocity field solving the "special" Navier-Stokes equation

$$dv/dt = \partial v/\partial t + (v\nabla)v = -\varrho^{-1}\nabla p - \nabla\Phi + \varrho^{-1}\eta\nabla^2 v \qquad (14)$$

where we have already assumed that the density ϱ as well as the viscosity η depend neither on the position nor on time. For $v = 0$ (14) reduces to the well-known condition for the hydrostatic equilibrium

$$\nabla p = -\varrho\nabla\Phi \qquad (15)$$

where p is the pressure of the neutron matter and Φ is the Newtonian gravitational potential. From the form of (14) we get a rough estimate of the dissipation time of the viscous forces :

$$\tau_{vis} := \rho L^2 \eta^{-1} \tag{16}$$

where L is the characteristic length scale of the velocity field. This time τ_{vis} is independent of the initial condition of the differential equation (14).

For $L \sim R$, where R is the radius of the neutron star and for a density $\rho \approx 10^{15}$ gcm^{-3} and a temperature $T = 10^8$ K we get for $\tau_{vis} \approx 10^8$ sec. $\approx$ 3 years. Afterwards the matter is rigidly corotating again.

We simplify (14) using the following ansatz (cf. Heintzmann et al. [17]) for the velocity field v (r,t)

$$v(r,t) = \Omega(r,t)\, r \sin\vartheta \; e_\varphi \; . \tag{17}$$

In spherical coordinates we obtain from (14) the following partial differential equation for $\Omega(r,t)$:

$$\partial\Omega(r,t)/\partial t = \rho^{-1}\eta \left\{ \partial^2\Omega(r,t)/\partial r^2 + 4 r^{-1}\, \partial\Omega(r,t)/\partial r \right\} . \tag{18}$$

Here we have already assumed $\nabla p \approx -\rho \nabla \Phi$. According to the results of Heintzmann et al. [17], [18] this assumption is justified.

For the complete solution of (18) we need initial and boundary conditions, which we define as follows :

<u>Initial condition :</u>

$$\Omega(r, 0+) =: \Omega_0$$

boundary condition :

α) The solution has to be regular at the origin $r = 0$.

β) $\Omega(R,t) = \Omega_S(t)$ is a function of time with $\Omega_S(0+) = \Omega_0$ which will be specified by the torque equilibrium condition below (Eq. (24)) .

R means the radial coordinate at the star's surface . The solution of the Laplace-transformed differential equation (18) reads

$$\Omega_L(r,s) = s^{-1}\Omega_0 + \left(\omega_0(s) - s^{-1}\Omega_0\right)\left(\frac{R}{r}\right)^3 \frac{\sinh\left(r(s/\nu)^{1/2}\right) - r(s/\nu)^{1/2}\cosh\left(r(s/\nu)^{1/2}\right)}{\sinh\left(R(s/\nu)^{1/2}\right) - R(s/\nu)^{1/2}\cosh\left(R(s/\nu)^{1/2}\right)} \tag{19}$$

with

$$\omega_0(s) := \mathcal{L}\{\Omega_S(t)\} \quad ,$$

and the quantity $\nu = \varrho^{-1}\eta$ is the kinematic viscosity . The torque at the surface of the sphere induced by the velocity field of the viscous fluid (19) can easily be given by (cf. Landau and Lifschitz [19])

$$D_L(s) = \int_0^{\pi} \eta R \left(\partial\Omega_L(r,s)/\partial r\right)_{|r=R} R\sin\vartheta\, 2\pi R^2 \sin^2\vartheta\, d\vartheta = 3^{-1} 8\pi R^4 \eta \left(\partial\Omega_L/\partial r\right)_{|r=R} . \tag{20}$$

Using (19) we explicitly get for $D_L(s)$

$$D_L(s) = 3^{-1} 8\pi R^3 \eta \left(\omega_0(s) - s^{-1}\Omega_0\right)\left\{\frac{-R^2(s/\nu)}{1 - R(s/\nu)^{1/2}\coth\left(R(s/\nu)^{1/2}\right)} - 3\right\} . \tag{21}$$

As already mentioned in the introduction we are interested in the influence of the viscous forces on the braking index $n(t)$. Using the balance condition for the torques we evaluate $\Omega_S(t)$ in the following way : The function $D_L(s)$ in (21) can not be transformed analytically into the original space of the Laplace-transform . Therefore we give the representation of D (t) in the two limiting cases :

$$\text{a)} \quad \delta_t \ll R \qquad \text{and} \qquad \text{b)} \quad \delta_t \gg R \ , \tag{22}$$

here δ_t means the depth of penetration of the velocity field at the time t (cf. Landau and Lifschitz [19]) .

For $\delta_t \ll R$ we get from (21)

$$D(t) = 3^{-1} \cdot 8\pi R^4 (\pi^{-1}\eta\varrho)^{1/2} \int_0^t d\tau \left(d\Omega_S(\tau)/d\tau\right) \cdot (t-\tau)^{-1/2} \tag{23}$$

which, likewise , represents the solution of the analogous "plane" problem [19] .
In Table 1 we give some values of times when the relation a) is valid . If we assume now that at the time t_o the braking at the surface of our fluid sphere is caused by the torque of a magnetic dipole field , i.e.

$$D_{elm}(\Omega_S) = -(2/3c^3)\,\Omega_S^3\,\mu^2 \sin^2\chi = D_{vis}(\Omega_S) \ , \tag{24}$$

we get the following integro-differential equation for $\Omega_S(t)$:

$$-(2/3c^3)\,\Omega_S^3\,\mu^2\sin^2\chi = 3^{-1} 8\pi R^4 (\pi^{-1}\eta\varrho)^{1/2} \int_0^t d\tau (d\Omega_S/d\tau)\cdot(t-\tau)^{-1/2} \ . \tag{25}$$

Here χ means the angle between the axis of rotation and the magnetic dipole axis, $\mu = B_o R^3$ is the magnetic dipole moment and c the velocity of light . Two-fold iterative integration of (25) with the starting ansatz $\Omega_S^{(1)}(t) = \Omega_o + \text{const.}\cdot(t)^{1/2}$ leads to the approximate solution for small values of t :

$$\Omega_S^{(2)}(t) = \Omega_o - (\Omega_o^3 / c_\eta \pi)\cdot t^{1/2}\left(1 - 2^{-1}\cdot 3\,\Omega_o^2 c_\eta^{-1} t^{1/2}\right) \tag{26}$$

with

$$c_\eta := 3^{-1} 8\pi R^4 (\pi^{-1}\eta\varrho)^{1/2} (3c^3/2\mu_0^2) \quad .$$

Hence follows the braking index n in the limit $t \to 0$

$$n(t) \doteq c_\eta \pi \Omega_0^{-2} t^{-1/2} - 1 \quad . \tag{27}$$

Obviously the braking index n diverges to $+\infty$ as t goes to 0. And putting $\Omega_0 \approx \Omega_{crit} = (R^{-3}GM)^{1/2} \approx 0.8 \cdot 10^4 \, sec^{-1}$ (Heintzmann et al. [18]) we see that n decreases from $+\infty$ to values of about 100 to 10 within the allowed time scales (cf. Table 1).

In the case b) the final state of a rotating viscous sphere of neutron star matter has already been studied by Heintzmann et al. [17]. We, however, are interested in a dynamical process which corresponds to this final state.

For $\delta_t \gg R$ the expansion of (21) and the following transformation into the original space leads to the result :

$$D(t) = (8/15)\pi R^5 \varrho \left\{ \dot{\Omega}_S(t) + (35)^{-1} R^2 \eta^{-1} \varrho \ddot{\Omega}_S(t) \right\} \quad . \tag{28}$$

In the limit $\eta \longrightarrow \infty$ we get the torque of a rigidly rotating sphere with the moment of inertia

$$I := (8/15)\pi R^5 \varrho \quad . \tag{29}$$

Using the torque equilibrium condition $D_{vis}(\Omega_S) = D_{elm}(\Omega_S)$ we now obtain an ordinary differential equation of the second order for $\Omega_S(t)$:

$$I \left\{ \dot{\Omega}_S(t) + a_\eta \ddot{\Omega}_S(t) \right\} = -\alpha \, \Omega_S^3(t) \tag{30}$$

where

$$a_\eta := (35)^{-1} R^2 \eta^{-1} \varrho \qquad \text{and} \qquad \alpha := (3c^3)^{-1} \cdot 2 \mu^2 \sin^2 \chi \quad . \tag{31}$$

Those times for which (30) is a valid approximation can be taken from Table 1 . It is useful to write (30) in terms of dimensionless parameters and variables . For that purpose we define the following quantities

$$x := t/t_A \; ; \quad \Omega_A := \Omega_S(t_A) \; ; \quad y := \Omega_S/\Omega_A \; ; \tag{32}$$

$$a := a_\eta^{-1} t_A \quad ; \qquad b := (a_\eta I)^{-1} \alpha \, (t_A \Omega_A)^2 \; .$$

The constant time t_A defines the onset of the validity for the limiting case b) . (30) now reads

$$d^2y/dx^2 + a \, dy/dx + b \, y^3 = 0 \quad . \tag{33}$$

The coefficients a and b are of the same order of magnitude, but both are very large compared to 1 (see Table 2) . For that reason we interpret the contribution due to the second derivative in (33) as a perturbation of the differential equation

$$a \, dy/dx + b \, y^3 = 0 \quad . \tag{33'}$$

Its solution corresponds to the slow down law of the rigidly rotating magnetic dipole and leads to the braking index $n = 3$.
The approximate solution of (33) is ($y_0' = -a^{-1} b \, y_0^3$)

$$y = y_0 + a^{-1} y_0^3 \left(1 + a^{-1} b \ln y_0^3 \right) , \tag{34}$$

whence we derive the braking index

$$n(t) = 3 \left(1 + (35)^{-1} 2 \alpha I^{-1} R^2 \eta^{-1} \varrho \, \Omega_S^2(t)\right) \quad . \tag{35}$$

Since we can not give the complete solution $\Omega_S(t)$ for all values of t , we are not able to give the exact initial value Ω_A . If we assume the period of the crab pulsar to have the value $\Omega_S(t)$ the additive term in (35) is of the order of 10^{-2} to 10^{-4} . It only gives a positive contribution of the order of percents to the braking index of the magnetic dipole .

Results and Discussion

As we have seen in the first section the first viscosity of neutron matter depends on the nuclear forces and on the density of the matter . It is proportional to T^{-2} . Especially the temperature influences the duration of viscous forces in the case of differential rotation of the neutron matter . Within a more detailed investigation in the second section we come to know that viscous forces can not be regarded as acting during astronomical time scales ($\tau_{vis}^{max} \approx$ 100 years) . From table 2 we see that the coupling of the differential rotation to the rigid rotation takes time scales of years to decades . However, in our above treatment we did not consider turbulences as a possible mechanism to destroy the velocity field [17] .
In section II we answered the question of how far the presence of differentially rotating viscous matter influences the slow down law of a pulsar . The resulting fact that the braking index n (t) is always larger than 3 is a consequence of our model calculation : While the surface of the star is already braked by the dipole radiation, the interior maintains its original angular velocity and therefore hurries on in advance of the surface (Fig. 1 b) . As a consequence angular momentum is transported from the inside to the surface due to collisions of the particles within the viscous fluid . This transported angular momentum leads to a braking index larger than 3 .

The interaction of the magnetic field in the interior of the star with the neutrons may be neglected because of the large Fermi energy of the neutrons in comparison to (μB) (cf. Pfarr [20]) . A more detailed discussion would have to include the existence of electrons, protons and negative myons as constituents of a neutron star together with their interactions with the magnetic field .

ACKNOWLEDGMENTS:

H.H. is grateful to Conselho Nacional de Pesquisas for financial support and specially to Mario Novello for his kind hospitality.

References

1. Ruderman, M., Annual Rev. of Astron. and Astrophys. 10 , 427 (1972)
2. Baym, G., Pethick, Ch., preprint , to be published in Annual Reviews of Nuclear Science (1975)
3. Krotscheck, E., Z. Physik 251 , 135 (1972)
4. Chao, N.C., Clark, J.W., Yang, E.-H. , Nucl. Phys. A 179 , 320 (1972)
5. Tamagaki, R., Prog. Theor. Phys. 44 , 905 (1970)
6. Takatsuka, T., Tamagaki, R., Prog. Theor. Phys. 46 , 114 (1971)
   Takatsuka , T., Prog. Theor. Phys. Lett. 47 , 1062 , (1972)
   Takatsuka, T., Prog. Theor. Phys. 48 , 1517 (1972)
7. Weyer , W., Dissertation Köln (1972)
8. Weyer, W., preprint, submitted for publication in Nucl. Phys. A (1975)
9. Heintzmann, H., Nitsch, J. , Astron. and Astrophys. 21 , 291 (1972)
10. Boynton, P.E., Groth, E.J., Hutchinson, D.P., Nanos, G.P., Partridge, R.B., Wilkinson, D.T., Astrophys. J. 175 , 217 (1972)
    Groth, E.J., Timing of the grab Pulsar II + III , Princeton Reprints , submitted to Astrophys. J. (1975)
11. Pfarr, J. , submitted for publication in Gener. Relat. and Grav. (1975)
12. Nitsch, J., Z. Physik 251 , 141 (1972)
13. Pines, D., Nozières, P., The Theory of Quantum Liquids, New York, Benj. Inc. (1966)
14. Mittelstaedt, P., Ristig, M., Z. Physik 193 , 349 (1966)
15. Gammel, J.L., Christian, R.S., Thaler, R.M., Phys.Rev. 105 , 311 (1959)
16. Abrikosov, A.A., Khalatnikiv, I.M., Rept. Progr. Phys. 22 , 329 (1958)
    Brooker, G.A., Sykes, J., Ann. Phys. (N.Y.) , 61 387 (1970)
    Feenberg, E., Theory of Quantum Fluid, Academic , New York (1969)
17. Heintzmann, H., Kundt, W., Schrüfer, E., Astron. and Astrophys. 27 , 45 (1973)
18. Heintzmann, H., Hillebrandt, W., Krotscheck, E., Kundt, W., Ann. of Phys. 81 , 625 (1973)
19. Landau, L.D., Lifschitz, E.M., Course of Theoretical Physics Vol. 6 , Pergamon, Oxford (1963)
20. Pfarr, J., Z. Phys. 251 , 152 (1972)

Table 1

$T[K]$	$\eta[10^{17}$ poise$]$	t_c	$n(t_c)$
10^6	$1.6 \cdot 10^6$	1/2 hour	$3 \cdot 10^6$
10^7	$1.6 \cdot 10^4$	2 days	$3 \cdot 10^4$
10^8	$1.6 \cdot 10^2$	1/2 year	$3 \cdot 10^2$
10^9	1.6	50 years	3

Table 2

$T[K]$	$t_A[sec]$	$\Omega_A[sec^{-1}]$	a	b/a
10^7	10^8	10^4	$6 \cdot 10^3$	1.5
	$3 \cdot 10^{10}$	$2 \cdot 10^2$	$2 \cdot 10^6$	$1.8 \cdot 10^{-1}$
10^8	10^8	10^4	$6 \cdot 10$	1.5
	$3 \cdot 10^{10}$	$2 \cdot 10^2$	$2 \cdot 10^4$	$1.8 \cdot 10^{-1}$

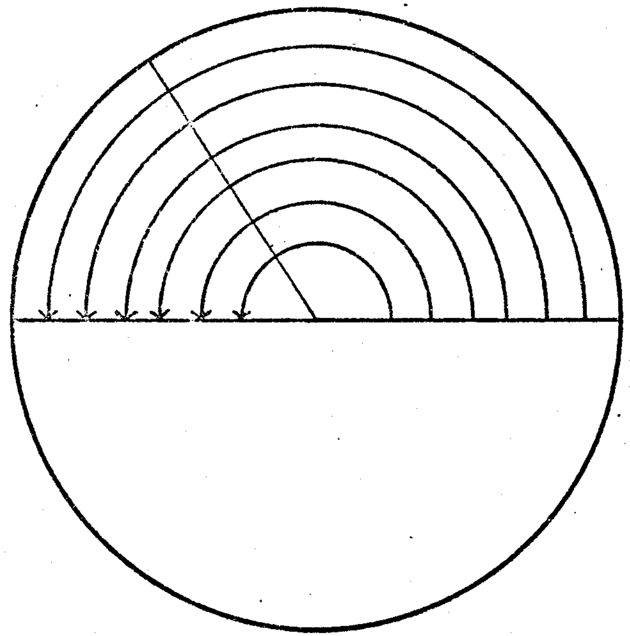

Fig. 1a

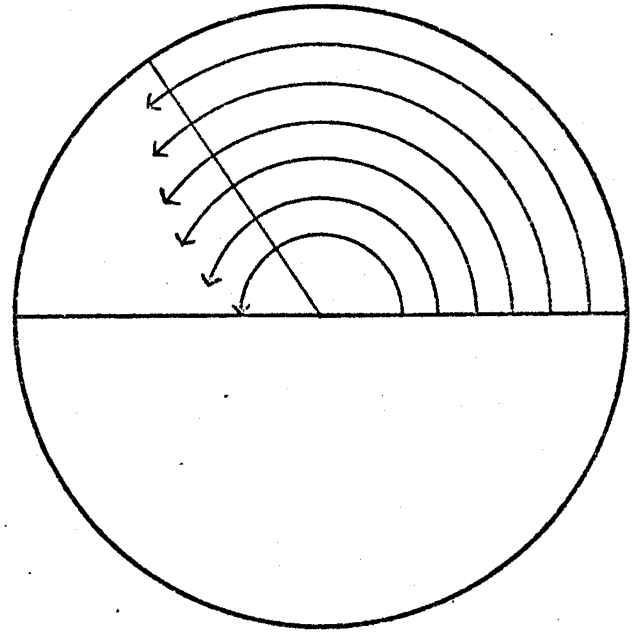

Fig.1b

Table captions :

Table 1

All values in Table 1 are given for constant density $\varrho = 10^{15}\ \mathrm{g\,cm^{-3}}$. The time parameter t_c is calculated by equating the radius R and the depth of penetration $\delta_t := 2\,(\eta\,\varrho^{-1} t)^{1/2}$:
$t_c = 4^{-1}\,\eta^{-1}\,\varrho\,R$. The two cases a) and b) in the text refer to $t \ll t_c$ and $t \gg t_c$, respectively .

Table 2

The coefficients a and b of the differential equation (33) are given for some special temperatures and initial data t_A and Ω_A .

<u>Fig. 1 :</u> Velocity fields ; before (a) and after (b) switching on the magnetic dipole field .