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Shear-free radiating collapse and conformal flatness

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Abstract

Here we study some general properties of spherical shear-free collapse. Its general solution when imposing conformal flatness is reobtained [1, 2] and matched to the outgoing Vaidya spacetime. We propose a simple model satisfying these conditions and study its physical consequences. Special attention deserve, the role played by relaxational processes and the conspicuous link betweeen dissipation and density inhomogeneity.

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1 Introduction

Gravitational collapse of stars is an important problem of astrophysics and building realistic models of collapse remains a formidable task. One interesting problem is to add heat flow to spherically symmetric models.

Indeed, dissipation due to the emission of massless particles (photons and/or neutrinos) is a characteristic process in the evolution of massive stars. In fact, it seems that the only plausible mechanism to carry away the bulk of the binding energy of the collapsing star, leading to a neutron star or black hole is neutrino emission [3].

In the diffusion approximation, it is assumed that the energy flux of radiation (as that of thermal conduction) is proportional to the gradient of temperature. This assumption is in general very sensible, since the mean free path of particles responsible for the propagation of energy in stellar interiors is in general very small as compared with the typical length of the object. Thus, for a main sequence star as the sun, the mean free path of photons at the centre, is of the order of $2\,cm$. Also, the mean free path of trapped neutrinos in compact cores of densities about $10^{12}\,g.cm.^{-3}$ becomes smaller than the size of the stellar core [4, 5].

Furthermore, the observational data collected from supernovae 1987A indicates that the regime of radiation transport prevailing during the emission process, is closer to the diffusion approximation than to the streaming out limit [6].

Many solutions of Einstein's field equations with dissipative fluids carrying heat flow have been studied (see [7] for references up to 1989 and [8, 10, 9] for more recent ones).

In this vein here we study dissipative spherical collapse with shear-free motion. Spherical conformally flat fluids undergoing dissipation in the form of radial heat flow where first considered in [1] and generalized in [2]. Here we reobtain the general conformally flat solution in a slightly different way. We match this spacetime to a radiating null field described by the outgoing Vaidya spacetime. A simple model is considered satisfying these conditions.

The paper is organized as follows. In section 2 the field equations are presented; in section 3 we reobtain the general solution by considering conformal flatness of spacetime; in section 4 we state the junction conditions to the external outgoing Vaidya null radiating field; section 5 presents a simple collapsing dissipative model and we finish with a brief conclusion.

2 Field equations

We assume a sphere of collapsing perfect fluid with heat flow. Its spherical surface Σ has center 0 and is filled with radially moving perfect fluid conducting heat flow, so having energy momentum tensor

$$T_{\alpha\beta} = (\mu + p)\, w_\alpha w_\beta + p g_{\alpha\beta} + q_\alpha w_\beta + w_\alpha q_\beta, \tag{1}$$

where μ and p are the proper density and pressure of the fluid, w_α its unit four-velocity, q_α the heat conduction satisfying $q_\alpha w^\alpha = 0$ and $g_{\alpha\beta}$ is the metric tensor of spacetime.

We choose comoving coordinates within Σ and impose shear-free fluid motion which allows the metric be written in the form [11]

$$ds^2 = -A^2 dt^2 + B^2 \left[dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right], \tag{2}$$

where A and B are only functions of r and t. We number the coordinates $x^0 = t$, $x^1 = r$, $x^2 = \theta$ and $x^3 = \phi$ and then we have the four-velocity given by

$$w_\alpha = -A\delta^0_\alpha, \tag{3}$$

and the heat flows radially,

$$q^\alpha = q\delta^\alpha_1, \tag{4}$$

where q is a function of r and t.

The rate of collapse $\Theta = w^\alpha{}_{;\alpha}$ of the fluid sphere is given, from (2) and (3), by

$$\Theta = 3\frac{\dot{B}}{AB}, \tag{5}$$

where the dot stands for differentiation with respect to t.

The spacetime described by (2) has the following non-null components of the Weyl tensor $C_{\alpha\beta\gamma\delta}$,

$$C_{2323} = \frac{r^3 B^2}{3} \sin^2\theta \left[\left(\frac{A'}{A} - \frac{B'}{B} \right) \left(1 + 2r\frac{B'}{B} \right) - r \left(\frac{A''}{A} - \frac{B''}{B} \right) \right], \tag{6}$$

and

$$\begin{aligned} C_{2323} &= -\frac{r^4 B^2}{A^2} \sin^2\theta C_{0101} = \frac{2r^2 B^2}{A^2} \sin^2\theta C_{0202} \\ &= 2r^2 A^2 B^2 C_{0303} = -2r^2 \sin^2\theta C_{1212} = -2r^2 C_{1313}, \end{aligned} \tag{7}$$

where the primes stand for differentiation with respect to r.

The non null components of Einstein's field equations $G_{\alpha\beta} = \kappa T_{\alpha\beta}$, where $G_{\alpha\beta}$ is the Einstein tensor and $T_{\alpha\beta}$ is given by (1), with metric (2) are

$$G_{00} = -\frac{A^2}{B^2}\left[2\frac{B''}{B} - \left(\frac{B'}{B}\right)^2 + \frac{4}{r}\frac{B'}{B}\right] + 3\left(\frac{\dot{B}}{B}\right)^2 = \kappa\mu A^2, \tag{8}$$

$$G_{11} = \left(\frac{B'}{B}\right)^2 + \frac{2}{r}\frac{B'}{B} + 2\frac{A'}{A}\frac{B'}{B} + \frac{2}{r}\frac{A'}{A}$$
$$+\frac{B^2}{A^2}\left[-2\frac{\ddot{B}}{B} - \left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right] = \kappa p B^2, \tag{9}$$

$$G_{22} = \frac{G_{33}}{\sin^2\theta} = r^2\left[\frac{A''}{A} + \frac{1}{r}\frac{A'}{A} + \frac{B''}{B} - \left(\frac{B'}{B}\right)^2 + \frac{1}{r}\frac{B'}{B}\right]$$
$$+r^2\frac{B^2}{A^2}\left[-2\frac{\ddot{B}}{B} - \left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right] = \kappa p r^2 B^2, \tag{10}$$

$$G_{01} = -2\left(\frac{\dot{B}}{AB}\right)' A = -\kappa q A B^2. \tag{11}$$

¿From (11) with (5) we obtain

$$\kappa q B^2 = \frac{2}{3}\Theta', \tag{12}$$

which shows that the outflow of heat, $q > 0$, imposes $\Theta' > 0$, meaning that, if $\Theta < 0$, dissipation diminishes the rate of collapse towards the outer layers of matter. If $q = 0$ then from (12) $\Theta' = 0$ which means that collapse is homogeneous [10].

The mass function $m(r,t)$ of Cahill and McVittie [12] is obtained from the Riemann tensor component ${R_{23}}^{23}$ and it is for metric (2)

$$m(r,t) = \frac{(rB)^3}{2}{R_{23}}^{23} = \frac{r^3 B}{2}\left[\left(\frac{\dot{B}}{A}\right)^2 - \left(\frac{B'}{B}\right)^2\right] - r^2 B'. \tag{13}$$

Differentiating $m(r,t)$ with respect to r and t and considering the field equations (8-11) we obtain

$$m' = \frac{\kappa}{2}\left[\mu(rB)^2(rB)' + q r^3 B^4 \frac{\dot{B}}{A}\right], \tag{14}$$

$$\dot{m} = -\frac{\kappa}{2}\left[p r^3 B^2 \dot{B} + q(rB)^2(rB)' A\right]. \tag{15}$$

¿From (14) and (15) we have that the heat flow diminishes the gradient and the time derivative of $m(r,t)$. This agrees with the discussion concerning (12), since dissipation diminishes the total amount of matter, it is expected that the rate of collapse slows down. Furthermore, this agrees too with the results obtained for the dynamical instability of nonadiabatical spherical collapse [18, 7] where it is proved that relativistically dissipation diminishes instability due to the decrease of matter content inside a collapsing sphere.

The Riemann curvature tensor can be split into the Weyl tensor and parts which involve only the Ricci tensor and the curvature scalar. This allows to say that the Weyl part is constructed only by the gravitational field. Considering the scalar of the Weyl tensor

$$\mathcal{C}^2 = C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}, \tag{16}$$

with (6), (8) and (13) we obtain after a long calculation

$$\mathcal{C}^2 = 48\left[\frac{m}{(rB)^3} - \frac{\kappa\mu}{6}\right]^2 = 48\frac{m_{\mathcal{C}}^2}{(rB)^3}, \tag{17}$$

where $m_{\mathcal{C}}$ is defined as the pure gravitational mass,

$$m_{\mathcal{C}} = m - \frac{\kappa}{6}\mu(rB)^3. \tag{18}$$

3 Conformally flat solution

Here we impose conformal flatness to the spacetime given by (2), i.e. all its Weyl tensor components must be zero valued. From (6) and (7) we see that if $C_{2323} = 0$ this condition is fulfilled, hence we have

$$r\left(\frac{A''}{A} - \frac{B''}{B}\right) - \left(\frac{A'}{A} - \frac{B'}{B}\right)\left(1 + 2r\frac{B'}{B}\right) = 0. \tag{19}$$

We can integrate (19) and after reparametrizing t we obtain

$$A = \left[C_1(t)\, r^2 + 1\right] B, \tag{20}$$

where C_1 is an arbitrary function of t. ¿From the isotropy of pressure, (9) and (10), equating $r^{-2}G_{22} - G_{11}$ to zero and using (20) we find

$$\frac{B''}{B'} - 2\frac{B'}{B} - \frac{1}{r} = 0, \tag{21}$$

which is easily integrated,

$$B = \frac{1}{C_2(t)r^2 + C_3(t)}, \tag{22}$$

where C_2 and C_3 are arbitrary functions of t. The solution found in [1] is a particular case of (20) and (22) with $C_1 = 0$. All conformally flat perfect fluid solutions with $q = 0$ have been obtained by Stephani [13, 14]

Conformal flatness imposes $\mathcal{C} = 0$ and from (17) we have that the pure gravitational mass $m_{\mathcal{C}} = 0$ and

$$m = \frac{\kappa}{6}\mu\,(rB)^3\,, \tag{23}$$

which is similar to the result obtained in [15] with $q = 0$. ¿From (14) with (23) we have

$$\mu' = qB^2\Theta, \tag{24}$$

which shows that for $q > 0$ and $\Theta < 0$ then $\mu' < 0$ implying that the density diminishes with increasing r. While from (5) with (12) we obtain

$$\kappa\mu' = \frac{1}{3}(\Theta^2)', \tag{25}$$

which can be integrated, giving

$$\kappa\mu = \frac{\Theta^2}{3} + g(t), \tag{26}$$

where g is a function only of t.

Substituting solution (20) and (22) into (9), (10) and (13) we obtain,

$$\kappa\mu = 3\left(\frac{\dot{C}_2 r^2 + \dot{C}_3}{C_1 r^2 + 1}\right)^2 + 12C_2C_3, \tag{27}$$

$$\begin{aligned}\kappa p = \frac{1}{(C_1r^2+1)^2}&\left[2(\ddot{C}_2 r^2 + \ddot{C}_3)(C_2r^2 + C_3) - 3(\dot{C}_2 r^2 + \dot{C}_3)^2\right.\\ &\left.-2\frac{\dot{C}_1}{C_1r^2+1}(\dot{C}_2r^2+\dot{C}_3)(C_2r^2+C_3)r^2\right]\\ &+\frac{4}{C_1r^2+1}\left[C_2(C_2-2C_1C_3)r^2 + C_3(C_1C_3-2C_2)\right],\end{aligned} \tag{28}$$

$$\kappa q = 4(\dot{C}_3C_1 - \dot{C}_2)\left(\frac{C_2r^2+C_3}{C_1r^2+1}\right)^2 r. \tag{29}$$

The expansion Θ of the fluid sphere given by (5) with (20), (22) and (29), is

$$\Theta = -3\frac{\dot{C}_2r^2+\dot{C}_3}{C_1r^2+1} = -3\left[\dot{C}_3 - \frac{\kappa q}{4}\frac{C_1r^2+1}{(C_2r^2+C_3)^2 r}\right]. \tag{30}$$

We see from (30) that if $q = 0$ the contraction is homogeneous, however if $q \neq 0$, dissipation produces inhomogeneous collapse, which has already been remarked in (12).

The density μ in (27) confirms the result (26) with $g(t) = 12C_2C_3$.

It is possible to prove, after a long calculation, that the fluid (27-29) does not satisfy an equation of state of the form $p = c\mu$, where c is a constant, with $q \neq 0$. A family of solutions with heat flux satisfying an equation of state is given in [16].

In the next section we consider the junction conditions of the collapsing dissipative fluid to a radiating field.

4 Junction conditions

If the collapsing fluid lies within a spherical surface Σ it must be matched to a suitable exterior. Since heat will be leaving the fluid across Σ the exterior is not vacuum, but the outgoing Vaidya spacetime which models the radiation and has metric

$$ds^2 = -\left[1 - \frac{2m(v)}{\rho}\right] dv^2 - 2dvd\rho + \rho^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{31}$$

where $m(v)$ is the total mass inside Σ and is a function of the retarded time v. In (31) ρ is a radial coordinate given in a non-comoving frame. The matching of these two spacetimes (2) and (31), using the field equations (9-11) and the mass function (13) satisfies [17, 7]

$$(rB)_\Sigma = \rho_\Sigma, \tag{32}$$

$$p_\Sigma = (qB)_\Sigma, \tag{33}$$

$$m(v) = \left\{\frac{r^3}{2}\left[\frac{\dot{B}^2 B}{A^2} - \frac{(B')^2}{B}\right] - r^2 B'\right\}_\Sigma . \tag{34}$$

¿From (28), (29) and (33) we have

$$\left\{\ddot{C}_2 r^2 + \ddot{C}_3 - \frac{3}{2}\frac{(\dot{C}_2 r^2 + \dot{C}_3)^2}{C_2 r^2 + C_3} - \frac{\dot{C}_1 r^2(\dot{C}_2 r^2 + \dot{C}_3)}{C_1 r^2 + 1} - 2(\dot{C}_3 C_1 - \dot{C}_2) r \right.$$
$$\left. + 2\frac{(C_1 r^2 + 1)}{C_2 r^2 + C_3}\left[C_2(C_2 - C_1 C_3) r^2 + C_3(C_1 C_3 - 2C_2)\right]\right\}_\Sigma = 0. \tag{35}$$

5 A simple model

A simple approximate solution for the functions $C_1(t)$, $C_2(t)$ and $C_3(t)$ satisfying the junction condition (35) is

$$C_1 = \epsilon c_1(t), \quad C_2 = 0, \quad C_3 = \frac{a}{t^2}, \tag{36}$$

where $0 < \epsilon \ll 1$ and $a > 0$ a constant. When $c_1 = 0$ then (36) describes a collapsing Friedmann dust sphere, with $k = 0$, whose radius diminishes from arbitrarily large values until, at $t = 0$, a singularity is formed. The time t runs from $-\infty$ to 0 and the constant a is proportional to the total mass inside the radius r. Substituting (36) into (35) we obtain up to $O(\epsilon)$,

$$\dot{c}_1 + \left(\frac{t}{r_\Sigma^2} + \frac{2}{r_\Sigma}\right) c_1 \approx 0, \tag{37}$$

which after integration yields,

$$c_1 \approx c_1(0) \exp\left(-\frac{t^2}{2r_\Sigma^2} - \frac{2t}{r_\Sigma}\right). \tag{38}$$

Substituting the solution (36,38) into (27-29) we obtain

$$\kappa\mu \approx \frac{12a^2}{t^6}\left(1-\epsilon 2c_1 r^2\right), \tag{39}$$

$$\kappa p \approx \epsilon\frac{4a^2c_1}{t^4}\left[1-\left(1+\frac{2r_\Sigma}{t}\right)\frac{r^2}{r_\Sigma^2}\right], \tag{40}$$

$$\kappa q \approx -\epsilon\frac{8a^3c_1 r}{t^7}, \tag{41}$$

which satisfy plausible physical conditions. It should be observed, however, that in the general case $\epsilon \neq 0$, the range of t is restricted by physical considerations. Thus for example if we want the the central pressure not to exceed the value of the central energy density, then we should have,

$$\frac{3}{t^2} > \epsilon c_1. \tag{42}$$

We see from (39) that the energy density diminishes to the outer regions due to dissipation; from (40) we have that pressure diminishes too towards the outer regions while from (41) we have that the heat flow increases in that same direction.

The mass function (13) inside a radius r with (36) and (38) becomes,

$$m(r,t) \approx \frac{2r^2}{a}\left(1-\epsilon 2c_1 r^2\right), \tag{43}$$

showing that dissipation diminishes the mass inside r. Now calculating the rate of collapse (5) with (36) and (38) we obtain

$$\Theta \approx \frac{6a}{t^3}\left(1-\epsilon c_1 r^2\right), \tag{44}$$

implying that dissipation slows down collapse. This result agrees with the fact that $m(r,t)$ is diminished by dissipation.

The effective adiabatic index

$$\Gamma = \frac{d\ln p}{d\ln\mu}, \tag{45}$$

gives a measure of the dynamical instability of the body at given instant of time. Calculating (45) for $r = 0$ and $r = r_\Sigma$ with (36) and (38) up to the order $O(\epsilon)$ in p we obtain,

$$\Gamma_{r=0} \approx \frac{2}{3}+\frac{t^2}{6r_\Sigma^2}+\frac{t}{3r_\Sigma}, \tag{46}$$

$$\Gamma_{r=r_\Sigma} \approx \frac{5}{6}+\frac{t^2}{6r_\Sigma^2}+\frac{t}{3r_\Sigma}. \tag{47}$$

We see from (46) and (47) that $\Gamma_{r=0} < \Gamma_{r=r_\Sigma}$ which shows that the centre is more unstable than the surface region of the collapsing body. This conclusion too agrees with our previous analysis.

5.1 Calculation of the temperature

Finally it is worth calculating the temperature distribution, $T(r,t)$, for our model, through the Maxwell-Cattaneo heat transport equation [19, 20, 21, 22, 23],

$$\tau h^{\alpha\beta} w^{\gamma} q_{\beta;\gamma} + q^{\alpha} = -K h^{\alpha\beta}(T_{,\beta} + T a_{\beta}), \tag{48}$$

where τ is the relaxation time, K the thermal conductivity and $h^{\alpha\beta} = g^{\alpha\beta} + w^{\alpha} w^{\beta}$ the projector orthogonal to w^{α}. Considering (2-4) then (48) becomes

$$\tau (qB)\dot{} B + qAB^2 = -K(TA)'. \tag{49}$$

Substituting (20), (22) and (29) into (49) and considering $C_2 = 0$ we obtain, up to order ϵ

$$\tau (C_1 C_3 \dot{C}_3)\dot{} r + C_1 \dot{C}_3 r = -\frac{\kappa K}{4}\left[T(C_1 r^2 + 1)\right]', \tag{50}$$

Now, in the non–dissipative case ($C_1 = \epsilon c_1 = 0$) it follows at once from (50) that $T = T_0(t)$, implying that in that case the temperature is homogeneous within the fluid distribution. Therefore, in the general dissipative case $C_1 \neq 0$, we shall have

$$T = T_0(t) + \epsilon T_{\epsilon}(r,t), \tag{51}$$

Then introducing (51) into (50) we obtain up to $O(\epsilon)$

$$T \approx T_c + \epsilon c_1 \left(\frac{4a}{\kappa K t^3} - T_0\right) r^2 - \epsilon \frac{4a^2 \tau c_1}{\kappa K t^5}\left(\frac{t}{r_{\Sigma}^2} + \frac{2}{r_{\Sigma}} + \frac{5}{t}\right) r^2. \tag{52}$$

where we have assumed for simplicity K =constant and $T_c(t)$ denotes the central temperature. The second term on the right hand side of expression (52) exhibits the influence of dissipation on the decreasing of temperature (remember that $t < 0$) with respect to the non–dissipative case, as calculated from the non–causal (Landau–Eckart) [24, 25] transport equation, whereas the last term describes the contribution of relaxational effects. The relevance of such effects have been brought out in recent works (see [16, 26] and references therein). In particular it is worth noticing the increasing of the spatial inhomogeneity of temperature produced by the relaxational term, an effect which has been established before [27].

6 Conclusion

We have presented the general field equations for a spherical dissipative shear-free collapse. Some general properties concerning the effects of dissipation on the collapsing body and its mass were discussed. By imposing conformal flatness we showed that the system is completely soluble in its radial part and producing three arbitrary time functions. Then we matched this solution to the outgoing Vaidya radiating spacetime. A simple model with a Friedmann limit is constructed satisfying the junction conditions.

Besides its simplicity, the merit of the model resides in the fact that it exhibits in a very clear way the influence of relaxational effects on the temperature, and thereby on the evolution of the system.

It is also worth noticing the appearance of density inhomogeneities directly related to dissipation, even though the space–time remains conformally flat. This reinforces doubts [28] on the proposal that the Weyl tensor [29] or some functions of it [30], could provide a gravitational arrow of time. The rationale behind this idea being that tidal forces tend to make the gravitating fluid more inhomogeneous as the evolution proceeds, thereby indicating the sense of time.

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