CBPF-NF-023/82

DO SPIN 3/2 LEPTONS EXIST ?

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Abstract

An elementary proof of the unicity of the wave equation for free spin 3/2 fields (and the corresponding Feynman integral equation) is given. Covariant forms are exhibited and a heuristic SU(2) ⊗ U(1) model is adopted for the construction of possible electromagnetic and weak currents involving spin 3/2 particles. The notions of Dirac and Majorana masses for spin 3/2 neutrinos are generalized from the spin 1/2 case.

Among the decays of charged massive spin 3/2 particles, the radiative decay into the spin 1/2 lepton of the same family is possible. A systematic experimental search for spin 3/2 leptons (and quarks) is urged.

ABSTRACT

An elementary proof of the unicity of the wave equation for free spin 3/2 fields (and the corresponding Feynman Integral equation) is given. Covariant forms are exhibited and a heuristic $SU(2) \otimes U(1)$ model is adopted for the construction of possible electromagnetic and weak currents involving spin 3/2 particles. The notions of Dirac and Majorana masses for spin 3/2 neutrinos are generalized from the spin 1/2 case.

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I. INTRODUCTION

The Fermi theory of weak interactions was formulated in order to describe the radioactive decay of nuclei and other weak reaction processes which were subsequently discovered.

Although the effective current-current lagrangean of Fermi's model was later found out to be not renormalizable, this theory was necessary for the calculation of first-order, non-divergent, processes ; it described experimental results and indicated the path for new experiments and discoveries such as, for instance, the prediction by Lee and Yang of parity non-conservation in weak interactions, the Feynman-Gell-Mann-Marshak-Sudarshan V-A current model and the whole of muon physics.

At present, the discovery[1)] of a new lepton, the tauon, which is heavier than the baryons of the SU_3-octet, leads us to ask whether leptons do not possibly have a structure or, at any rate, new kinds of interactions somehow hidden or suppressed at present energies. We may therefore ask whether leptons do not exist in an excited state, with spin 3/2.

We do not possess any experimental results so far which would suggest the occurence of such spin 3/2 leptons ; but these particles have not been systematically searched for either. The theory of electromagnetic and weak processes with spin 3/2 fields[2)] which we dispose of is not renormalizable. We may, however, use appropriate effective current-current interaction amplitudes to calculate first order, non-divergent, processes involving possible spin 3/2 leptons in order to have criteria and effects which might be submitted to experimental tests.

We, therefore, propose to reverse the strategy which was used in the case of the familiar weak decays of spin 1/2 particles : the theoretical model was there suggested by experimental facts in the low-energy domain ; for the possible, speculated, high energy weak phenomena involving spin 3/2 leptons we use a theoretical model to guide us in the search for such events. And the non renormalizable spin 3/2 lagrangean might well result from an appropriate renormalizable model for lepton constituents.

Perhaps, the mystery of the occurence of the three lepton families and of the large muon and tauon masses, relative to the electron mass, is deeper than a naive model of lepton structure might suppose.

In the meantime, it may be worthwhile to ask the question : do spin 3/2 leptons exist ?

In this note, we give a simple, elementary proof of the unicity of the equation of free spin 3/2 fields, the corresponding Feynman integral equation, and the well-known propagator. Covariant forms are exhibited for the construction of phenomenological interactions involving spin 3/2 fields. A heuristic SU(2) $\otimes$ U(1) model for these fields determines the corresponding electromagnetic and weak currents. A spin 3/2 neutrino may acquire a Dirac or Majorana mass in a way similar to the spin 1/2 case.

Among the possible decays[3] of spin 3/2 charged paricles we point out the radiative decay into the spin 1/2 charged lepton with the same leptonic number the branching ratio of which is estimated.

II. FEYNMAN PROPAGATION EQUATION FOR SPIN 3/2 FIELDS

As a spin 3/2 free field is described by a vector-spinor $\psi^{\mu}_{a}(x)$, which obeys Dirac's equation and two subsidiary conditions

$$(i\gamma\cdot\partial - m)_{aa'}\,\psi^{\mu}_{a'}(x) = 0 \quad , \qquad \mu = 0,1,2,3$$
$$(\gamma_{\mu})_{aa'}\,\psi^{\mu}_{a'}(x) = 0 \qquad , \qquad a = 1,2,3,4 \qquad (2.1)$$
$$\partial_{\mu}\,\psi^{\mu}_{a}(x) = 0$$

the initial problem consists in finding a lagrangean with appropriate auxiliary fields, which will lead to a single equation in ψ^{μ}_{a}, capable of generating the three equations (2.1).

With the vector-spinor ψ^{μ}_{a}, $g^{\mu\nu}$ the matrices γ^{μ}, γ^{5} and the differential operator ∂^{μ}, we may define the following tensor-spinors of the second rank (omit the spinor index a) :

$$F_{\alpha\beta} = \partial_{\beta}\psi_{\alpha} - \partial_{\alpha}\psi_{\beta} \qquad (2.2)$$

which is a Maxwell-like tensor, and its dual and pseudo-tensor-spinor associates :

$$*F^{\mu\nu} = \frac{1}{2}\,\epsilon^{\mu\nu\alpha\beta}\,F_{\alpha\beta}\ ; \qquad (2.3)$$

$$F_5^{\mu\nu} = \gamma^5\,F^{\mu\nu}\ ; \qquad (2.4)$$

And a Rarita-Schwinger-like tensor spinor

$$\mathcal{G}_{\alpha\beta} = \gamma_{\alpha}\,\psi_{\beta} - \gamma_{\beta}\,\psi_{\alpha}\ ; \qquad (2.5)$$

together with the corresponding dual and pseudo variables which are associated to it :

$$*\mathcal{G}^{\mu\nu} = \frac{1}{2}\,\epsilon^{\mu\nu\alpha\beta}\,\mathcal{G}_{\alpha\beta}\ ; \qquad (2.6)$$

$$\mathcal{G}_5^{\mu\nu} = \gamma^5\,\mathcal{G}_{\mu\nu}\ .$$

We next postulate that the scalar lagrangean to be constructed with the above tensors must generate a first order wave equation for the field ψ^{μ} such that : 1) for $m \neq 0$, it gives the equations (2.1) ; 2) for $m = 0$ it will be spinor-gauge invariant i.e., invariant under the transformation

$$\psi^{\mu}(x) \rightarrow \psi^{\mu}(x) + \partial^{\mu}\varphi(x)$$

where $\varphi(x)$ is an arbitrary spinor.

The lagrangean which satisfies this postulate is unique[4] and has the form :

$$L = -\frac{1}{2}\,\overline{\mathcal{G}}^{\mu\nu}_{5}\,\left({}^{*}F_{\mu\nu} - \frac{i}{2}\,m\,{}^{*}\mathcal{G}_{\mu\nu}\right) \qquad (2.8)$$

In terms of the spinor-vector ψ^{μ}, this is :

$$L = -\,\overline{\psi}_{\alpha}\,\epsilon^{\alpha\mu\nu\beta}\,\gamma^{5}\gamma_{\mu}\left(\partial_{\nu} + \frac{im}{2}\,\gamma_{\nu}\right)\psi_{\beta} \qquad (2.8a)$$

and gives rise to the equation :

$$\epsilon^{\alpha\mu\nu\beta}\,\gamma^{5}\gamma_{\mu}\left(\partial_{\nu} + \frac{im}{2}\,\gamma_{\nu}\right)\psi_{\beta} = 0 \qquad (2.9)$$

This is the equation adopted in supergravity when $m = 0$ and ψ_{β} is a Majorana field.

The integral equation which describes the Feynman propagation of the vector-spinor defined by equation (2.9) is the following :

$$\theta(t_2 - t_1)\,\psi^{\alpha}_{(+)}(x_2) = i\int_{t_2>t_1} S_F^{\alpha\mu}(x_2 - x_1)\,\epsilon_{\mu\nu\beta\lambda}\,\gamma^{5}\gamma^{\nu}\delta^{\lambda}_{o}\,\psi^{\beta}_{(+)}(x_1)\,d^3x_1 \qquad (2.10)$$

for positive energy solutions and

$$\theta(t'_1 - t_2)\,\psi^{\alpha}_{(-)}(x_2) = -\,i\int_{t_2<t'_1} S_F^{\alpha\mu}(x_2 - x'_1)\,\epsilon_{\mu\nu\beta\lambda}\,\gamma^{5}\gamma^{\nu}\delta^{\lambda}_{o}\,\psi^{\beta}_{(-)}(x'_1)\,d^3x'_1 \qquad (2.10a)$$

for negative energy solutions. $S_F^{\alpha\mu}$ is the Feynman propagator for massive spin 3/2 fields obeying the equation :

$$\epsilon_{\alpha\mu\nu\beta}\gamma^{5}\gamma^{\mu}\left(\partial^{\nu} + \frac{im}{2}\,\gamma^{\nu}\right)S_F^{\beta\lambda}(x_2 - x_1) = i\,\delta^{\lambda}_{\alpha}\,\delta^{4}(x_2 - x_1) \qquad (2.10\,b)$$

It is given by :

$$S_F^{\mu\nu}(x_2 - x_1) = -i(i\gamma\partial_2 + m)\Delta_F^{\mu\nu}(x_2 - x_1),$$

$$\Delta_F^{\mu\nu}(x_2 - x_1) = \left(g^{\mu\nu} - \frac{1}{3}\gamma^{\mu}\gamma^{\nu} - \frac{i}{3m}\,\gamma^{\mu}\partial^{\nu} + \frac{i}{3m}\,\gamma^{\nu}\partial^{\mu} + \frac{2}{3m^2}\,\partial^{\mu}\partial^{\nu}\right)\Delta_F(x_2 - x_1). \qquad (2.11)$$

This expression results also from the definition involving the time ordered vacuum expectation value of the product $\psi^\mu \overline{\psi^\nu}$, namely :

$$S_F^{\mu\nu}(x_2 - x_1) = - <0|\psi^\mu(x_2)\overline{\psi^\nu}(x_1)|0>\theta(t_2 - t_1) + <0|\overline{\psi^\nu}(x_1)\psi^\mu(x_2)|0>\theta(t_1 - t_2)$$

and the commutation rules :

$$\{\psi_a^\mu(x_2), \overline{\psi_b^\nu}(x_1)\} = - i(i\gamma\cdot\partial_2 + m)_{ab}\, \Delta^{\mu\nu}(x_2 - x_1)$$

$\Delta^{\mu\nu}$ is given by $\Delta_F^{\mu\nu}$ in (2.11) where one replaces Δ_F by the Cauchy delta-function $\Delta(x_2 - x_1)$.

In view of the occurrence of the double differentiation of Δ_F in equation (2.11) the propagator $S_F^{\mu\nu}$ attributes to processes involving this propagator non-renormalizable divergences similar to those of massive vector fields.

III. COVARIANT FORMS FOR PHENOMENOLOGICAL INTERACTIONS

In order to build possible interactions between spin 3/2 fields and other fields we have at our disposal the covariant forms as given in Table I :

TABLE I. Covariant forms for spin 3/2 fields

Two scalars	:	$\bar{\psi}_\alpha \psi^\alpha$ and $\bar{\psi}_\alpha \gamma^\alpha \gamma^\beta \psi_\beta$
Two pseudoscalars	:	$\bar{\psi}_\alpha \gamma^5 \psi^\alpha$ and $\bar{\psi}_\alpha \gamma^\alpha \gamma^5 \gamma^\beta \psi_\beta$
Four four-vectors	:	$\bar{\psi}_\alpha \gamma^\mu \psi^\alpha$;
		$\bar{\psi}_\alpha \gamma^\alpha \gamma^\mu \gamma^\beta \psi_\beta$;
		$\bar{\psi}^\mu \gamma^\beta \psi_\beta$;
		$\bar{\psi}^\alpha \gamma_\alpha \psi^\mu$;
Four axial-vectors	:	$\bar{\psi}_\alpha \gamma^\mu \gamma^5 \psi^\alpha$;
		$\bar{\psi}_\alpha \gamma^\alpha \gamma^\mu \gamma^5 \gamma_\beta \psi^\beta$;
		$\bar{\psi}^\mu \gamma^5 \gamma^\alpha \psi_\alpha$;
		$\bar{\psi}^\alpha \gamma_\alpha \gamma^5 \psi^\mu$
Four tensors	:	$\bar{\psi}_\alpha \sigma^{\mu\nu} \psi^\alpha$
		$i\, \bar{\psi}_\alpha \epsilon^{\alpha\mu\nu\beta} \gamma^5 \psi_\beta$
		$-\, i(\bar{\psi}^\mu \psi^\nu - \bar{\psi}^\nu \psi^\mu)$
		$\frac{i}{2}\left[\bar{\psi}_\alpha \gamma^\alpha (\gamma^\mu \psi^\nu - \gamma^\nu \psi^\mu) + (\bar{\psi}^\mu \gamma^\nu - \bar{\psi}^\nu \gamma^\mu) \gamma_\alpha \psi^\alpha\right]$

Convenient linear combinations of these are :

$$S = \frac{i}{2}\,\overline{\psi}_\alpha\, \epsilon^{\alpha\mu\nu\beta}\,\gamma^5\gamma_\mu\gamma_\nu\psi_\beta$$

$$P = -\frac{1}{2}\,\overline{\psi}_\alpha\, \epsilon^{\alpha\mu\nu\beta}\,\gamma_\mu\gamma_\nu\psi_\beta$$

$$V^\mu = -\,i\,\overline{\psi}_\alpha\, \epsilon^{\alpha\mu\nu\beta}\,\gamma^5\gamma_\nu\psi_\beta$$

$$A^\mu = +\,i\,\overline{\psi}_\alpha\, \epsilon^{\alpha\mu\nu\beta}\gamma_\nu\psi_\beta$$

$$S^{\mu\nu} = \frac{i}{2}\,\overline{\psi}_\alpha\left[\epsilon^{\alpha\mu\lambda\beta}\,g_{\lambda\eta}\,\sigma^{\eta\nu} - \epsilon^{\alpha\nu\lambda\beta}\,g_{\lambda\eta}\,\sigma^{\lambda\mu}\right]\gamma^5\psi_\beta$$

which, regarded as normal products with ψ_α as operator, satisfy the charge conjugation properties :

$$S^c = S\ ;\ P^c = P\ ;\ (A^\mu)^c = A^\mu$$

$$(V^\mu)^c = -\,V^\mu\ ;\ (S^{\mu\nu})^c = -\,S^{\mu\nu}$$

We may take as vector current either V^μ above or $\overline{\psi}_\alpha\gamma^\mu\psi^\alpha$ or $\overline{\psi}_\alpha\gamma^\alpha\gamma^\mu\gamma^\beta\psi_\beta$ or the sum $\overline{\psi}^\mu\gamma^\beta\psi_\beta + \overline{\psi}^\alpha\gamma_\alpha\psi^\mu$ since under charge conjugation the latter sum changes sign like the former two currents.

All terms containing $\gamma_\alpha\psi^\alpha$ vanish if one constructs forms with vector spinors in momentum space which represent asymptotically free spin 3/2 particles.

IV. ELECTROMAGNETIC AND WEAK INTERACTIONS OF SPIN 3/2 FIELDS AS DEDUCED FROM A HEURISTIC STANDARD SU(2) ⊗ U(1) MODEL

In order to find out possible forms of weak and electromagnetic couplings of spin 3/2 particles we may use, as a heuristic method, the standard SU(2) ⊗ U(1) model. This means that we postulate, besides the familiar Higgs and vector gauge fields, a left handed isospinor with spin 3/2 fields, a_α and b_α :

$$L_\alpha = \begin{pmatrix} \frac{1}{2}(1 - \gamma^5)\, a_\alpha \\ \frac{1}{2}(1 - \gamma^5)\, b_\alpha \end{pmatrix} \equiv \begin{pmatrix} a_{\alpha L} \\ b_{\alpha L} \end{pmatrix}$$

and two right handed isoscalars :

$$a_{\alpha R} = \frac{1}{2}(1 + \gamma^5) a_\alpha \;;\; b_{\alpha R} = \frac{1}{2}(1 + \gamma^5)\, b_\alpha$$

which transform as :

$$L_\alpha \to \exp(ig\vec{\Lambda}(x)\cdot\frac{\vec{\tau}}{2})\exp(-\frac{ig'}{2}(1 - 2\eta)\theta(x))L_\alpha$$

$$a_{\alpha R} \to \exp(ig'\eta\,\theta(x))a_{\alpha R}$$

$$b_{\alpha R} \to \exp(ig'(\eta - 1)\theta(x))b_{\alpha R}$$

g and g' are the Salam-Weinberg coupling constants. The charges of a_α and b_α are respectively η and $\eta - 1$ in units of e ; if $\eta = 0$ we have a doublet of a spin 3/2 neutrino and a negatively charged spin 3/2 lepton. If $\eta = \frac{2}{3}$, we have a doublet of spin 3/2 particles charged like quarks.

Apart from the terms corresponding to the Higgs and vector gauge fields the lagrangean will contain a term for the spin 3/2 particles in interaction with the Higgs field ϕ of the form :

$$\begin{aligned} \mathcal{L} = &- \bar{L}_\alpha\, \epsilon^{\alpha\mu\nu\beta}\, \gamma^5\gamma_\mu\, D_\nu L_\beta - \bar{a}_{\alpha R}\, \epsilon^{\alpha\mu\nu\beta}\, \gamma^5\gamma_\mu\, D_\nu a_{\beta R} - \\ &- \bar{b}_{\alpha R}\, \epsilon^{\alpha\mu\nu\beta}\, \gamma^5\gamma_\mu\, D_\nu b_{\beta R} - \frac{iG_a}{2}(\bar{L}_\alpha\, \epsilon^{\alpha\mu\nu\beta}\gamma^5\gamma_\mu\gamma_\nu a_{\beta R})\phi - \\ &- \frac{iG_b}{2}(\bar{L}_\alpha\, \epsilon^{\alpha\mu\nu\beta}\, \gamma^5\gamma_\mu\gamma_\nu b_{\beta R})\phi^c \\ &+ h.c. \end{aligned}$$

where $\phi^c = i\,\tau_2\,{}^t\phi^+$ is the charge conjugate of ϕ. The mass terms derived from the Higgs coupling are :

$$\mathcal{L}_m = -\frac{i}{2}\left[m_a(\bar{a}_\alpha\,\epsilon^{\alpha\mu\nu\beta}\,\gamma^5\gamma_\mu\gamma_\nu a_\beta) + m_b(\bar{b}_\alpha\,\epsilon^{\alpha\mu\nu\beta}\,\gamma^5\gamma_\mu\gamma_\nu b_\beta)\right]$$

where

$$m_a = G_a\,\lambda\;, \qquad m_b = G_b\,\lambda$$

and λ is the vacuum expectation value of the neutral Higgs field.

In the case in which $\eta = 0$ the lepton a_α is a neutrino with spin 3/2. A Majorana mass may be attributed to this neutrino by introduction of a coupling with a Higgs triplet[5)] $\vec{H}$ of the form :

$$L_M = -\frac{i}{2}\,G_M\,\bar{L}_\alpha\,\epsilon^{\alpha\mu\nu\beta}\,\gamma^5\gamma_\mu\gamma_\nu(\vec{\tau}\cdot\vec{H})L^c_\beta + \text{h.c.}$$

where

$$L^c_\alpha = i\,\tau_2\begin{pmatrix}(\nu_{\alpha L})^c\\(b_{\alpha L})^c\end{pmatrix}$$

If we call w the vacuum expectation value of the neutral component of $\vec{H}$, $H^{(o)} \equiv H_1 - i\,H_2$ and call $m_M = G_M\,w$ then the Majorana mass term is

$$-\frac{i}{2}\,m_M\left[\bar{\nu}_{\alpha L}\,\epsilon^{\alpha\mu\nu\beta}\,\gamma^5\gamma_\mu\gamma_\nu(\nu^c_\beta)_R + \overline{(\nu^c_\alpha)_R}\,\epsilon^{\alpha\mu\nu\beta}\,\gamma^5\gamma_\mu\gamma_\nu\,\nu_{L\beta}\right]$$

The kinematic term for the neutrino will have the form :

$$L_{kin} = -\bar{\chi}_\alpha\,\epsilon^{\alpha\mu\nu\beta}\,\gamma^5\gamma_\mu(\partial_\nu + \frac{i}{2}\,m_M\,\gamma_\nu)\chi_\beta$$

where

$$\chi_\alpha = \nu_{\alpha L} + (\nu_{\alpha L})^c$$

is a Majorana field the left-handed component of which enters the weak couplings.

V. RADIATIVE DECAY OF SPIN 3/2 LEPTONS

The electromagnetic current of spin 3/2 particles is then in the SU(2) ⊗ U(1) model :

$$j^{\mu}_{\gamma} = \eta\, i(\bar{a}_{\alpha}\, \epsilon^{\alpha\lambda\mu\beta}\, \gamma^{5}\gamma_{\lambda}a_{\beta}) + (\eta - 1)\, i\, (\bar{b}_{\alpha}\, \epsilon^{\alpha\lambda\mu\beta}\, \gamma^{5}\gamma_{\lambda}b_{\beta})$$

The charged weak current is :

$$j^{\mu}_{W} = i(\bar{b}_{\alpha}\, \epsilon^{\alpha\lambda\mu\beta}\, \gamma^{5}\gamma_{\lambda}(1 - \gamma^{5})a_{\beta})$$

and the neutral weak current

$$j^{\mu}_{Z} = \frac{i}{2}\left[\bar{a}_{\alpha}\, \epsilon^{\alpha\lambda\mu\beta}\, \gamma^{5}\gamma_{\lambda}(1 - \gamma^{5})a_{\beta} - \bar{b}_{\alpha}\, \epsilon^{\alpha\lambda\mu\beta}\gamma^{5}\gamma_{\lambda}(1-\gamma^{5})b_{\beta}\right] - 2\sin^{2}\theta_{W}j^{\mu}_{\gamma}$$

We thus expect that if charged spin 3/2 particles exist with charge η they may be produced in electron-positron collisions and subsequently suffer competing weak decays as calculated previously .

As is well-known, the ratio between the cross sections for quark pair production to μ-pair production in the $e^{+}e^{-}$ annihilation, increases by $3Q_q^2$ each time a new quark with charge $Q_q e$ is produced, as the center of mass energy increases. Such a ratio in the case of production of spin 3/2 quark pairs would be energy dependent[3a] and this would be a characteristic signature for the production of such particles.

If such processes are detected it would be of importance to know the form of the couplings which give rise to them.

Besides the previously[3] calculated production and possible decays of heavy and charged spin 3/2 particles M, we wish to point out that the production of spin 3/2 leptons accompanied of a spin 1/2 lepton of the same family is possible

$$e^{+} + e^{-} \longrightarrow \begin{array}{l} T^{-} + \tau^{+} \\ M^{-} + \mu^{+} \\ L^{-} + e^{+} \end{array}$$

if a Pauli - like interaction of the form :

$$\frac{e}{M}\,\bar{b}_\alpha\,\epsilon^{\alpha\mu\nu\beta}\,\gamma^5\gamma_\mu D_\nu D_\beta\varphi + \text{h.c.}$$

is assumed, where b_α is the Rarita-Schwinger field of a spin 3/2 lepton with mass M and φ describes a charged spin 1/2 lepton. This coupling has the form :

$$-\frac{e}{2M}\left[\bar{b}^\nu\gamma^\beta - \bar{b}^\beta\gamma^\nu - (\bar{b}^\alpha\gamma_\alpha)\,\gamma^\nu\gamma^\beta\right]F_{\nu\beta}\,\varphi + \text{h.c.}$$

and may lead to γ-decay of a spin 3/2 lepton into the spin 1/2 lepton of its family (with the same leptonic number) :

$$L \to \ell + \gamma$$

The branching ratio ρ for this decay as compared to β-decay :

$$L \to \nu_L + \ell + \bar{\nu}_\ell$$

is estimated to be :

$$\rho = \frac{\Gamma(L \to \ell + \gamma)}{\Gamma(L \to \nu_L + \ell + \bar{\nu}_\ell)} \cong \frac{e^2}{G_F^2\,M^4} \simeq 10^8\left(\frac{m_p}{M}\right)^4$$

For $M \gg 100\,m_p$ we expect $\rho \ll 1$.

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