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A SIMPLE GAUGE-FIXING CONDITION IN YANG-MILLS THEORY FOR ANY GAUGE GROUP

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ABSTRACT:

The gauge-fixing conditions $A^{o}_{a} \approx 0$, $A^{3}_{a} \approx 0$ are proposed in Yang-Mills theory for any gauge group. They are ghost-free and fix the gauge even for arbitrary strong fields. Using the formalism of the phase-space functional integral we obtain a description of the theory which involves physical degrees of freedom only.

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The Lagrangians of gauge theories in their local and Poincaré invariant formulation are singular containing non-physical degrees of freedom. Dirac's method[1] for constrained dynamical systems may be used to construct Hamiltonian which, however, contains arbitrary functionals multiplying the surviving first class constraints. These constraints are generators of infinitesimal gauge transformations. In order to express the theory in terms of physical degrees of freedom only we must impose supplementary gauge-fixing constraints. In the context of canonical quantization they must be chosen in such a way that the arbitrary functional are unambiuously determined and Heisenberg equations of motion come out free from this arbitrariness. In the context of quantization by Feynman functional integral[2] they should give unambiguous meaning to the integral and be simple enough to handle the integral itself. It was discovered recently by Gribov[3] that in the case of Yang-Mills theory the commonly used Coulomb gauge becomes singular for sufficiently strong gauge fields and thus fails to fix the gauge. We must then look for other gauges which are free of ghosts and ambiguities. The temporal gauge, $A^o \approx 0$, is a simple choice to start with. The canonical brackets of independent variables here coincide withthe Dirac brackets and thus are easy to work with. Goldstone

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and Jackiw[4] used this gauge and solved the constraint equation for the wave functional Ψ in Yang-Mills theory for SU(2) gauge group in momentum representation ($\vec{\pi}_a = -\vec{E}_a$ diagonal) and obtained a formulation without any non-physical degrees of freedom. Faddeev, Izergin, Korepin, Semenov-Tian-Shansky[5] in a recent publication showed that the same results are obtained in the context of the functional integral by imposing gauge-fixing conditions on the canonical momenta but not on the vector potential $\vec{A}_a$. These authors could also generalize their method for an arbitrary gauge group as well.

We present here an alternative very simple gauge-fixing condition in Yang-Mills theory which works for any gauge group. The gauge is ghost-free and we give an expression for the generating functional of the S-matrix which involves physical degrees of freedom only. The same can ofcourse be done also in the context of canonical quantization[6].

Let us look at the equations of motion, in temporal gauge, for the gauge potentials. They are given in the usual notation* by

$$\dot{\vec{A}}_a = \frac{\delta H}{\delta \vec{\pi}_a} = \vec{\pi}_a - (\delta_{ab}\vec{\nabla} + g f_{abc}\vec{A}_c)u_b \tag{1}$$

Here u_a are the arbitrary functionals in the general Hamiltonian

$$H = \int d^3x \left[\frac{1}{2}\vec{\pi}_a\cdot\vec{\pi}_a + \frac{1}{4}F_a^{kl}F_{kl}^a + u_a\chi_a \right] \tag{2}$$

where

$$\chi_a \equiv \vec{D}^{ab} \cdot \vec{\pi}_b \equiv (\delta_{ab} \vec{\nabla} + g f_{abc} \vec{A}_c) \cdot \vec{\pi}_b \approx 0 \tag{3}$$

are the surviving first class Gauss' law constraints for the non-abelian case and $\dot{\chi}_a \approx 0$. It follows from the Eq.(1) that the gauge condition, say, $A^3{}_a \approx 0$ has the very interesting property

$$\Pi_3^a = \partial_3 u_a \tag{4}$$

since we must require also $\dot{A}^3{}_a \approx 0$ for the gauge condition to hold for all times. There are no fields present on the right hand side of Eq.(4). The functionals u_a are thus determined without any ambiguity contrary to the case of Coulomb gauge. This holds true even when interactions are present[6]. For the economy of space we will write below expressions only for pure Yang-Mills fields.

It follows that $A^3_a \approx 0$ gauge is a very convenient choice for any gauge group. We note

$$\{ \chi_a(\vec{x},t) , A^3_b(\vec{y},t) \} \approx -\delta_{ab} \, \partial_3^x \, \delta^3(\vec{x}-\vec{y}) \tag{5}$$

so that $\det \| \{ A^3_a , \chi_b \} \| = \det (\partial_3)$ is constant independent of the fields. Our gauge is ghost-free.

The generating functional for the S-matrix may be expressed in our gauge as the following functional integral[2]

$$Z = N \int e^{iS} \prod d\pi^a_k \, dA^k_a \;\; \delta(A^3_a)\, \delta(\chi_a) \det \| \{ A^3_a , \chi_b \} \| \tag{6}$$

where N is a normalization factor and S is the action given by

$$S = -\int d^4x \left[\dot{\vec{\Pi}}_a \cdot \vec{A}_a + \frac{1}{2} \vec{\Pi}_a \cdot \vec{\Pi}_a + \frac{1}{4} F_a^{kl} F_{kl}^a \right] \tag{7}$$

The determinant being constant it may be absorbed inthe normalization and we may integrate over A^3_a using delta functional. Writing $\delta(\chi_a)$ in exponential representation the action takes the following form

$$S = \int d^4x \left[\bar{\Pi}_a \cdot \dot{\bar{A}}_a - \frac{1}{2} \vec{\Pi}_a \cdot \vec{\Pi}_a - \frac{1}{4} F_a^{kl} F_{kl}^a - u_a (\partial_3 \Pi_3^a + \bar{D}^{ab} \cdot \bar{\Pi}_b) \right] \tag{8}$$

where $\bar{A} = (A^1, A^2)$, $\bar{\nabla} = (\partial_1, \partial_2)$, $\bar{D}^{ab} = (\bar{\nabla}\delta_{ab} + g f_{abc} \bar{A}_c)$ etc. and we have to perform functional integration over u_a in addition Integration on $\Pi^a{}_3$ now simply replaces these variables by $\partial_3 u_a$. The integration over u_a is then done by the usual shift transformation. The functional integral factor depending solely on u_a is absorbed in the normalization and the generating functional takes the form

$$Z = N \int e^{iS} \prod d\Pi_1^a \, d\Pi_2^a \, dA_a^1 \, dA_a^2 \tag{9}$$

where

$$S = \int d^4x \left[\bar{\Pi}_a \cdot \dot{\bar{A}}_a - \frac{1}{2} \bar{\Pi}_a \cdot \bar{\Pi}_a - \frac{1}{4} F_a^{kl} F_{kl}^a + \frac{1}{2} \int d^3y \, \bar{D}^{ab} \cdot \bar{\Pi}_b(\vec{y},t) K(\vec{x},\vec{y}) \bar{D}^{ac} \cdot \bar{\Pi}_c(\vec{x},t) \right] \tag{10}$$

Here[§] $K(\vec{x},\vec{y}) = G(x^3,y^3)\, \delta^2(\bar{x}-\bar{y})$ and $G(t,t')$ is the Green's function satisfying $(\partial^2 G / \partial t^2) = \delta(t-t')$.In the presence of a spinor field source ($\bar{D}.\bar{\pi}$)$_a$ is replaced by $(\bar{D}.\bar{\Pi})_a - g\psi^\dagger t_a\psi$ in the last terms of Eqns.(8) and (10). For the abelian case it reduces to $(\bar{\nabla}.\bar{\pi}) - g\psi^\dagger\psi$.

The gauge-fixing conditions $A^0_a \approx 0$, $A^3_a \approx 0$ thus fix the gauge uniquely even for arbitrary strong fields and we obtain a description of the Yang-Mills field for any gauge group in terms of the physical degrees of freedom only. The same may be shown true also in the context of canonical quantization. For the abelian case the Hamiltonian in Eq.(10) may be rewritten in terms of the transverse components alone and the Coulomb self-energy interaction term is seperated out. The kinetic terms here are handled by a unitary transformation[6]. It is also clear from the equations of motion for canonical momenta that any gauge condition on them will bring in difficult[5] constraint equations specially when interactions are present and we are dealing with an arbitrary gauge group.

We finally remark that, having seen that our gauge conditions in phase space do fix the gauge, we may as well use the exponential representation for $\delta(\chi_a)$ in Eq.(6). The functional integrations over the variables $\vec{\pi}_a$ may then be readily performed by using the shift transformations $\pi_3^a \rightarrow \pi_3^a + \partial_3 u_a$, $\bar{\pi}_a \rightarrow \bar{\pi}_a + \bar{\bar{\pi}}_a$ where $\bar{\bar{\pi}}_a = (\dot{\bar{A}}_a + \bar{D}^{ab} u_b)$. A Gaussian functional integral solely over variables $\vec{\pi}_a$ may be absorbed in the normalization and we obtain the following expression for the generating functional

$$Z = N \int e^{iS} \prod dA^k_a \, dA^0_a \, \delta(A^3_a) \tag{11}$$

where

$$S = -\frac{1}{4}\int [\partial^\mu A^\nu_a - \partial^\nu A^\mu_a + g f_{abc} A^\mu_b A^\nu_c]^2 d^4x \tag{12}$$

and we have rewritten u_a as A^0_a. This representation may then be used to go over to other convenient gauge conditions[7] where Feynman rules are given in manifestly covariant form.

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* $\vec{\pi}_a = -\vec{E}_a$ are canonical momenta, a=1,2...n are the indices of gauge group and f_{abc} are completely antisymmetric structure constants and g is the coupling constant.

§ We may have taken for an arbitrary constant unit vector $\vec{n}$ the gauge condition to be $\vec{n}.\vec{A}_a \approx 0$. x^3 is then replaced by $\vec{n}.\vec{A}$ and $\bar{A}$ represents a vector transverse to the direction $\vec{n}$.