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by

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# EFFECT OF NON-LOCALITY IN FERMI INTERACTIONS DUE TO VECTOR MESONS ON THE DECAYS $\Sigma \to p + \gamma$ AND $\mu \to e + \nu + \bar{\nu}$ *@

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SUMMARY: The effect of non-locality in Fermi interaction introduced by vector mesons on the decays $\Sigma \to p + \gamma$ and $\mu \to e + \nu + \bar{\nu}$ are discussed. It is found that the branching ratio of $\Sigma \to p + \gamma$ to $\Sigma \to p + \pi^0$ can be of the order $\sim 10^{-4}$ and that in the muon decay to get Michel's parameter $\lesssim \frac{3}{4}$, we need a vector meson mass greater than 10 Proton masses.

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## INTRODUCTION

A Universal Vector minus Axial Vector coupling for the Fermi interaction among four fermions was proposed by several authors[1]. It was suggested by Feynman and Gell-Mann that such Universal Interaction may arise from the exchange of electrically charged vector mesons of very high mass between the two currents formed by the two pairs of fermions[2]. Leite Lopes[3] suggested the possibility of including the neutral counterpart of vector meson in the theory also, provided that we assume that the neutral meson enters the coupling with the fermion field in such a way that the current which creates the neutral meson is conserved. The neutral meson then leads to an effective non-electromagnetic potential between neutron (or neutrino) and electron.

In a theory which allows only direct four fermion interaction the radiative decays of the type $\mu \to e + \gamma$, $\Sigma \to p + \gamma$ etc., involve at least square of the Fermi coupling constant G in the matrix element and consequently, are very slow. However, if the interaction is due to the existence of vector mesons[4] these decays are possible with only first power of coupling constant (or rather eG) in the matrix element. These decays may not be then very small compared to the corresponding other modes of decays known experimentally and involving only the first power of G in the matrix element. Feinberg[5] has shown that, through the vector meson, the branching ratio[6] of $\mu \to e + \gamma$ to $\mu \to e + \nu + \bar{\nu}$ is $\sim 10^{-4}$ while that of $\mu^+ \to e^+ + e^+ + e^-$ arising due to the internal conversion of the photon is $\sim 10^{-6}$.

In this note it is our purpose to discuss the implications that the existence of such vector mesons would imply in certain decays like $\Sigma \to p + \gamma$ and $\mu \to e + \nu + \bar{\nu}$. We show that the non locality introduced in muon decay by these vector mesons is consistent with $\rho > \frac{3}{4}$ or $\rho \leq \frac{3}{4}$ where $\rho$ is the Michel parameter, depending on the choice of the mass of the vector mesons.

First we calculate the branching ratios for the decay[7] $\Sigma \to p + \gamma$ which is also possible through the intermediary of the charged vector meson. The $\Sigma$ decays virtually into a charged meson and a neutral baryon which then recombines to create a proton. The photon may be emmitted due to the interaction of the electromagnetic field with the charged meson or with the in-going or the outgoing charged particles.

From the experimental point of view such an event which, as suggested by M. Goldhaber, could be interpreted as the decay $\Sigma \to p + \gamma$ was found by George, Herz, Noon and Solnsteff (Nuovo Cimento 3, 94 (1955); see also R. E. Behrends, Phys. Rev. 111, 1691, (1958) Footnote.

## TRANSITION PROBABILITY

The interaction Hamiltonion for the coupling of fermion fields with the vector meson field is given by[8]

$$\mathcal{H}_I(x) = g \; \bar{\psi}_n \gamma_\rho (1 + \gamma_5) \psi_\Sigma \; \phi_\rho + g \; \bar{\psi}_P \gamma_\rho (1 + \gamma_5) \psi_n \; \phi_\rho^+ + \text{h.c.}$$

The electromagnetic interaction is given by

$$\mathcal{H}_I^{e.m.}(x) = e \; \bar{\psi}_\Sigma \gamma_\mu \psi_\Sigma \; A_\mu + e \; \bar{\psi}_P \gamma_\mu \psi_P \; A_\mu + ie \left[ \phi^+_{\mu\nu} \phi_\nu - \phi^+_\nu \phi_{\mu\nu} \right] A_\mu$$

where

$$\phi_{\mu\nu} = (\partial_\mu \phi_\nu - \partial_\nu \phi_\mu) \quad \text{and} \quad \phi^+_{\mu\nu} = (\partial_\mu \phi^+_\nu - \partial_\nu \phi^+_\mu)$$

$\phi_\rho$ is the vector meson field ($\rho = 1,2,3,4$) and $\psi_n$ represents the field of the intermediate neutral particle $n$, $\Lambda^0$ or $\Sigma^0$ etc. $\psi_P$ and

$\Psi_\Sigma$ are proton and $\Sigma$ fields respectively.

The graphs to be considered are:

The corresponding matrix elements are given by:

$$M_a \doteq e\, g^2 \int d^4_q\, \bar{u}^P (p_1)\, (\gamma \cdot \varepsilon)\, (i\gamma \cdot p_2 + m_2)^{-1}\, \gamma_\rho (1 + \gamma_5)\, (i\gamma \cdot q + m)^{-1} \gamma_\lambda\, (1 + \gamma_5)\, P_{\lambda\rho}\, (p_2 - q)\, u^\Sigma\, (P_2)$$

$$M_b \doteq e g^2 \int d^4_q\, \bar{u}^P (p_1)\, \gamma_\rho\, (1 + \gamma_5)\, (i\gamma \cdot q + m)^{-1} \gamma_\lambda\, (1 + \gamma_5)\, (i\gamma \cdot p_1 + m_1)^{-1}\, (\gamma \cdot \varepsilon)\, P_{\lambda\rho}\, (p_1 - q)\, u^\Sigma\, (p_2)$$

$$M_c \doteq e g^2 \int d^4_q\, \bar{u}^P (p_1)\, \gamma_\rho\, (1 + \gamma_5)\, P_{\rho\nu}\, (p_1 - q)\, C_{\lambda\rho}\, P_{\sigma\lambda}\, (p_2 - q) (i\gamma \cdot q + m)^{-1} \gamma_\sigma\, (1 + \gamma_5)\, u^\Sigma\, (p_2)$$

where

$$P_{\mu\nu} (q) = \left(\delta_{\mu\nu} + \frac{q_\mu\, q_\nu}{M_x^2}\right) \frac{1}{(q^2 + M_x^2)}$$

$$C_{\lambda\rho} = \left\{2(p_2 - q) \cdot \varepsilon\, \delta_{\lambda\rho} - \left[(p_1 - q)_\lambda \varepsilon_\rho + (p_2 - q)_\rho \varepsilon_\lambda\right]\right\}$$

and

$$P_2 = p_1 + k$$

Here $p_1$ and $p_2$ are the four-momenta of the proton and the $\Sigma$

of masses $m_1$ and $m_2$ respectively. q is the four-momenta of the intermediate baryon, say neutron of mass m and k is the four momenta of the photon of polarization four-vector $\varepsilon$ .

The quadratically divergent term as well as the non-gauge invariant terms disappear when contributions from the three diagrams are summed and the final result is only logarithmically divergent and is also gauge invariant.

Then $|M = |M_e + |M_b + |M_c$

$$\simeq e\left(\frac{g^2}{M_x^2}\right)\bar{u}^P(p_1)\left\{\frac{i\pi^2}{2}D\left[(1-\gamma_5)m_2+(1+\gamma_5)m_1\right]\sigma_{\mu\nu}(\varepsilon_\mu K_\nu - \varepsilon_\nu K_\mu) + \right.$$

$$\left. + F\right\} u^\Sigma(p_2)$$

where $D = \frac{1}{i\pi^2}\int d^4q/(q^2+M_x^2)^2$ is the logarithmically divergent integral and F is the finite part. When the mass of the vector meson is assumed high (say ten proton mass) F can be shown to be proportional to the inverse of the square of the vector meson mass and can therefore be neglected.

The transition probability for the decay is given by

$$P = e^2\left(\frac{g^2}{M_x^2}\right)^2 \frac{1}{2^6(2\pi)^5}(m_1^2+m_2^2)\left(\frac{m_2^2 - m_1^2}{m_2}\right)^3 D^2$$

For the branching ratios (summing over the various neutral baryons) we obtain, for a cut-off of the order of the mass of the vector meson:

$$\frac{P(\Sigma \to p + \gamma)\text{theo.}}{P(\Sigma \to p + \pi^0)\text{exp.}} \simeq 1{,}4 \times 10^{-4}$$

and

$$\frac{P(\Sigma \to p + \gamma)\text{theo.}}{P(\Sigma \to p + \mu + \nu)\text{theo.}} \simeq 4{,}3 \times 10^{-3}$$

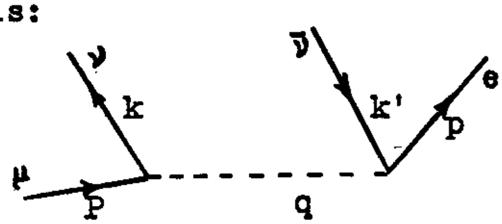
Leite Lopes[3] has shown that $P(\Sigma \to p + \gamma)$ through pions would be of the

the order of $10^3$ sec.$^{-1}$ while through the intermediate vector meson, we obtain it to be $\approx$ $10^6$ sec.$^{-1}$

## THE MUON DECAY THROUGH A VECTOR MESON

Here we study the non-local effects which arise because of the introduction of the intermediate vector meson in the muon decay.

The Feynman graph is:




The matrix element is given by:

$$M = \frac{1}{q^2 + M_x^2}\left\{\delta_{\mu\nu} + \frac{q_\mu q_\nu}{M_x^2}\right\}\left(\bar{u}_k \gamma_\mu (1+\gamma_5) u_P\right)\left(\bar{u}_p \gamma_\nu (1+\gamma_5) u_{k'}\right)$$

Where P = Momentum of the incoming muon

p = " " " outgoing electron

k = " " " outgoing neutrino

k' = " " " antineutrino

q = " " " intermediate vector meson

In the approximation $q^2 \ll M_x^2$ we get:

$$\rho \simeq \frac{3}{4}\left(1 + \frac{m_\mu^2}{M_x^2} - 4\frac{m_e^2}{m_\mu^2}\right)$$

for the Michel parameter, and

$$\tau \simeq (2{,}26 \times 10^{-6}) \,/\, \left(1 + \frac{3}{5}\frac{m_\mu^2}{M_x^2} - 4\frac{m_e^2}{m_\mu^2}\right) \text{ sec.}$$

for the lifetime. Here $m_\mu$, $m_e$ are the muon and electron mass respectively. Then, to get the experimental $\rho \lesssim \frac{3}{4}$, we must have the vector meson mass higher than about ten proton masses, which was already anticipated in reference 3.

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## APPENDIX

## EVALUATION OF DIVERGENT INTEGRALS

In the decay $\Sigma \rightarrow p + \gamma$ there appear divergent integrals of the form:

$$I = \int \frac{k^n d^4k}{((k-\ell)^2 + a^2)^m} \qquad \text{(A-1)}$$

We want to make a variable change $k \rightarrow k + \ell$. In general when I is divergent $I - I' \neq 0$, where I' is given by

$$I' = \int \frac{(k+\ell)^n d^4k}{(k^2 + a^2)^m} \qquad \text{(A-2)}$$

$S = I - I'$ are the well known surface terms, which for simple cases are calculated in Jauch-Rohrlich.[10]

As we have more complicated integrals, we shall report here a method which appears in Enatsu[11]. Let us consider the integrals:

$$\int f(k_\mu, \ell_\mu)\, d^4k \qquad \text{(A - 3)}$$

and let us make the translation $k_\mu \rightarrow k_\mu + \ell_\mu$. For this purpose we make the development:

make the development:

$$f(k_\mu - \ell_\mu + \ell_\mu, \ell_\mu) = f(k_\mu + \ell_\mu , \ell_\mu) - \ell_\mu \left(\frac{\partial f}{\partial k_\mu}\right)_{k \to k+\ell} +$$

$$+ \frac{1}{2!} \ell_\mu \ell_\nu \left(\frac{\partial^2 f}{\partial k_\mu \partial k_\nu}\right)_{k \to k+\ell} + \ldots \qquad \text{(A-4)}$$

the various terms with the derivatives will give the surface terms. When the divergence in (A-5) is logarithmic we do not need to take any term with derivative in (A-4). The surface term is then zero. If the divergence is linear, in (A-4), we take the development until the first derivative, if ir is quadratic until the second derivative, and so on. The remaining terms give vanishing result.

In this way we reduce the calculation to the evaluation of the integrals of the form

$$I_{\lambda \sigma \ldots \rho} = \int \frac{k_\lambda k_\sigma \ldots k_\rho}{(k^2 + a^2)^n} d^4k \qquad \text{(A-5)}$$

For this we postulate a symmetric integration in k-space, which means the substituions:

$$k_\alpha \to 0 \; , \quad k_\alpha \; k_\beta \; k_\gamma \to 0 \; , \quad k_\alpha k_\beta k_\gamma k_\delta k_\rho \to 0$$

$$k_\alpha \; k_\beta \to \frac{1}{4} k^2 \delta_{\alpha\beta} \qquad \text{(A-6)}$$

$$k_\alpha \; k_\beta \; k_\gamma \; k_\delta \to \frac{1}{24} (k^2)^2 \left[\delta_{\alpha\beta} \; \delta_{\gamma\delta} + \delta_{\alpha\gamma} \; \delta_{\beta\delta} + \delta_{\alpha\delta} \; \delta_{\beta\gamma}\right]$$

$$k_\alpha k_\beta k_\gamma k_\delta k_\varepsilon k_\eta \to \frac{1}{192} (k^2)^3 \left[\delta_{\alpha\beta} \; \delta_{\gamma\delta} \delta_{\varepsilon\eta} + \delta_{\alpha\beta} \; \delta_{\gamma\varepsilon} \; \delta_{\delta\eta} + \ldots \text{15 terms}\right]$$

and so on. Then, instead of integrals of the type (A-5) we have essentially to consider integrals of the type:

$$I_{mn} = \int \frac{(k^2)^{m-2}}{(k^2 + a^2)} \; d^4k \qquad \text{(A-7)}$$

The convergent forms of (A-7) are given in reference

$$I_{mn} = \frac{i\pi^2}{(a^2)^{n-m}} \quad \frac{\Gamma(m)\,\Gamma(n-m)}{\Gamma(n)} \tag{A-8}$$

for $n > m > o$.

If $I_{mn}$ is divergent, we shall reduce it to divergent forms of the type:

$$D_1 = \int \frac{d^4k}{(k^2+a^2)^2} \qquad D_2 = \int \frac{d^4k}{(k^2+a^2)} \tag{A-9}$$

This can be done with the help of the following substitutions in the numerators of the integrals (A-7)

$$k^2 \rightarrow (k^2 + a^2) - a^2$$

$$k^4 \rightarrow (k^2 + a^2)^2 - 2\,a^2(k^2 + a^2) - a^4 \tag{A-10}$$

------------------ etc.

For the justification of these replacements see reference 11.

Then we can construct the following table of integrals

$$\int \frac{k_\lambda\, k_\mu d^4k}{[(k-\ell)^2 + a^2]^3} = \int \frac{(k+\ell)_\lambda\ (k+\ell)_\mu}{(k^2+a^2)^3} d^4k = \frac{1}{4}\delta_{\lambda\mu}D_1 - \frac{i\pi^2}{8}\delta_{\lambda\mu} + $$

$$+ \quad \ell_\lambda \ell_\mu \frac{i\pi^2}{2\,a^2}$$

$$\int \frac{k^2\ k_\lambda\ d^4k}{[(k-\ell)^2 + a^2]^3} = \int \frac{(k+\ell)^2 (k+\ell)_\lambda}{(k^2+a^2)^3} d^4k - \frac{i\pi^2}{2}\ell_\lambda$$

(A-11)

$$= \frac{3}{2}\ \ell_\lambda\ D_1 + \ell^2\,\ell_\lambda\ \frac{i\pi^2}{2\,a^2} - \frac{5}{4}\ i\pi^2\,\ell_\lambda$$

$$\int \frac{k^2 k_\lambda\ k_\mu d^4k}{[(k-\ell)^2 + a^2]^3} = \int \frac{(k+\ell)^2 (k+\ell)_\lambda\ (k+\ell)_\mu}{(k^2+a^2)^3} d^4k - \frac{i\pi^2}{3}\Big[-\frac{7}{2}\ \ell_\lambda \ell_\mu - \ell^2\delta_{\lambda\mu}$$

$$= \frac{1}{4}\delta_{\lambda\mu} D_2 + \left\{ -\frac{a^2}{2}\delta_{\lambda\mu} + 2\,\ell_\lambda \ell_\mu + \frac{\ell^2}{4}\delta_{\lambda\mu} \right\} D_1 +$$

$$+ i\pi^2 \left\{ \frac{a^2}{8}\delta_{\lambda\mu} - \frac{13}{6}\ell_\lambda \ell_\mu - \frac{11}{24}\ell^2\delta_{\lambda\mu} + \frac{\ell^2 \ell_\lambda \ell_\mu}{2a^2} \right\}$$

$$\int \frac{k^4 k_\lambda \, d^4k}{[(k-\ell)^2 + a^2]^3} = \int \frac{(k+\ell)^4 (k+\ell)_\lambda \, d^4k}{(k^2+a^2)^3} - \frac{1}{2}\ell_\lambda D_2 - \frac{1}{2}a^2 \ell_\lambda D_1 +$$

$$+ i\pi^2 (a^2 - \frac{13}{3}\ell^2)\ell_\lambda$$

$$= \left[ \frac{3}{2} D_2 + (4\ell^2 - \frac{9}{2}a^2) D_1 + 2i\pi^2 a^2 - \frac{19}{3} i\pi^2 \ell^2 + \right.$$

$$\left. + \frac{i\pi^2}{2a^2}\ell^4 \right] \ell_\lambda$$

Another type of important integral is given by

$$A = \int \frac{k^2 k_\sigma \, d^4k}{[(k-\ell)^2 + a^2]^2} \qquad \text{(A-12)}$$

It has a cubic divergence. But we can calculate it by use of (A-11) remembering that

$$\int \frac{k^2 k_\sigma \, d^4k}{(k^2+a^2)^2} = 0$$

by symetric integration. Then:

$$A = \int k^2 k_\sigma \left\{ \frac{1}{[(k-\ell)^2 + a^2]^2} - \frac{1}{[k^2+a^2]^2} \right\} d^4k =$$

$$= (-2) \int d^4k \int_0^1 dz \; \frac{(\ell^2 - 2k\ell) k^2 k_\sigma \, d^4k}{[(k-\ell z)^2 + a^2 + \ell^2 z(1-z)]^3}$$

and by use of (A-11) we get:

$$A = \left[ D_2 + ( \ell^2 - 2a^2 ) D_1 + \frac{i\pi^2}{2} a^2 - \frac{11}{6} i\pi^2 \ell^2 \right] \ell_\sigma \quad (A-13)$$


1. R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193, (1958); E. C. G. Sudarshan and R. E. Marshak, Padua Venice Conference on "Mesons and Recently Discovered Particles", September 1957; J. J. Sakurai, Nuovo Cimento 8, 649 (1958).

2. Intermediate mesons of spin zero coupled to fermion fields directly or with derivative give effective scalar or pseudo-scalar interaction in disagreement with experiments.

3. J. Leite Lopes, "A model of the Universal Fermi Interaction", Nuclear Physics, 8 234, (1958).

4. The vector meson must be of very high mass in order to be unstable for the decay into the known particles (Reference 3). The high mass is also required for the non-local effects to be sufficiently small which is implied by the close agreement of the experiments with the direct four fermion interaction theory.

5. G. Feinberg, Phys. Rev. 110, 1482 (1958).

6. The branching ratio $\mu \to e + \gamma$ to $\mu \to e + \nu + \bar{\nu}$ is given by $1,5 \cdot 10^{-5}$ for a cut off the order of the mass of the vector meson.

7. For a similar decay of $\Lambda^0$ into a neutron and a photon the photon may be emitted by the intermediate charged meson or the baryon only. The decay $\Sigma^0 \to \Lambda^0 + \gamma$ should be treated differently since it conserves strangeness and is very fast.

8. The coupling constant $g$ is related to the coupling constant $G$ involved in the direct Fermi interaction written in the form $\mathcal{H}_I(x) = (G/\sqrt{2})[\bar{\psi}_a \gamma_\rho (1 + \gamma_5) \psi_b][\bar{\psi}_c \gamma_\rho (1 + \gamma_5) \psi_d] + h.c.$ by $(g^2/M_x^2) = G/\sqrt{2}$ where $M_x$ is the mass of the vector meson (see Feynman and Gell-Mann, reference 1).

9. The leptonic decay of $\Lambda^0 \to p + \mu^- + \nu$ has recently been confirmed experimentally. The theoretical branching ratio to the pion modes $\Lambda^0 \to p + \pi^-, n + \pi^0$ is $\sim 10^{-2}$. Similar leptonic modes of decay are expected for $\Sigma$ with the corresponding branching ratios of the order $\sim 10^{-2}$ (J. Leite Lopes, An. Acad. Bras. Ci. vol. 29, nº 4, 521, (1957); see also M. Gell-Mann and A. Rosenfeld, Annual Review of Nuclear Sciences, vol.7 (1957).

10. Jauch-Rohrlich - The theory of Photons and Electrons - Appendix V.

11. H. Enatsu - Progress of Theoretical Physics, Vol VI, Nº 5, p. 643, (1951).