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ENERGY SPECTRUM OF PIONS AND POLARIZATION OF MUONS AND ELECTRONS IN THE $K_{\mu 3}$ AND K_{e3} DECAY* +

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INTRODUCTION

The nature of the interactions involved in $K^+_{\mu 3}$ and K^+_{e3} decay has so far been investigated through the energy spectrum of the μ -mesons and electrons emitted.

Since these are three-body decays, more information can be obtained from the determination of the complete sharing in energy

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amongst the particles. Although the pion energy has not been measured so far, we hope that this information may become available.

On the other hand, an important consequence of the non-conservation of parity in weak interactions is that μ -mesons and electrons from $K_{\mu 3}$ and K_{e3} decay will in general be partially polarized along their direction of motion. The orientation and energy dependence of the polarization depend on the nature of the interactions.

In this paper we are concerned with a theoretical treatment of these two subjects.

I. GENERAL ASSUMPTIONS

Assuming that there exists only one K-meson having different decay modes (parity not being conserved in some of them) present evidence strongly favours the assignment of spin zero. Therefore we restrict our analysis to the case of a spinless K-meson. Our discussion of the polarization is carried out on the basis of the two-component theory of the neutrino as formulated by Yang and Lee[1]. In this theory, the spin of the neutrino is always parallel to its momentum while the spin of the anti-neutrino is always antiparallel to its momentum. This theory is equivalent to the conventional one if the neutrino field, ψ_ν , is restricted to eigenfunctions of γ_5 belonging to the eigenvalue -1, so that $\frac{1-\gamma_5}{2}\psi_\nu=\psi_\nu$. We further assume that in a definite decay process either a neutrino or an anti-neutrino is produced. The magnitude of the polarization should be the same in either case, but would have different sign.

If the results of β-decay[2,3] are interpreted according to the two component theory, one concludes that a neutrino is produced with the positron[4] while an antineutrino is produced with the electron, in accordance with the principle of conservation of leptons[5].

Applying this principle to the process:

$$\mu^+ \xrightarrow{\qquad\qquad} e^+ + \nu + \bar{\nu} \quad ,$$

one concludes that μ^+ like e^+ should be an anti-particle. Experiments on the polarization of μ-mesons and electrons from K-meson decay will provide a check on the validity of the principle of conservation of leptons. On the basis of this principle the processes should be:

$$K_{\mu 3} \xrightarrow{\qquad\qquad} \mu^+ + \nu + \pi^0$$

$$K_{e 3} \xrightarrow{\qquad\qquad} e^+ + \nu + \pi^0$$

These combinations are assumed in our analysis. All conclusions on polarization refer to them.

It should be noted that as far as energy distribution is concerned the spin state of the neutrino is irrelevant.

In principle, the only requirement upon the interactions is that they must be invariant under proper Lorentz transformations. However, we assume that the fermion pair is produced through a Fermi interaction, the whole process involving closed baryon loops of which an example is given in Fig. 1.

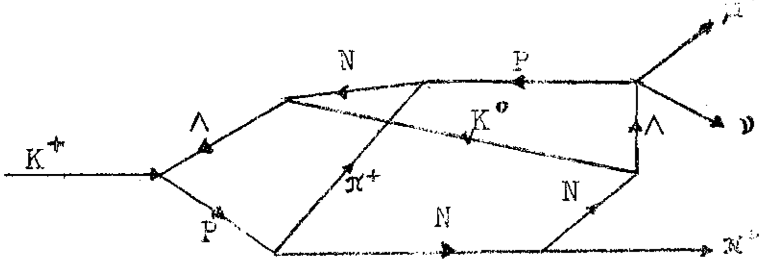

Fig. 1

Radiative corrections are negligible. Moreover, we assume that the factor in the transition matrix element arising from the internal processes can be expanded in a power series in the initial and final momenta; we retain only the leading term of this expansion for each type of Fermi interaction. This is the procedure adopted by Furuichi et al[7] in their analysis of the μ-meson and electron energy spectra.

We summarise our assumptions as follows:

I. The K-meson is a spinless particle.

II. The neutrino is described by the two-component theory of Yang and Lee, or equivalently in terms of the Dirac spinor ψ_ν satisfying the condition:

$$\frac{1 - \gamma_5}{2} \psi_\nu = \psi_\nu \qquad (1)$$

III. The decay modes of $K^+_{\mu 3}$ and K^+_{e3} are:

$$K^+_{\mu 3} \longrightarrow \mu^+ + \nu + \pi^0 \qquad (2A)$$

$$K^+_{e 3} \longrightarrow e^+ + \nu + \pi^0 \qquad (2B)$$

IV. A primary Fermi interaction is involved in the production of the fermion pair. An expansion of the matrix element in powers of the four-momenta of the decay products is assumed possible and only the leading term for each type of Fermi interaction is retained.

II. MATRIX ELEMENTS

The transition matrix elements for processes in which μ-mesons (electrons) are produced with spin parallel or anti-parallel.

to their momenta can be written as:

$$M_{\pm} = (2\pi)^4 \delta^4(p_K - p_\pi - p - p_\nu)\, \bar{\psi}_\nu\, O_j\, \psi_{\pm}\, A^j(p) \qquad (3)$$

where ψ_ν is a spinor for a neutrino with momentum p_ν and $\psi_{\pm}$ is a spinor for a muon (electron) with momentum p and spin parallel (+) or anti-parallel (-) to $\underline{p}$. The O_j's are covariant Dirac matrices and $A^j(p)$ are covariant functions of the momenta, which, taking into account hypothesis IV and the conservation laws, reduce to:

$$\text{Scalar} \quad (O_j = I) \qquad A^j = \frac{1}{M} g_S \qquad (4A)$$

$$\text{Vector} \quad (O_j = i\gamma_\alpha) \qquad A^j = -\frac{i}{M^2}\,(g_V p_K{}^\alpha + g_{V'} p_\pi{}^\alpha) \qquad (4B)$$

$$\text{Tensor} \quad (O_j = i\sigma_{\alpha\beta}) \qquad A^j = \frac{1}{M^3}\, g\, p_K^\alpha\, p_\pi^\beta \qquad (4C)$$

for the three possible types of Fermi interaction; axial vector and pseudo-scalar couplings are equivalent to vector and scalar by virtue of condition (I). The g's are dimensionless constants and M is the K-meson mass. Throughout this paper we use natural units such that $\hbar = c = 1$. The transition rate into an element of phase space is given by:

$$\begin{aligned} dT_{\pm} &= (2\pi)^4 \delta^4(p_K - p_\pi - p - p_\nu)\ \cdot \\ &\cdot \mathrm{Tr}\left[\not{p}_\nu\, O_j\, \frac{1 \pm \sigma \cdot \epsilon}{2} (\not{p} + m) \bar{O}_i\, \frac{1 - \gamma_5}{2} \right] \times \\ &\times \frac{A^j A^{i*}}{(2\pi)^9}\ \frac{d^3 p_\pi\, d^3 p\, d^3 p_\nu}{2M \cdot 2E_\pi \cdot 2E \cdot 2E_\nu} \end{aligned} \qquad (5)$$

where $\frac{1 \pm \sigma \cdot \epsilon}{2}$ is the spin projection operator for the state $\psi_{\pm}$, ϵ is a unit vector in the direction of motion of the muon (electron). As is shown in the appendix, invariance under time reversal implies that all g's must be either real or pure imaginary depending on the parity of the K-meson. In general, they may be complex. However, since the strong interactions are invariant under time reversal, it is evident from the assumed decay mechanism that $g_V\, g_{V'}^{*}$ is real.

III. ENERGY DISTRIBUTION

Integrating expression (5) over the whole of momentum space one obtains

$$T_{\pm} = \iint \mathrm{Tr.} \left[\not{p}_{\nu}\ O_j \frac{1 \pm \sigma \cdot e}{2} (\not{p} + m)\ \bar{O}_i \frac{1 - \gamma_5}{2} \right] \times$$

$$\times \ \frac{A^j A^{i*}}{(4\pi)^3} \frac{1}{M}\, dE\, dE_{\pi} \qquad (6)$$

where the integrand is a function of E and E_{π} only, because of the conservation of energy and momentum.

The total transition rate is given by:

$$T = T_{+} + T_{-} = \iint \rho(E, E_{\pi})\, dE\, dE_{\pi} \qquad (7)$$

where $\rho(E, E_{\pi})$ is the probability density for an event to take place in which the energy of the pion and muon (electron) are respectively E_{π} and E . We give below the expressions for $\rho(E, E_{\pi})$ in the K-meson rest system for each type of Fermi interaction:

$$\rho_S = 2|g_S|^2 (W_{\pi} - E_{\pi})/(4\pi)^3 M^2 \qquad (8A)$$

$$\rho_V = 2\Big[|g_V + g_{V'}|^2 [2EE_{\nu} - M(W_{\pi} - E_{\pi})]\, M +$$

$$+ |g_{V'}|^2 m^2 (W_{\pi} - E_{\pi}) - 2(g_V + g_{V'}) g_{V'}^{*} m^2 E_{\nu}\Big]/(4\pi)^3 M^4 \qquad (8B)$$

$$\rho_T = 2\,|g_T|^2\left[\left[(E-E_\nu)^2-m^2\right](W_\pi-E_\pi)M+2m^2E_\nu^2\right]/(4\pi)^3M^5 \quad (8C)$$

where $W_\pi = \frac{1}{2M}(M^3+\mu^2-m^2)$ is the maximum pion energy, μ and m are the pion and muon (electron) masses. Numerical values are given in tables 1 and 2. The numbers represent the relative probabilities of finding events inside the corresponding energy intervals. The last row and column give the frequency distribution of pions and muons (electrons). The vector interaction has been calculated for $g_{V'} = 0$. We have taken $M = 965$ m , $\mu = 264.5$ m, $m_\mu = 207$ m .

One observes very distinct behaviour for the probability densities corresponding to the various interactions. For a given μ-meson or electron energy, ρ_S decreases linearly whereas ρ_V increases linearly with pion energy. For a given pion energy, ρ_S is constant. Vector coupling suppresses probability of the electron (also, to a lesser extent, of the μ-meson) and neutrino to go off in opposite directions, hence the probability density is small near the extreme end of the electron (μ-meson) spectrum and greatest in between; just the opposite trend is evidenced by ρ_T since, due to the factor $(E - E_\nu)^2$ the probability density is large at extreme energies and small at intermediate ones.

In calculating the tables 1 and 2, we have omitted the second term of (4B) because it can be split into two terms: one of the form of the first term, the other giving a contribution of the form of a scalar interaction. Using conservation of momentum and the Dirac equation, one obtains the result that

$$\bar{\psi}_\nu \gamma_\alpha \psi p_\pi^{\ \alpha} = \bar{\psi}_\nu \gamma_\alpha \psi p_K^{\ \alpha} - m\bar{\psi}_\nu \psi \quad .$$

It is important to realise that in the vector interaction the constants g_V and $g_{V'}$ are likely to be of the same order of magnitude. The difference between ρ_V calculated for each term separately is small of order of magnitude of $\left(\frac{m}{M}\right)^2$ (cf. 8B) and therefore negligible for K_{e3} - decay; for $K_{\mu 3}$ - decay the V'-coupling favours highly energetic pions even more than V-coupling (see Fig. 2). For the most general combination of interactions, the probability density is given by

$$\rho\,(E,E_\pi) = \rho_S + \rho_V + \rho_T + 2\mathrm{Re}(\rho_{SV} + \rho_{VT} + \rho_{TS}) \tag{9}$$

where

$$\rho_{SV} = 2m\left[g_S(g_V+g_{V'})^* E_\nu - g_S g^*_{V'}(W_\pi - E_\pi)\right] / (4\pi)^3 M^3 \tag{10A}$$

$$\begin{aligned}\rho_{VT} = 2m\Big[(g_V+g_{V'})g^*_T\left[M(W_\pi - E_\pi) - E_\nu(M - E_\pi)\right] M -& \\ - g_{V'}g^*_T\left[(E - E_\nu)(W_\pi - E_\pi)M - m^2E_\nu\right]\Big] / (4\pi)^3 M^5&\end{aligned} \tag{10B}$$

$$\rho_{TS} = 2g_T g^*_S\left[(E - E_\nu)(W_\pi - E_\pi)M - m^2 E_\nu\right] / (4\pi)^3 M^4 \tag{10C}$$

The total pion spectrum shows much greater differences in the case of the K_{e3} - decay due to the fact that the electron energies are almost completely in the relativistic range. Because of this, the behaviour of the matrix elements for the different types of interaction are very different and influence the shape of the spectra decisively. On the other hand, in the $K_{\mu 3}$ case the phase space factor is more effective. Curves for the pion spectrum due to S, V, V' and T coupling and to phase space alone, are shown in Figs. 2 and 3. In general, the pion spectrum $\frac{dT}{dE_\pi} = \int \rho \, dE$ is given by[8]

$$\frac{dT}{dE_\pi} = \frac{2}{(4\pi)^3} \left\{ |g_S - \frac{m}{M} g_{V'}|^2 Q_S + |g_V + g_{V'}|^2 Q_V + |g_T|^2 Q_T + 2\mathrm{Re}\left[(g_S - \frac{m}{M} g_{V'})(g_V + g_{V'})^* Q_{SV} + (g_V + g_{V'})^* g_T Q_{VT} \right] \right\} \quad (11)$$

where

$$Q_S = (W_\pi - E_\pi)\delta E/M^2 \quad (12A)$$

$$Q_V = \left[2\bar{E}\bar{E}_\nu - \frac{1}{6}(\delta E)^2 - M(W_\pi - E_\pi) \right]\delta E/M^3 \quad (12B)$$

$$Q_T = \left\{ \left[(\bar{E} - \bar{E}_\nu)^2 + \frac{1}{3}(\delta E)^2 - m^2 \right] M(W_\pi - E_\pi) + 2m^2 \left[\bar{E}_\nu^2 + \frac{1}{12}(\delta E)^2 \right] \right\} \delta E/M^5 \quad (12C)$$

$$Q_{SV} = m\bar{E}_\nu \delta E/M^3 \quad (12D)$$

$$Q_{VT} = m\left[M(W_\pi - E_\pi) - \bar{E}_\nu (M - E_\pi) \right]\delta E/M^4 \quad (12E)$$

$$Q_{TS} = 0 \quad (12F)$$

and $\bar{E} = M - E_\pi - \bar{E}_\nu$ and $\bar{E}_\nu = M\frac{(W_\pi - E_\pi)(M - E_\pi)}{M^2 - 2ME_\pi + \mu^2}$ are the average energies of the muon (electron) and neutrino for a given pion energy and $\delta E = \frac{2M(W_\pi - E_\pi)}{M^2 - 2ME_\pi + \mu^2}(E_\pi^2 - \mu^2)^{1/2}$ is the phase space factor.

Numerical values for the expressions (12) are given in tables 3 and 4. The electron mass was neglected in these calculations.

IV. POLARIZATION OF MUONS AND ELECTRONS

An important consequence of non-conservation of parity in $K_{\mu 3}$ and K_{e3}- decay is that the muons and electrons will in general be partially polarized along their direction of motion. Polarization in other directions should arise if the motion of the

TABLE 3

Numerical values (x 10^2) for pion spectrum in $K_{\mu 3}$-decay

E_π/M	Q_S	Q_V	Q_T	Q_{SV}	Q_{VT}
.30	2.36	.29	.014	.75	-.016
.33	3.02	.54	.042	1.05	-.054
.36	3.15	.83	.074	1.22	-.105
.39	2.92	1.12	.099	1.30	-.163
.42	2.42	1.44	.114	1.28	-.224
.45	1.69	1.70	.109	1.14	-.265
.48	.815	1.65	.071	.53	-.237
.51	.032	.24	.0053	.06	-.030

TABLE 4

Numerical values (x 10^2) for pion spectrum in K_{e3}-decay

E_π/M	Q_S	Q_V	Q_T
.30	3.16	.06	.019
.33	3.94	.21	.047
.36	4.26	.42	.082
.39	4.22	.71	.115
.42	3.85	1.08	.138
.45	3.20	1.52	.143
.48	2.31	2.04	.124
.51	1.20	2.65	.077

pion is observed.

If $\frac{dT_+}{dE}$, $\frac{dT_-}{dE}$ are the transition rates to states in which the muon (electron) spin and momentum are respectively parallel and anti-parallel, the polarization is given by

$$P = \left(\frac{dT_+}{dE} - \frac{dT_-}{dE}\right) \Big/ \left(\frac{dT_+}{dE} + \frac{dT_-}{dE}\right) \qquad (13)$$

In the two component theory of the neutrino, the polarization and, in particular, its sign, depends mainly on the commutators of the Dirac operators O_j involved in the covariant product $\bar{\psi}_\nu O_j \psi$ with the operator γ_5.

Let us consider

$$\bar{\psi}_\nu O_j \psi = \bar{\psi}_\nu \frac{1+\gamma_5}{2} O_j \psi = \bar{\psi}_\nu O_j \frac{1 \pm \gamma_5}{2} \psi \quad ,$$

where the upper sign corresponds to the scalar and tensor operators which commute with γ_5 and the lower one to the vector operator which anti-commutes with γ_5. Remembering that for plane waves

$$\gamma_5 \psi = \frac{\underline{\sigma} \cdot \underline{p}}{E + \gamma_0 m} \psi \quad ,$$

so that

$$\frac{1 \pm \gamma_5}{2} \psi = \frac{1 \pm \underline{\sigma} \cdot \underline{p} / (E + \gamma_0 m)}{2} \psi \quad , \qquad (14)$$

one can see that for sufficiently large values of the energy, the matrix element M_- is small in the case of scalar or tensor coupling and M_+ in the case of vector coupling. Therefore, the polarization should be large in either case but oriented in opposite senses.

These considerations apply particularly to the electrons from K_{e3} - decay. For most of the spectrum, $E \gg m$ and the polarization is essentially equal to the velocity of the electron. In fact, for each type of coupling, the magnitude of the electron polarization (derived from (16) below) is of the form $\frac{p}{E}[1+0(\frac{m}{M})]$. Since $\frac{m}{M} = .001$, the second term is negligible and the polarization just equals the velocity of the electron even at low energies. (The same result has been obtained for the electrons in β-decay[9,10]). Therefore, for S and T or V coupling the polarization rises from zero to a value close to unity within a few MeV, and apart from a small region at low energies, is practically complete.

However, a mixture of vector with scalar and (or) tensor interactions will produce only partial polarization.

On the other hand, for $K_{\mu 3}$ - decay, since the maximum energy of the muon is only about 2 m, its mass cannot be neglected and from (14) only, one cannot draw a definite conclusion. For the scalar and vector interactions, the polarization is given by

$$P_S = Mp/(ME - m^2)$$

$$P_{V'} = - Mp/(ME - m^2)$$

$$P_V = - (M - 2E)p/[(M - 2E)E + m^2] .$$

The absolute values of P_S and $P_{V'}$ increase monotonically with energy to a value of .996 at maximum energy. For V-coupling, the polarization reaches a maximum of about 56% at $E \simeq .36\ M$ and then decreases to 21% at maximum energy. The expression for the polarization for tensor coupling is somewhat involved. The peculiar behaviour exhibited in this case (see Fig.4) can be understood by observing that

$$i\,\psi_\nu\,\sigma_{\alpha\beta}\psi\; p_K^\alpha p_\pi^\beta = \bar{\psi}_\nu\psi[p_K^\alpha(p_\alpha - p_{\nu\alpha}) - m^2] - - m\,\bar{\psi}_\nu\,\gamma_\alpha\psi\; p_\pi^\alpha . \qquad (15)$$

The first term is a scalar coupling multiplied by a factor which reduces to $[M(E - E_\nu) - m^2]$ in the K-meson rest system. This term gives a small contribution in the intermediate energy region where $\bar{E}_\nu = E - m^2/M$. (As has already been pointed out, the energy spectrum has a minimum in this region). The second term is then predominant, and, as it is of vector type V', the polarization is negative. At higher energies, as $\overline{E}_\nu$ approaches zero, the first term increases causing the polarization to become positive and large.

Curves for the polarization corresponding to these interactions are shown in Fig.4. It is interesting that at high energies the polarization is large and positive for S and T, large and negative for V'-coupling, but small for V-coupling. This can be explained in the following way. As is shown in Fig.5, the angular correlation between fast muons and neutrinos due to phase space alone is strongly peaked backwards. From conservation of angular momentum when the muons move in opposite direction to the neutrinos, they are completely polarized in their direction of motion. Therefore, the phase space factor enhances the contribution of the matrix element giving a large right-handed polarization at maximum energy in the case of S and T coupling [cf.(14)]. However, for vector coupling the matrix element and phase space factor favour opposite spin alignments. The net result is small polarization for V-coupling. For V'-coupling the matrix element strongly dominates which accounts for the large polarization.

For the most general combination of interactions $P\frac{dT}{dE} = \left(\frac{dT_+}{dE} - \frac{dT_-}{dE}\right)$ is given by an expression analogous to (11), viz:

$$P\frac{dT}{dE} = \frac{2}{(4\pi)^3}\Big\{|g_S - \frac{m}{M}g_{V'}|^2 D_S + |g_V + g_{V'}|^2 D_V + |g_T|^2 D_T + 2\mathrm{Re}\Big[(g_S - \frac{m}{M}g_{V'})(g_V + g_{V'})^* D_{SV} + (g_V + g_{V'})g_T^* D_{VT} + g_T(g_S - \frac{m}{M}g_{V'})^* D_{ST}\Big]\Big\} \qquad (16)$$

where

$$D_S = \left\{ \frac{p}{E}(W_\pi - \bar{E}_\pi)M + \frac{1}{2} m^2 \frac{\delta E_\pi}{E} \right\} \delta E_\pi / M^3 \tag{17A}$$

$$\begin{aligned} D_V = \Big\{ & - \frac{p}{E} \left[2E\bar{E}_\nu - (W_\pi - \bar{E}_\pi)M \right] + \\ & + \frac{1}{2} m^2 \frac{\delta E_\pi}{E} \Big\} \delta E_\pi / M^3 \end{aligned} \tag{17B}$$

$$\begin{aligned} D_T = \Bigg\{ & \frac{p}{E} \Big(\left[(E - \bar{E}_\nu)^2 - m^2 \right] (W_\pi - \bar{E}_\pi) M + \\ & + 2 m^2 \bar{E}_\nu^2 + \frac{1}{3} \left(\frac{\delta E_\pi}{2} \right)^2 \Big[(W_\pi - E_\pi) - \\ & - 2 \Big(E - \bar{E}_\nu - \frac{m^2}{M} \Big) \Big] M \Big) + \frac{1}{2} m^2 \frac{\delta E_\pi}{E} \cdot \\ & \cdot \Big(p^2 - 2 \bar{E}_\pi \bar{E}_\nu - \frac{1}{3} \Big[p \delta E_\pi + \bar{E}_\nu^2 - \left(\frac{\delta E_\pi}{2} \right)^2 \Big] \Big) \Bigg\} \delta E_\pi / M^5 \end{aligned} \tag{17C}$$

$$D_{SV} = m \, \delta \, E_\pi^2 / 2M^3 \tag{18A}$$

$$\begin{aligned} D_{VT} = m \Big\{ & \frac{p}{E} \left[2E\bar{E}_\nu - (W_\pi - \bar{E}_\pi)M \right] - \frac{1}{2} m^2 \frac{\delta E_\pi}{E} + \\ & + \frac{1}{2} \Big(E - \frac{4}{3} \bar{E}_\nu \Big) \delta E_\pi \Big\} \delta E_\pi / M^4 \end{aligned} \tag{18B}$$

$$\begin{aligned} D_{ST} = \Big\{ & \frac{p}{E} \Big[(E - \bar{E}_\nu)(W_\pi - \bar{E}_\pi) - \frac{1}{3} \left(\frac{\delta E_\pi}{2} \right)^2 \Big] - \\ & - \frac{2}{3} m^2 \frac{\bar{E}_\nu}{M} \frac{\delta E_\pi}{E} \Big\} \delta E_\pi / M^3 \end{aligned} \tag{18C}$$

In these expressions,

$p = (E^2 - m^2)^{1/2}$; $\bar{E}_\pi = M - E - \bar{E}_\nu$; $\bar{E}_\nu = M(M-E)(W-E) \cdot (M^2 - 2\,ME + m^2)^{-1}$; $\delta E_\pi = 2Mp(W-E)/(M^2 - 2ME + m^2)$ and $W = M - W_\pi = (M^2 + m^2 - \mu^2)/2M$ is the maximum energy of the muon (electron). The absolute values of the coefficients of $\frac{p}{E}$ in the first terms of the expressions (17) are the energy distributions of the mesons or electrons for the corresponding couplings. They have been obtained and tabulated by Furuichi et al[7].

TABLE 5

Numerical values (x 10^2) for the polarization and energy distribution of μ-mesons in $K_{\mu 3}$ - decay. The upper figures correspond to $P\,\frac{dT}{dE}$, the lower ones to $\frac{dT}{dE}$.

E/M	D_S	D_V	D_T	D_{SV}	D_{VT}	D_{TS}
.27	.43	-.23	.0035	.09	.030	-.072
	.78	.70	.0664	.66	-.210	-.220
.30	1.52	-.61	-.0123	.33	.103	-.110
	1.84	1.21	.0602	1.09	-.259	-.262
.36	2.47	-.69	-.0186	.53	.164	.063
	2.68	1.26	.0349	1.17	-.156	-.038
.42	2.65	-.43	.0468	.57	.194	.390
	2.74	.84	.0715	.91	.015	.348
.48	.149	-.006	.0224	.032	.015	.0586
	.151	.023	.0227	.039	.012	.579

The muon spectrum is also given by formula (16) provided the lower figures for the D's in the above table are used.

V. FOUR COMPONENT THEORY

The predictions of the two component theory are equivalent to a particular case ($f = 0$) of a more general interaction for which the transition matrix is proportional to:

$$g\,\bar{\psi}_\nu \frac{1+\gamma_5}{2} O_j \psi + f\,\bar{\psi}_\nu \frac{1-\gamma_5}{2} O_j \psi, \qquad (19)$$

where ψ_ν is here a four-component Dirac spinor. The transition rate to a state $\psi_\pm$ corresponding to the second term is equal to the transition rate to the state $\psi_{\mp}$ corresponding to the first. Since there is no interference between these two terms, the energy spectrum is simply obtained by adding to the expression we have given an identical expression with the g's substituted by f's , and the generalisation of (16) is

$$P \frac{dT}{dE} = \left(P \frac{dT}{dE} \right)_g + \left(P \frac{dT}{dE} \right)_f \quad , \qquad (20)$$

where on the right hand side $P \frac{dT}{dE}$ is to be calculated by means of (16) with the constants g or f as indicated.

CONCLUDING REMARKS

The following discussion is based on the validity of our assumptions I and IV. If higher powers of momenta were not neglected in (3), the correct expression for the probability density corresponding to each coupling should differ from that we have calculated only by a factor of the form $[1+f(E_\pi)]$. Therefore, the dependence on E (the muon or electron energy) is correctly given by our expressions[11]. Moreover, since the internal processes essenctially involve nucleon propagators, one might expect the magnitude of $f(E_\pi)$ to be small compared with unity and the results of our approximation not to differ appreciably from those of a more rigorous treatment.

The expressions for ρ given in (8) and (10) are linearly independent functions of E and E_π. Also, integrating with respect to either of these variables, one obtains linearly independent functions of the other. Therefore, in principle, from the $K_{\mu 3}$ -decay it is possible to determine the magnitudes of $(g_S - \frac{m}{M} g_V)$,

$(g_V+g_{V'})$ and g_T, and the relative phases of these parameters. For K_{e3} since only the interference between the scalar and tensor interactions is appreciable, one can determine only the relative phase of g_S and g_T.

If our parameters are found to be complex (note that a small relative phase is not significant), one has two alternatives for interpreting this result:

(i) Invariance under time reversal is violated.

(ii) The two component theory or the law of conservation of leptons is not valid (in both cases the number of real parameters at our disposal is doubled). With the values of our parameters determined from the energy distribution, one can use formula (16) to calculate the polarization. If the result of such a calculation is inconsistent with the experimentally observed polarization, this implies a failure of the two component theory or of the conservation of leptons (even if the parameters are real). On the other hand if there is a good agreement between the theoretical predictions and the experimental results, then complex values of the parameters implies violation of time reversal invariance.

One notes that the foregoing considerations provide a more powerful test of the two-component theory or conservation of leptons than of time reversal invariance.

The author is very glad to express his thanks to Dr. R.H. Dalitz for his stimulating assistance, to Professor R.E. Peierls for most helpful discussions and to Dr. Nina Byers for many valuable suggestions.

APPENDIX

Invariance under time reversal

Since $S = 1 - iM$ it follows from the unitarity of the S - matrix that

$$i(M_{ab} - M_{ba}^{*}) = \sum_{c} M_{ac} M_{bc}^{*} \quad .$$

Taking a and b to be (K) and (π, μ, ν) states, the order of magnitude of the right-hand side is given by the square of the coupling constant for weak interactions or else the product of such a coupling constant and e^4 (an electromagnetic interaction between μ^+ and π^o is of fourth order in e). Therefore, one can neglect it and take

$$M_{ab} = M_{ba}^{*} \tag{21}$$

Invariance under time reversal implies that

$$M_{ba} = M_{-a-b} \quad , \tag{22}$$

where $-a$, $-b$ are the time reversed states corresponding to a,b. Using the Majorama representation of the Dirac matrices, under time reversal one has

$$\underline{p} \longrightarrow -\underline{p} \quad ; \psi \longrightarrow \gamma_o \gamma_5 \psi^{*} \quad ;$$

$$\overline{\psi} \longrightarrow \overline{\psi}^{*} \gamma_5 \gamma_o \tag{23}$$

Therefore, for a particular choice of coupling, it follows that

$$M_{ba} \sim \overline{\psi}_{\nu}^{*} \gamma_5 \gamma_o O_j \gamma_o \gamma_5 \psi^{*} A^{j}(-\underline{p}) =$$
$$= \overline{\psi}_{\nu}^{*} \gamma_5 O_j \gamma_5 \psi^{*} A^{j}(\underline{p}) = \epsilon [\overline{\psi}_{\nu} \hat{O}_j \psi A^{j}(\underline{p})^{*}]^{*} \; ,$$

where $\epsilon = 1$ if the coupling is scalar or tensor and $\epsilon = -1$

if it is vector. The first step follows from parity conservation in strong interactions. The last one is a consequence of our definitions of O_j in (4) according to which, in this representation, the O_j's are all real. Therefore invariance under time reversal requires that

$$\epsilon A^j(\underline{p})^* = A^j(\underline{p}) \quad , \tag{24}$$

which implies that all g's are real. Remembering that for pseudo-scalar bosons the field operators $\phi(t)$ transform under time reversal into $-\phi^+(-t)$ while for scalar bosons $\phi(t) \longrightarrow \phi^+(-t)$ one sees that the above result is valid only for odd parity K-meson. For even parity K-meson, the above expression for M_{ba} must be multiplied by (-1) and, as a consequence, it follows that the g's must be pure imaginary.

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8. In its centre of mass system the fermion pair is produced in the following states of angular momentum and parity:

   Scalar coupling : $0^{\pm}$

   Vector coupling : $0^{\pm}$, $1^{\mp}$

   Tensor coupling : $1^{\mp}$

   This explains why the interference term Q_{TS} vanishes.

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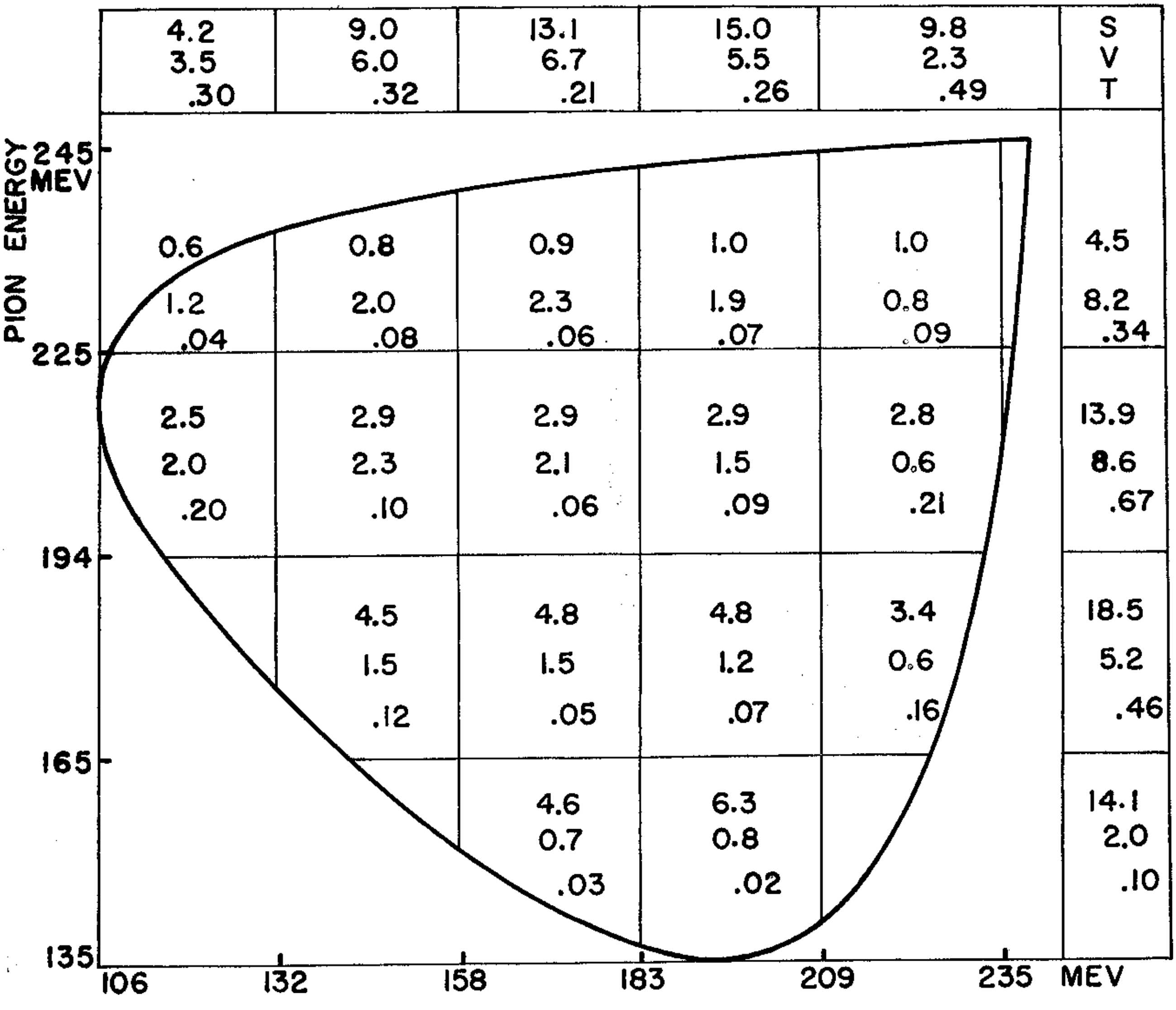
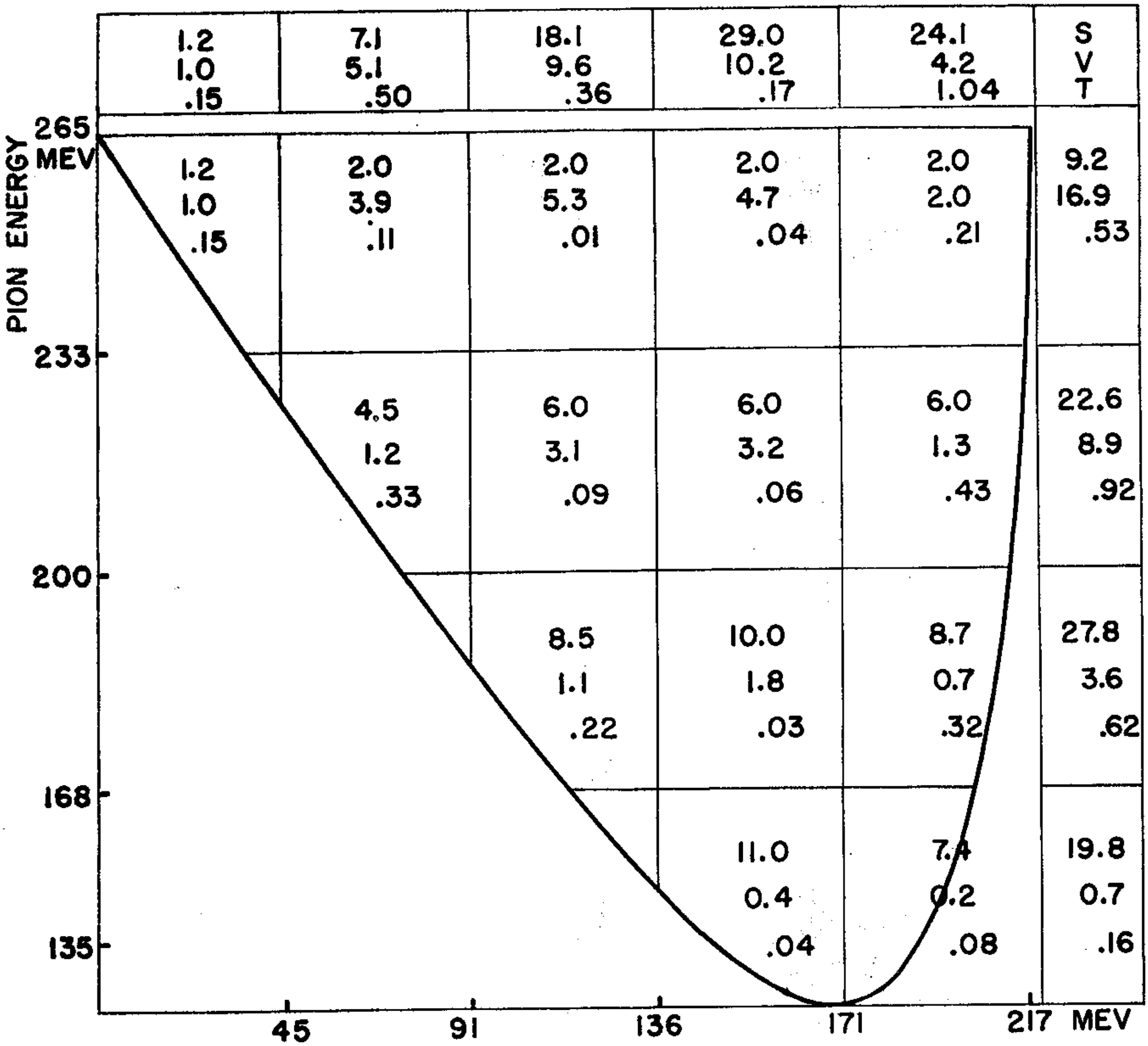


4.2 3.5 .30	9.0 6.0 .32	13.1 6.7 .21	15.0 5.5 .26	9.8 2.3 .49	S V T
0.6 1.2 .04	0.8 2.0 .08	0.9 2.3 .06	1.0 1.9 .07	1.0 0.8 .09	4.5 8.2 .34
2.5 2.0 .20	2.9 2.3 .10	2.9 2.1 .06	2.9 1.5 .09	2.8 0.6 .21	13.9 8.6 .67
	4.5 1.5 .12	4.8 1.5 .05	4.8 1.2 .07	3.4 0.6 .16	18.5 5.2 .46
		4.6 0.7 .03	6.3 0.8 .02		14.1 2.0 .10

TABLE -1-



1.2 1.0 .15	7.1 5.1 .50	18.1 9.6 .36	29.0 10.2 .17	24.1 4.2 1.04	S V T
1.2 1.0 .15	2.0 3.9 .11	2.0 5.3 .01	2.0 4.7 .04	2.0 2.0 .21	9.2 16.9 .53
	4.5 1.2 .33	6.0 3.1 .09	6.0 3.2 .06	6.0 1.3 .43	22.6 8.9 .92
		8.5 1.1 .22	10.0 1.8 .03	8.7 0.7 .32	27.8 3.6 .62
			11.0 0.4 .04	7.4 0.2 .08	19.8 0.7 .16

TABLE - 2 -

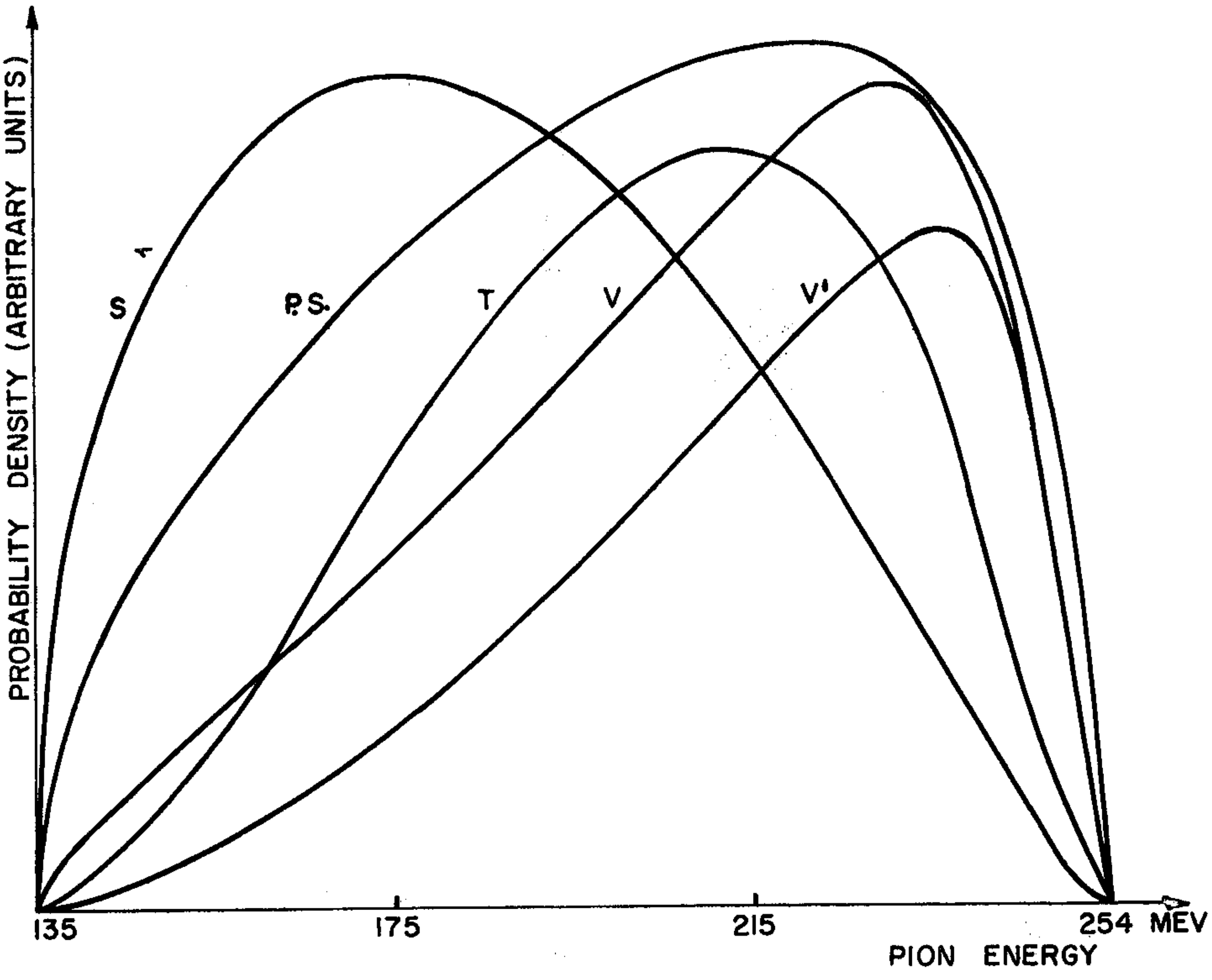


FIG. - 2 - ENERGY DISTRIBUTION OF PIONS IN $K_{\mu 3}$ - DECAY

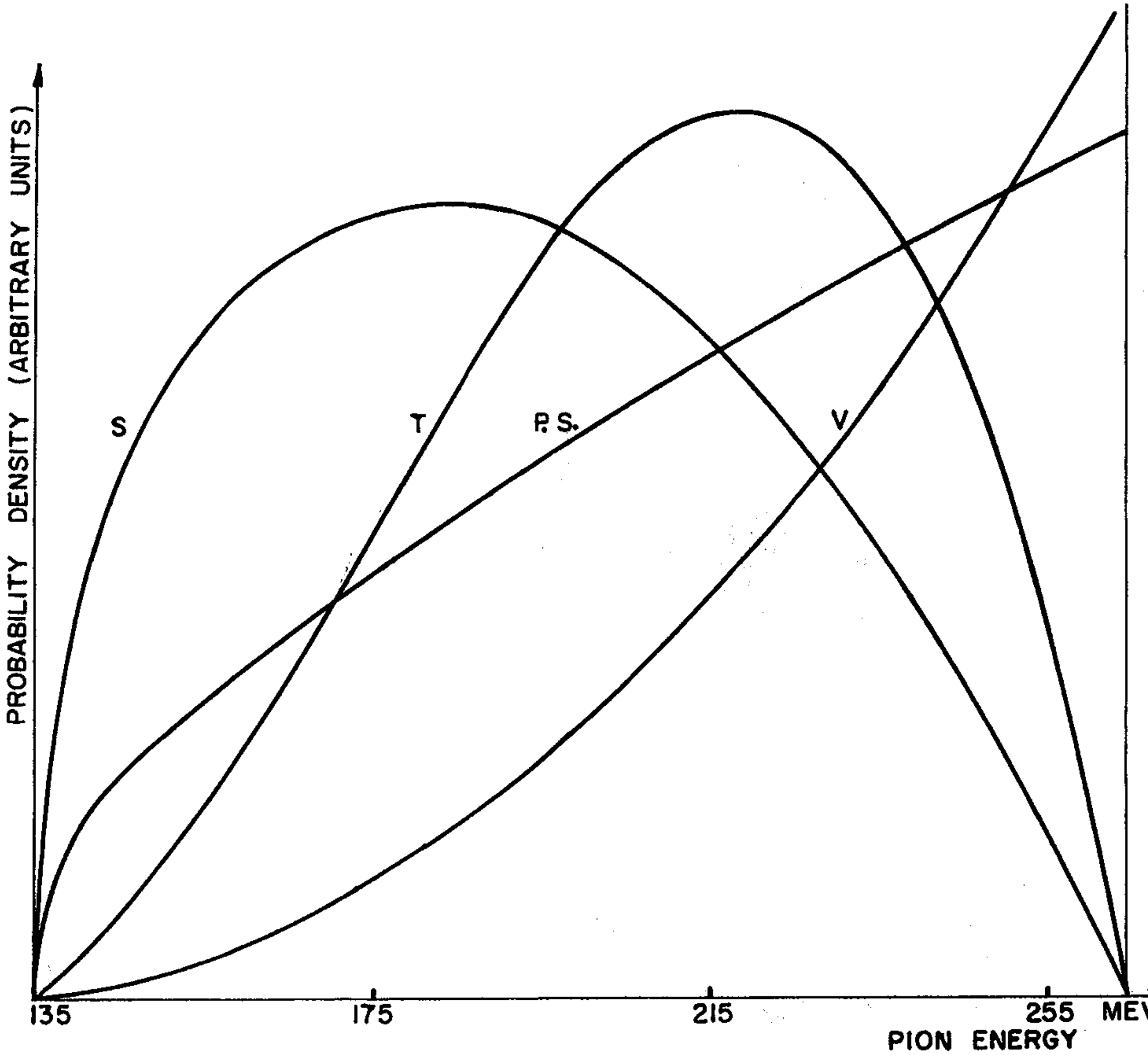


FIG.-3 - ENERGY DISTRIBUTION OF PIONS IN K_{e3} - DECAY

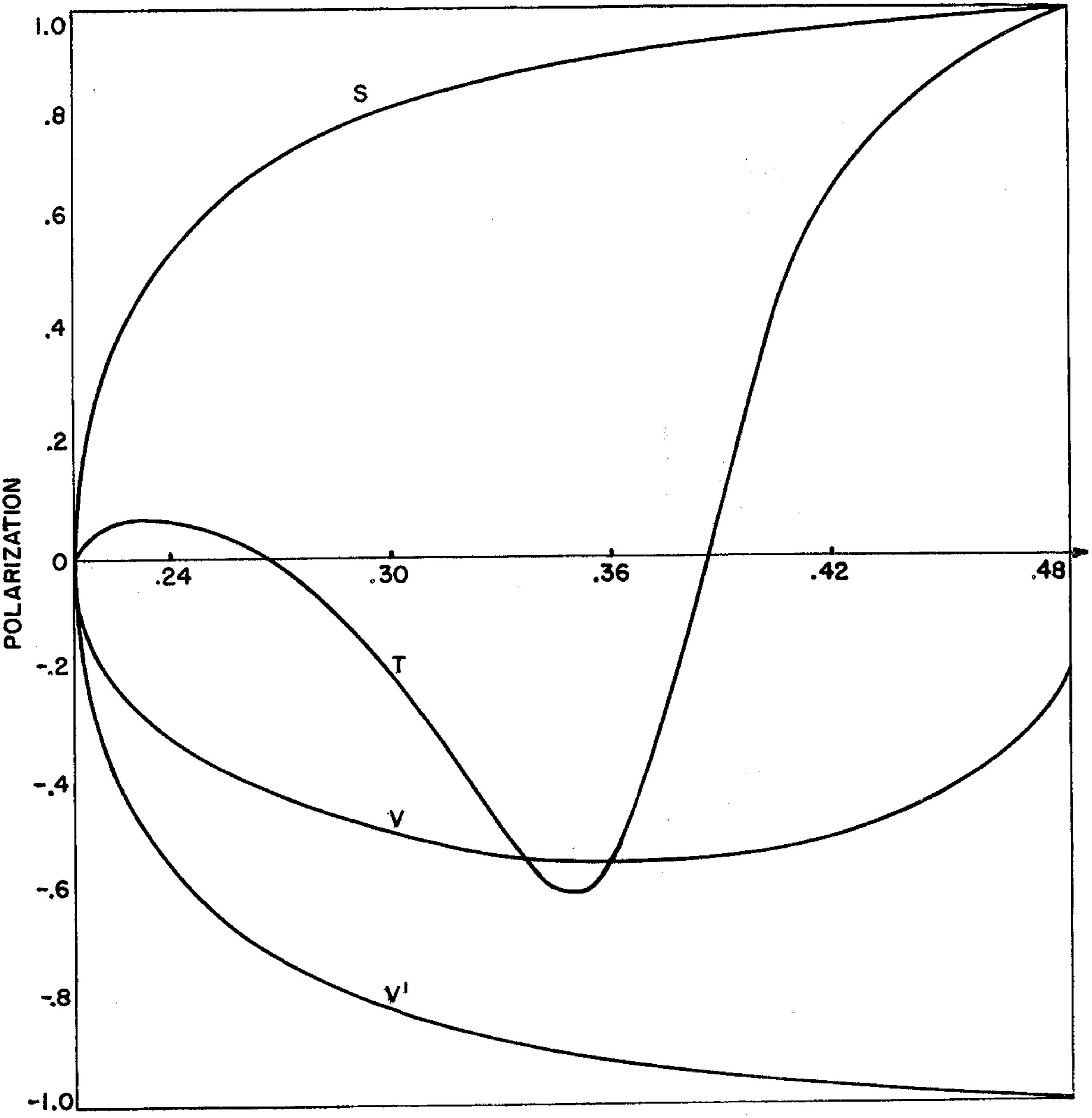


FIG. 4 – POLARIZATION OF MUONS IN $K_{\mu 3}$ – DECAY

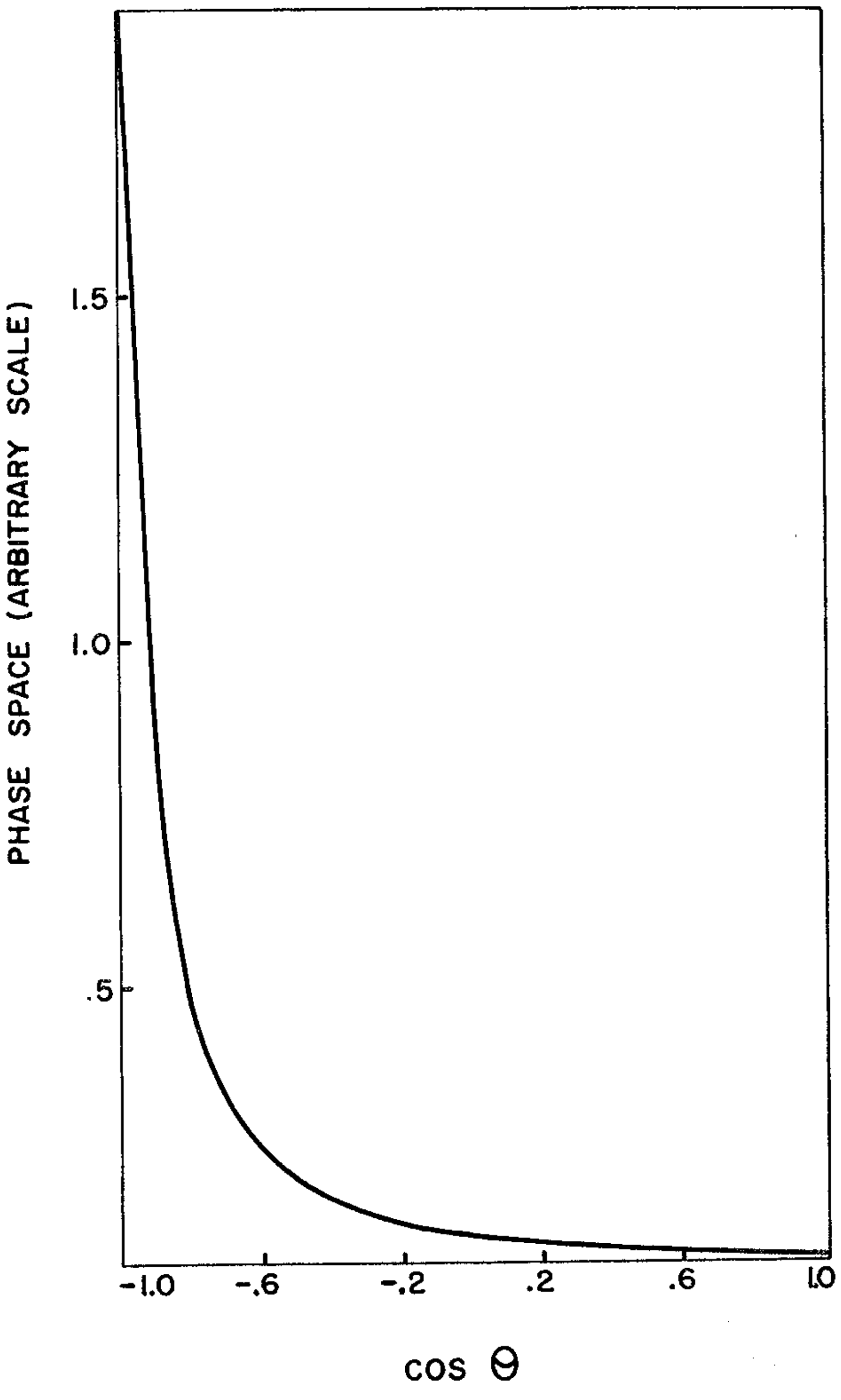


FIG. 5 ANGULAR CORRELATION BETWEEN THE DIRECTIONS OF MOTION OF FAST MUONS AND NEUTRINOS DUE TO PHASE SPACE ALONE.