

# HUBBLE DIAGRAM OF GAMMA-RAY BURSTS AND PREVALENCE OF ENERGY CONDITIONS IN FRIEDMANN-LEMAÎTRE-ROBERTSON-WALKER COSMOLOGY

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## ABSTRACT

The 1997 february 28 Beppo-SAX discovery of the X-ray afterglow of a gamma-ray burst (GRB) allowed the first precise determination of a GRB redshift. This major breakthrough came to confirm the long-standing suspicion in the high energy astrophysics community that GRBs arrive from, and their sources lie at cosmological whereabouts. Since then, the possibility of using them as actual cosmological probes has stimulated the search for self-consistent methods of bringing them into the realm of cosmology. A handful of attempts have been on trial. Since the introduction of the *Amati relation* (Amati et al. 2002), other largely promising techniques have appeared in an attempt to turn GRBs into reliable cosmological probes. These include the *Ghirlanda relation* (Ghirlanda et al. 2004), the *Liang-Zhang relation* (Liang & Zhang 2005), and the recently discovered *Firmani et al. relation* (Firmani et al. 2006a), all of which take into account the most relevant physical properties of gamma-ray bursts (GRBs) as peak energy, jet opening angle, and both time lag and variability. Such discoveries hints at the long-sought *Holy Grail* of creating a cosmic ruler from GRBs observables to be achievable. In this *Letter* we construct the Hubble diagram (HD) for the standard Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological model after enforcing it with the general relativistic energy conditions, heeding to investigate whether it still stands on as the leading scenario for cosmology in face of the distance modulus-redshift relation of a sample of GRBs that had their redshifts properly estimated and corrected upon applying on the data analysis the above quoted relations. Our  $\chi^2$  analysis support the view that FLRW plus the strong energy condition (SEC) is what better fits the GRB data. But this is not the whole story, since for a cosmological constant  $\Lambda \neq 0$  the FLRW+SEC analysis with undefined  $p = p(\rho)$  suggests that  $\Lambda$  is not constant anyhow, because it does not follows the  $p = \omega\rho$  HD, with  $\omega = -1$ . Thenceforth, one concludes that either the cosmological constant does not exist at all, and consequently one is left with the FLRW+SEC which fits properly the GRBs HD and there is no late-time acceleration, or it does exist but should be time-varying. This all is contrary to current views based on SNIa observations that advocate for an actual constant  $\Lambda$  and an accelerating universe. In connection to the results above, this last argumentation would imply that either there is something wrong with the SNIa interpretation regarding cosmic late-time acceleration (Middleditch 2006; Vishwakarma 2005; Santos, Alcaniz & Rebouças 2006) or we will have to move away from general relativity. We would better to think that general relativity is still the most correct theory of gravity.

*Subject headings:* Cosmology: standard model :: distance modulus :: redshift :: HD — gamma-ray-bursts: general — general relativity: energy conditions

## 1. INTRODUCTION: GRB STANDARDIZED CANDLES FOR ACCURATE COSMOLOGY

Gamma-ray bursts (GRBs) are the biggest explosions in the universe. The major breakthrough in understanding of GRBs came with the Beppo-SAX satellite discovery of the X-ray afterglow of GRB970228 (Costa et al. 1997). A swift follow-up of this event allows the subsequent detection of residual optical and radio emissions from this transient (van Paradijs et al. 1997; Frail et al. 1997). From these afterglows the first measurement of the redshift of a GRB was done, what definitely confirmed a long-standing suspicion that GRBs have cosmological origin. Such realization opens-up the possibility of using GRBs for a couple of fundamental researches in contemporary astrophysics and cosmology: a) as dust absorption-free tracers of the massive star formation history in the universe (Totani 1997; Paczynski 1998; Wijers et al. 1998; Blain & Natarajan 2000, Lamb & Reichart 2000; Bromm & Loeb 2002, Lloyd-Ronning & Ramirez Ruiz 2002; Lloyd-Ronning, Fryer & Ramirez Ruiz 2002; Firmani

et al. 2004; Yonetoku et al. 2004; Friedman & Bloom 2005; Schaefer 2006<sup>3</sup>), and b) as standard candles and hence as cosmic probes with which one can follow back in time upto very high redshifts the expansion history of the universe (Schaefer 2003; Ghirlanda et al. 2004; Dai, Liang & Xu 2004; Firmani et al. 2005; Liang & Zhang 2005; Xu et al. 2005).

Unfortunately, to accomplish both of these key investigations is not an easy task since both face the problem of the small statistics of events with measured readshift ( $z$ ) because, to determine  $z$ , is required deep-space optical/infrared or X-ray spectra. With regard to the item b), the large dispersion of the energetics of GRBs precludes of granting GRBs the status of cosmic rulers (Bloom, Frail & Kulkarni 2003). Nonetheless, a major effort is being pursued to overcome both the above mentioned drawbacks with significant achievements after realizing that there exist tight relations that connect GRB intrinsic energetics and/or luminosity with other physical observables (Ghirlanda et al.

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<sup>3</sup> Regarding the recent claim by Schaefer (2006) on the exclusion of the cosmological constant  $\Lambda$  after confronting it with the GRB HD, a very interesting counter-argument was provided by Friedman (2006). He argues that the correct HD should also show data calculated for a cosmology with  $\Lambda$ , and that only a comparison of relative  $\chi^2$  can determine the favored cosmology. We agree with this statement, and as we demonstrate below that condition should be supplemented with the need for the energy conditions to be satisfied, too.

2004; Liang & Zhang 2005; Firmani et al. 2006a). These relations become powerful distance indicators, and in events such supernovae (Woosley & Bloom 2006); for which  $z$  can be inferred via an energetics-independent procedure, GRBs become standard candles for practical cosmographic studies (see Firmani et al. 2006a for a full accounting of this impressive progress).

Presently, after the first series of controversial statements on the viability of granting to GRBs the status of standard cosmic rulers (Bloom 2002; Schaefer 2002, 2003; Bloom, Frail & Kulkarni 2003; Friedman & Bloom 2005), a definite consensus appears to be arising, and the hope to have a *Holy Grail* to do cosmology upon GRBs is renaissance (Ghisellini & Lazzati 2004 (GGL2004); Friedman & Bloom 2005; Mörtzell & Sollerman 2005; Schaefer 2006). GGL2004 gave to the search for a *Holy Grail* a renaissance after finding that the Amati et al. (2002) correlation becomes much tighter if one corrects  $E_{\text{iso}}$  for the collimation factor of the GRB opening angle  $\theta_j$ , which is the rest-frame time of the achromatic break in the light curve of an afterglow.<sup>4</sup> Thus the angle-corrected  $\gamma$ -energetics reads:  $E_\gamma = (1 - \cos \theta_j)E_{\text{iso}}$ . The  $E_\gamma - E_{\text{peak}}$  correlation<sup>5</sup> have been used to put constraints on some cosmological parameters avoiding the *circularity problem* (Ghirlanda et al. 2004; Firmani et al. 2005). Besides, Firmani et al. (Firmani et al. 2006a) recently discovered new tight correlation among prompt emission properties in long GRBs, which involves the isotropic peak luminosity,  $L_{\text{iso}}$ , the peak energy,  $E_{\text{peak}}$ , in the spectrum,  $\nu L_\nu$ , of the time-integrated prompt emission and the ‘‘high signal’’ time-scale,  $T_{0.45}$ , that was previously used to characterize the variability behaviour of GRBs. In the source rest-frame, this relation follows as:  $L_{\text{iso}} = \tilde{K} E_{\text{peak}}^{1.62} T_{0.45}^{-0.49}$ , with  $\tilde{K}$  a constant. It uses the GRB opening angle-corrected redshifts into  $L_{\text{iso}}$ .

Here we investigate whether the energy conditions (ECs) of general relativity (GR) when enforced on the standard cosmological FLRW model can bring it a pace with the Hubble diagram (E. P. Hubble, 1929, Proc. Nat. Acad. Sci. USA, 15, 168) constructed from the sample of GRBs that have redshifts measured through spectroscopy or other methods (like photometry). For the present analysis we shall use a set of three samples of GRBs that had their redshifts estimated: a) Bloom, Frail & Kulkarni (2003; BFK2003), as shown in Table-1 and Figure 1a). b) The Schaefer (2006) GRB set analyzed with five methods, shown in Figure 1b). c) the  $\theta_j$ -corrected GRBs sample, as from the discovery by Firmani et al. (2006a), plotted in Figure 1c).

## 2. ENERGY CONDITIONS IN GENERAL RELATIVITY

It is well-known in GR that the energy conditions (ECs) [at least seven types are currently invoked in GR (Visser 1996; Carrol & Hoffman 2003)] provide definite constraints on the physical properties of any matter field that could be present in the universe. Any degree of arbitrariness in the energy-momentum tensor  $T_{\alpha\beta}$ , or in other words; in the equation of state (EoS)<sup>6</sup> describing a given dynamical field, can be

<sup>4</sup> It has been pointed out that the presence of a collimated break in GRB afterglow light curve is a strong evidence that the GRB emission is collimated into a cone of semi-aperture angle  $\theta_j$ .

<sup>5</sup> This is a power-law relation  $E_{\text{peak}} - E_\gamma$ , where:  $E_{\text{peak}} = \tilde{\kappa} \left( \frac{E_\gamma}{E_0} \right)^\eta$ , that relates the energy at the maximum ( $E_{\text{peak}}$ ) of a given GRB with its total energy in  $\gamma$ -rays ( $E_\gamma$ ), with the constant  $E_0 = h^{-3/2}$  yielding the best-fit parameters  $\tilde{\kappa}$  and  $\eta$  independent of the value of the Hubble parameter  $h$ .

<sup>6</sup> The relation between the energy density ( $\rho$ ) content and the pressure ( $p$ ).

TABLE 1  
SELECTED SAMPLE OF 24 GRB EVENTS WITH MEASURED REDSHIFT, COMPUTED DISTANCE MODULUS AND ERROR BARS, AS TABULATED BY BLOOM, FRAIL & KULKARNI (2003).

GRB event	Redshift $z$	Dist.-Modulus $\mu(z)$	error $\pm$	$\theta_j$ -Corrected <sup>1</sup> $\bar{z}$
970508	0.8349	45.10	0.47	-
970828	0.9578	43.86	0.45	1.212
971214	3.4180	<47.83	0.37	4.801
980613	1.0969	< 47.03	0.54	-
980703	0.9662	44.31	0.45	1.104
990123	1.6004	44.00	0.53	1.358
990510	1.6187	47.13	0.42	1.210
990705	0.8424	43.98	0.43	1.109
990712	0.4331	<42.13	0.33	-
991208	0.7055	>43.35	0.86	-
991216	1.0200	44.02	0.83	0.823
000131	4.5110	>47.06	0.44	4.957
000210	0.8463	<43.90	0.29	-
000301C	2.0335	46.30	0.40	-
000418	1.1181	42.97	1.43	-
000911	1.0585	>44.00	0.46	1.173
000926	2.0369	45.93	0.92	-
010222	1.4768	45.28	0.21	-
010921	0.4509	41.71	0.62	-
011121	0.3620	<41.35	1.08	-
011211	2.1400	47.54	0.45	-
020405	0.6899	43.99	0.67	-
020813	1.2540	45.00	1.07	-
021004	2.3320	46.48	1.80	-

<sup>1</sup> Redshifts in this column were corrected by using the *Firmani et al. relation*

thoroughly restricted by imposing on its dynamics the ECs. As a rule of practice in GR, such conditions are stated in a coordinate-invariant fashion by considering the timelike, null-like or spacelike nature of the proper  $T_{\alpha\beta} = \text{diag}(\rho, p, p, p)$  and any vector field  $A_\gamma$  that may exist. However, it is pertinent to this discussion to point out that despite being physically sound, the ECs were not always universally accepted and issues regarding how fundamental, as a whole, the ECs are have been since long a matter of concern. Indeed, by the late 1970’s researchers realize that not any matter field accomplish with the ECs. A classical outlier is the Einstein cosmological constant  $\Lambda$  which, *if being indeed a constant with  $\omega = -1$  in the EoS*, violates the strong energy condition (SEC) while keeps on fulfilling the WEC. Other examples are the *Casimir effect* that violates the WEC and DEC, *Hawking evaporation* which violates the NEC and *cosmological anti-de Sitter* ( $\Lambda < 0$ ) *inflation* which violates WEC and DEC (Visser 1996).

The ECs of relevance for the present analysis can be expressed as follows:<sup>7</sup>

<sup>7</sup> We notice that recently Santos, Alcaniz & Rebouças (2006) have shown that the GR ECs are relevant to constraint general energy-momentum tensors (or equations of state) of current use in most cosmological models. It was shown that in the case of the FLRW scenario, ECs provide model-independent bounds on the distance-modulus of cosmic sources with dependence on the redshift. Data from SNIa were analyzed and the question regarding a possible violation of the ECs in FLRW model was addressed. They conclude that SNIa seems to violate all the ECs. Our analysis below in some sense come across with this conclusion. In similar lines, Pérez Bergliaffa (2006) has demonstrated that the WEC is a very good tool to constraint theories of gravity of the kind  $f(R)$ .

- Null energy condition (NEC): the quantity  $T_{\alpha\beta}\mathcal{N}^\alpha\mathcal{N}^\beta \geq 0$  for any null vector  $\mathcal{N}^\gamma \exists T_a(M)$  lying at the point  $a$  in the tangent space  $T(M)$  of a real four-dimensional manifold  $M$ . Physically, it means that the *local* energy density as measured by any timelike observer has to be positive.

- Weak energy condition (WEC): the inequality  $T_{\alpha\beta}T^\alpha T^\beta \geq 0$  holds for all timelike vector  $T^\gamma \exists T_a(M)$ . By continuity it implies the NEC bound  $\rho + p \geq 0$ . It follows that

$$\text{WEC} \implies \rho \geq 0 \quad \text{and} \quad \rho + p \geq 0. \quad (1)$$

- Dominant energy condition (DEC): the relation  $T_{\alpha\beta}T^\alpha T^\beta \geq 0$  holds for all timelike vector  $T^\gamma \exists T_a(M)$  whenever it is supplemented by demanding that  $T_{\alpha\beta}T^\alpha$  be a non-spacelike vector. It states in physical terms the positivity of the *local* energy density, and that the energy flux is time-like or null. It implies the WEC and by continuity the NEC. It follows that

$$\text{DEC} \implies \rho \geq 0 \quad \text{and} \quad -\rho \leq p \leq \rho. \quad (2)$$

- Strong energy condition (SEC): the quantity  $(T_{\alpha\beta} - \frac{1}{2}Tg_{\alpha\beta})T^\alpha T^\beta \geq 0$ , should be held for any timelike vector  $T^\gamma$ , being  $T$  the trace of the tensor  $T_{\alpha\beta}$ . It follows that

$$\text{SEC} \implies \rho + 3p \geq 0 \quad \text{and} \quad \rho + p \geq 0. \quad (3)$$

Next we impose the ECs to the FLRW model to build the HD they predict so as to compare it with the one constructed upon the GRB data. We will check for any putative violation of ECs by the observed GRBs. The best fit to the GRB  $\mu(z)$  vs.  $z$  relation (HD) will select the cosmological model one must seriously should take for cosmography studies.

### 3. FLRW COSMOLOGICAL MODEL: DISTANCE MODULUS VS. REDSHIFT RELATION

Relativistic cosmology is founded on three pillars known as:

a) *The Cosmological Principle*, which states that the universe is homogeneous and isotropic at extremely large scales. This leads to the FLRW line element [with  $c = 1$ , scale factor  $R(t)$ , curvature  $k = 0, \pm 1$  and metric signature  $(-, +, +, +)$ ] (D'Inverno 1993)

$$ds^2 = -dt^2 + R^2(t) \left( \frac{dr^2}{1-kr^2} + r^2[d\theta^2 + \sin^2\theta d\phi^2] \right). \quad (4)$$

b) *The Weyl's Postulate*, which demands that the universe's substratum be a perfect fluid, namely:

$$T_{\alpha\beta} = (\rho + p)U_\alpha U_\beta + pg_{\alpha\beta}, \quad (5)$$

where  $U_\gamma$  is the fluid 4-velocity.

c) *The General Theory of Relativity*, that is; it should be described by Einstein's equations ( $G = 1$ ,  $\kappa_G = 8\pi$ )

$$G_{\alpha\beta} - \Lambda g_{\alpha\beta} = \kappa_G T_{\alpha\beta}. \quad (6)$$

Thus in a preferred coordinate frame the couple of independent (Friedmann's) equations reads

$$3 \frac{\dot{R}^2 + k}{R^2} - \Lambda = 8\pi\rho \quad \therefore \quad \dot{R} = \frac{dR}{dt}, \quad (7)$$

$$\frac{2R\ddot{R} + \dot{R}^2 + k}{R^2} - \Lambda = -8\pi p \quad \therefore \quad \ddot{R} = \frac{d\dot{R}}{dt}, \quad (8)$$

where  $\dot{R}$  and  $\ddot{R}$  are the expansion velocity and acceleration of the universe. Putting the cosmological constant  $\Lambda = 0$ , from Eqs.(7, 8) one can recast the ECs (1,2,3) as dynamical constraints on the scale factor ( $R$ ) and its derivatives ( $\dot{R}$ ,  $\ddot{R}$ ) whichever the curvature ( $k$ ) come to be. The resulting set of equations reads

$$\text{WEC} \implies -\frac{\ddot{R}}{R} + \left[ \frac{\dot{R}^2 + k}{R^2} \right] \geq 0 \implies \dot{R} \geq H_0 R \Big|_{k=0}, \quad \forall R < R_0, \quad (9)$$

$$\text{DEC} \implies -2 \left( \frac{\dot{R}^2 + k}{R^2} \right) \leq \frac{\ddot{R}}{R} \leq \left( \frac{\dot{R}^2 + k}{R^2} \right) \quad (10)$$

$$\implies \dot{R} \leq H_0 \left[ \frac{R_0^3}{R^2} \right] \Big|_{k=0} \quad \text{and} \quad \dot{R} \geq H_0 R \Big|_{k=0}, \quad \forall R < R_0,$$

$$\text{SEC} \implies \frac{\ddot{R}}{R} \leq 0 \implies \dot{R} \geq H_0 R_0, \quad \forall R < R_0. \quad (11)$$

Notice that physically the SEC means that in a FLRW universe the expansion is always decelerating whatever the curvature  $k$  is. This also assures the pace with the attractiveness of gravity. We have also used the cosmological definition of the redshift  $\frac{R_0}{R} = 1 + z$ . Above  $R_0$ , and  $H_0 = \frac{\dot{R}(t_0)}{R(t_0)}$ , are respectively the scale factor and Hubble parameter presently (subscript 0).

#### 3.1. Distance-modulus vs. redshift relation in FLRW

For an outlying source of apparent  $m$  and absolute  $M$  magnitudes, distance estimates are made through the *distance-modulus*  $m - M$ , which relates to the luminosity distance (given in units of Gpc below)

$$d_L = R_0(1+z) \int_R^{R_0} \frac{dR}{R\dot{R}}, \quad (12)$$

through

$$\mu(z) \equiv m - M = 5 \log_{10} d_L(z) + 25. \quad (13)$$

By assuming a flat universe, one can obtain model-independent bounds on the distance-modulus  $\mu(z)$  of a further-lying GRB source as a function of the redshift  $z$  for each of the ECs enforced on an expanding FLRW universe model. Hence, whenever any EC gets obeyed its corresponding distance indicator should take values not larger than those allowed by the constraints given by Eqs.(9, 10, 11).

##### 3.1.1. ECs + FLRW $\mu(z)$ - $z$ relations

After integrating the  $d_L$  of Eq.(12) for each of the ECs dynamical relations provided in Eqs.(9,10,11) one obtains the distance-modulus vs. redshift relations as follows:

WEC + FLRW Hubble diagram

$$\mu(z) - z \implies \left\{ \mu(z) \leq 5 \log_{10} \left[ H_0^{-1} z(1+z) \right] + 25 \right.$$

DEC + FLRW Hubble diagram

$$\mu(z) - z \implies \left\{ \begin{array}{l} \mu(z) \geq 5 \log_{10} \left[ \frac{H_0^{-1} z(2+z)}{2(1+z)} \right] + 25 \quad (\text{DEC1}) \\ \mu(z) \leq 5 \log_{10} \left[ H_0^{-1} z(1+z) \right] + 25 \quad (\text{DEC2}). \end{array} \right.$$

SEC + FLRW Hubble diagram

$$\mu(z) - z \implies \left\{ \mu(z) \leq 5 \log_{10} \left[ H_0^{-1} (1+z) \ln(1+z) \right] + 25. \right.$$

Next we use the values  $H_0 = 1/3h$  Gpc<sup>-1</sup>, with  $h = 0.73$ , to discuss the implications of these relations to the FLRW cosmology.

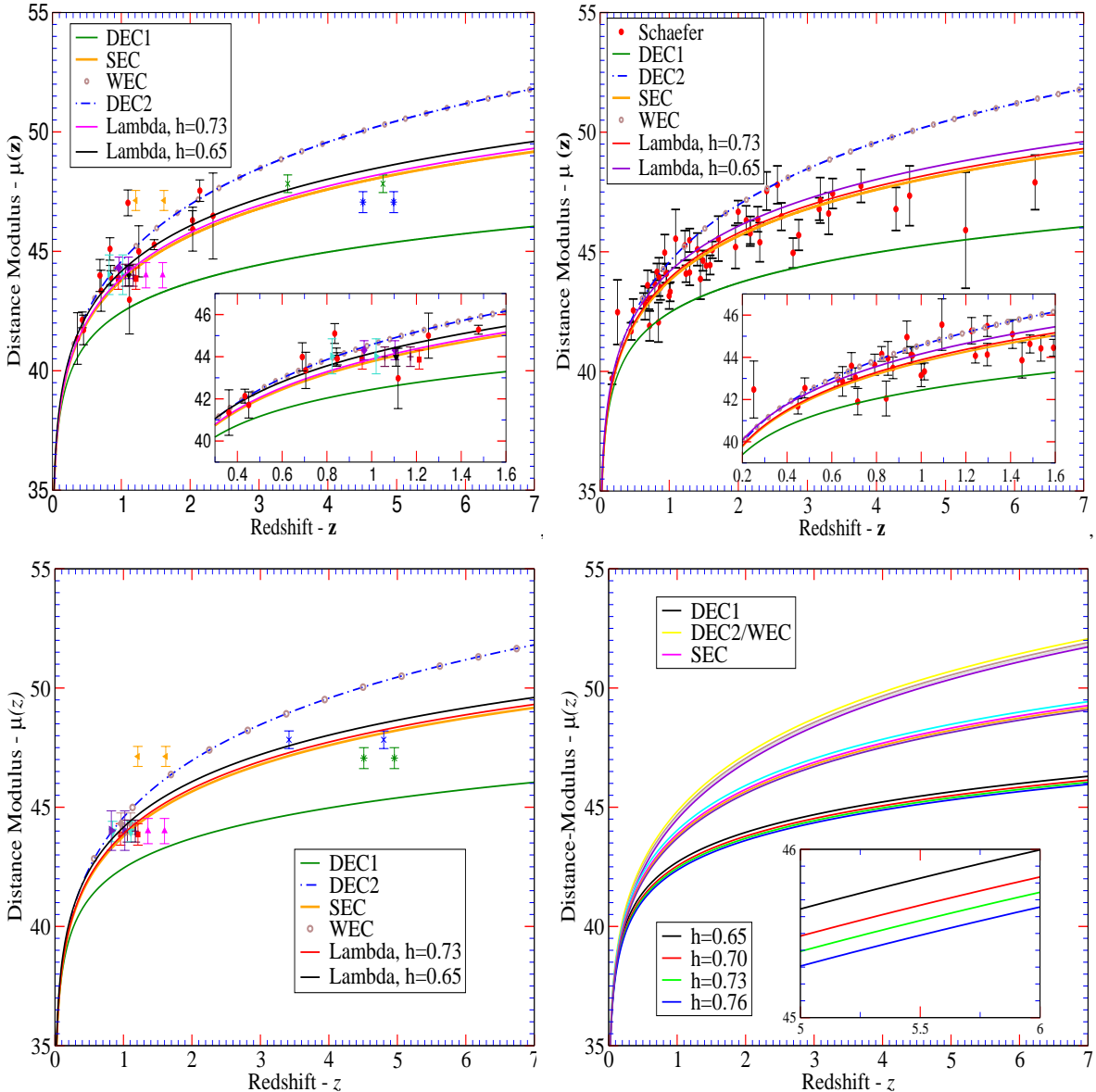


FIG. 1.— COLOR ONLINE - HDs for the samples used in this analysis: a) The 24 GRBs listed in Table-1 [Data taken from Bloom, Frail & Kulkarni (2003)]. Notice the pairs of events with identical colors indicating those GRBs that had their  $z$   $\theta$ -corrected with the Firmani et al. relation. b) The Schaefer's (2006) 52 GRBs events, which were analyzed by 5 different methods [Data taken from Schaefer presentation at the AAS Meeting (January 2006)]. c) The 9 GRBs listed in the last column of Table-1 that had their redshift corrected (shifted - the same symbol/color pairs) by using the Firmani et al. relation [Data taken from Firmani et al. (2006a)]. d) Behavior of the  $\mu(z)$  vs.  $z$  relation as a function of the Hubble parameter  $h$  for values as indicated. No relevant changes ( $\leq 1\%$ ) are observed for the interval depicted.

#### 4. DISCUSSION AND CONCLUSIONS

In Figures 1a), 1b) and 1c), we present the main results of this investigation. First, after looking at Figures 1a) and 1b) one can verify that the upper DEC bound (dash-dotted line) matches perfectly the WEC  $\mu(z)$  prediction (open circle line). Besides, one can also realize that both the DEC upper and lower bounds are respected by the GRBs observations, that is, the bulk of the bursts that have been observed has its representative point in the FLRW HD inside the limits of DEC1 and DEC2 in Figures 1a), 1b) [the outliers may indicate systematic errors, as is also observed in the case of some SNIa]. Besides, there is no clear violation of either the WEC or the SEC. This results go almost contrary to conclusions achieved by Santos, Alcaniz & Rebouças (2006); who showed, after confronting the ECs

with SNIa observations, that practically each of the ECs seems to have been violated in the recent past history of the cosmic evolution. We argue that as GRBs are almost free of any effect as absorption or the like, one is inclined to conclude that perhaps the astrophysics of SNIa has not been exhaustively understood (Middleditch 2006), and hence the interpretation of luminosity-dimmed SNIa as messengers of late-time acceleration should decidedly be rethought, perhaps in the lines of Middleditch (2006).

Further, it is quite easy to see that the SEC prediction seems to fit more properly the GRBs HD than any another of the ECs+FLRW. This is quite a surprising result. It seems to indicate, contrarily to most of the current expectations (see Firmani et al. 2006b), that the cosmological constant  $\Lambda$  (i.e. EoS with  $\omega = -1$ ) is not the driving force pushing out the

universe in the late times since it “naturally” violates the SEC, as pointed out earlier. In fact, looking at Figure 1b), where Schaefer’s (2006) 52 GRBs are plotted, this behavior seems to extend down to lower redshifts  $z \sim 0.1 - 0.5$ . Thus our ECs+FLRW plus GRB HD seem to exclude any need for dark energy.

To bring our results into consistency, we have performed a  $\chi^2$ -square analysis to compare the distance-modulus vs. redshift relation as predicted by each of the ECs+FLRW cosmological scenarios with the HD obtained from GRBs observations. Here we define

$$\chi^2 = \sum_i \frac{[\mu_i^0(z) - \mu_i^t(z)]^2}{\sigma_{\mu_i^0}^2},$$

where respectively  $\mu_i^0(z)$  and  $\mu_i^t(z)$  define the measured and calculated (theoretical) value for the distance-modulus, and  $\sigma_{\mu_i^0}$  represents the dispersion in the  $\mu_i^0(z)$  (see Table 1). Despite the  $\mu(z)$  uncertainties, the data clearly selects the SEC+FLRW as the best fit. Our calculations for the 24 GRBs from BFK2003 show that the average  $\chi^2$  (per event) for the WEC 13.32, for the SEC 4.864, and for the DEC 26.47. For the 52 Schaefer (2006) GRBs the  $\chi^2$  figures go as follows: for the WEC 12.17, for the SEC 1.36, and for the DEC 10.78. This therefore confirms what a visual inspection of Figures 1a) and 1b) allows to conclude. Hence, as the SEC implies that the universe is all the time decelerating irrespective of the type of curvature it may have, then one is forced to conclude that the cosmological constant is playing no role at all in the universe dynamics since it naturally violates the SEC. Nor it is being the fundamental field driving the purported universe late-time acceleration since there seems to be not such a transition down redshifts  $z \sim 0.5 - 1.0$ . This in passing would imply that there is no need for the own dark energy field, as it appears to be ruled out by the GRBs HD. Besides, DE should comply with the ECs, and they (in particular the SEC) show that such a transition-to-acceleration at low redshifts seems to have not occurred.

Meanwhile, to elucidate any undisputable role of the cosmological constant we now impose the SEC  $\rho + 3p \geq 0$  bound on the Friedmann’s equations with  $\Lambda \neq 0$  but unspecified  $p = p(\rho)$  EoS<sup>8</sup>, one obtains the dynamics (for  $k = 0$ )

$$\frac{\ddot{R}}{R} \leq \frac{\Lambda}{3} \implies \left( \frac{\dot{R}}{R_0} \right)^2 \geq H_0^2 - \frac{\Lambda}{3} \left[ \frac{z(z+2)}{(1+z)^2} \right]. \quad (14)$$

(One can verify that for  $\Lambda = 0$  one recovers Eq.(11), above). Thus, the luminosity-distance becomes

$$d_L \leq \frac{(1+z)}{\left( H_0^2 - \frac{\Lambda}{3} \left[ \frac{z(z+2)}{(1+z)^2} \right] \right)^{1/2}} \ln(1+z), \quad (15)$$

and therefore, the  $\mu(z)$ - $z$  HD reads:

$$\text{SEC + FLRW + } \Lambda \text{ Hubble diagram} \\ \mu(z) - z \implies \left\{ \mu(z) \leq 5 \log_{10} \left( \frac{(1+z) \ln(1+z)}{\left( H_0^2 - \frac{\Lambda}{3} \left[ \frac{z(z+2)}{(1+z)^2} \right] \right)^{1/2}} \right) + 25. \right. \quad (16)$$

Putting  $\Lambda = 2.036 \times 10^{-35} \text{ s}^{-2} = 0.214 \text{ Gpc}^{-2}$ , and  $h = 0.73$  one verifies that according to Figures 1a), 1b) and 1c) it gets close to the SEC limit of Eqs.(7,8) without  $\Lambda$ . The  $\chi^2$  analysis for this case gives 1.33 for the 52 GRBs and 4.36 for the 24 GRBs. It is a bit better than the SEC limit alone. However, the error bars do not allow to claim that there is any actual difference between the  $\chi^2$  value for SEC+FLRW+ $\Lambda$  and the sole for SEC+FLRW. Thus, this result can be interpreted in the lines of our discussion above as either indicating that there is no need for  $\Lambda$ , or that  $\Lambda$  is time-variable. As the HD for the case  $\Lambda \neq 0$  plus SEC+FLRW does not coincide with the one corresponding to the case  $p = -\rho$  (footnote 8) then one is led to conclude that the cosmo-(i)logical constant does not exist at all or it is not constant either. Figures 1 illustrate the behavior of a  $\Lambda$ +SEC+FLRW for different values of the Hubble parameter  $h$ .

In summary, despite the relatively large uncertainties still present in the luminosity distance of most GRBs and the small statistics of GRB with low redshifts, the approach introduced here appears to be more conclusive than an analysis standing on SNIa data alone. The main reason is that the sample of GRBs that we are using for the present analysis already reaches farther out, upto redshifts  $z \sim 7$  where the early-days cosmology was playing hard, while SNIa are naturally limited to distance scales no farther out than  $z \sim 1$ . (Surely, the set of both lower (about  $z \sim 0.1 - 0.5$ ), and larger (upto  $z \sim 10 - 20$ ) redshifts will raise in the near future when more GRBs are to be observed by the SWIFT satellite). On physical grounds, the outcome of this inedit test suggests that the FLRW model when faced with the GRB HD guarantees the prevalence of the WEC, SEC and DEC bounds, with the SEC being the better cosmological fit to the GRBs observational data. As SNIa are facing problems to go a pace with the ECs+FLRW, we think that it is the SNIa astrophysics that should be reviewed instead of general relativity (Middleditch 2006).

To the last, the GRBs HD has been built to demonstrate the reliability of GRBs as an independent, but nonetheless complementary tool to SNIa (and perhaps to CMB observations), to confront current cosmological models. Thus, everything now allows to foresee a promising future for a high-precision era in GRBs cosmology, a field which is by-now at its infancy (Lazzati et al. 2006), based on these novel relations that promise to become a major breakthrough in GRBs cosmology. This is the first paper of a thorough research of this key issue in current Cosmology. In forthcoming papers other cosmological models, specially those purporting modifications of gravity at very larger scales, will be analyzed in face of GRBs data to cross-check them for consistency with the HD of GRBs.

<sup>8</sup> Notice that DE scenarios with  $\Lambda$  constant! have EoS  $p = \omega\rho$ , with  $\omega = -1$ . This model clearly violates the SEC bound since  $\text{SEC} \rightarrow p \geq -1/3\rho$ . However, it provides a dynamics:  $\dot{R} \geq H_0 R$ , that follows the WEC bound

given by the top curves of HDs in Figure 1.

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