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THE TWELVE COLOURFUL STONES*

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*Revised version.

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Abstract

The gauge symmetry is extended. It is associated differents matter and gauge fields to the same group. A group of gauge invariant Lagrangians is established. A gauge invariant mass term is introduced. A massive Yang Mills is obtained. A dynamics with twelve colourful stones is created based on the concepts of gauge and colour. Structures identified as quarks and leptons are generated. A discussion about colour meaning is presented.

Key-word: Colour.

INTRODUCTION

It seems that a major consequence of the development of quark physics was the appearance of the colour concept. It is based on experimental results.

There are many indications for three colours. At low and high energies appear indications of colour. The low energy process $\pi^0 \to 2\gamma$ which has been precisely measured, would be predicted too slow with respect to measurement but colour effect saves the situation for the number of colours $N_c = 3$. Then the colour symmetry was added to isospin. The high energy processes concern the rate of hadron production in electron-positron colliders, which again would require the theory to have the colour degree of freedom with $N_c = 3$ to match the data. Cancellation of the triangle anomaly in the lepton-quark sector also requires $N_c = 3$. The observed value for the branching ratios in the τ-decay are in good agreement with $N_c = 3$. To satisfy the Fermi statistics is another motivation for colour. Still another type of manifestation of pure colour, even without the accompanying quarks, is the glueballs.

The consequence is that there is a new fundamental parameter to analyse the nature process with. Such dynamics has previously been built in terms of phenomenological entities like space, time, mass and charge. Yet, the surprising fact is that while this new parameter influences experimental results, it cannot be observed. We thus suppose that the galilean physics where all the parameters, independently of theory, are directly measured, becomes argued.

The idea that we are motivated to develop is that colour and gauge concepts are enough to build up a dynamics. A Lagrangian based only on it, as massless QCD, can predict properties. Therefore the first attitude of this work is to understand the consequences in manipulating elementary particles made by colour only. These particles will be called colourful stones. They will appear as the most primitive way to create dynamics. We would say that it is a big-bang made in terms of a colourful verb. Another aspect is by noticing that just quarks and leptons are performing the present physics scenario. Thus these twelve stones will appear as building blocks for this particles. The concept of unity would be comming from behind.

The paper contain two parts. In part I, gauge symmetry is extended for abelian and non-abelian cases. The method is to include different gauge and matter fields in the same group. A mass term for intermediate particles based on gauge invariance is introduced. In part II, a colourful model in terms of twelve colourful stones is organized. A context for this insight is presented in the conclusion. A model to generate asymptotic freedom and confinement is also proposed.

PART I: EXTENDED GAUGE SYMMETRY

Historically, the symmetry principles have proved to be very powerful in summarizing the properties of the laws of nature. Gauge symmetry principle is based in the matter field "rotation". Particles, may fall into multiplets that seem to reflect underlying internal symmetries. These internal aspects are related through group theory. They are not necessarily connected with fields nature. The motivation here is to extend the gauge concept for cases where different matter and gauge fields are included in the same group. Compare with four-vector and four-momentum. They are different entities but rotate under the same Lorentz group.

1. ABELIAN CASE

a) One matter field and two gauge fields in the same group.

Consider two different gauge fields A_μ and B_ν associated to the same matter field. There are two possibilities to rotate the matter field,

$$\psi(x) \rightarrow e^{i\alpha(x)}\,\psi(x) \tag{1}$$

and

$$\psi(x) \rightarrow e^{ig\alpha(x)}\,\psi(x) \tag{2}$$

Associating the covariant derivatives,

$$D_\mu(A) = \partial_\mu + ig_A A_\mu \tag{3}$$

$$D_\nu(B) = \partial_\nu + ig_B B_\nu \tag{4}$$

yields for (1),

$$A'_\mu = A_\mu - \frac{1}{g_A}\partial_\mu\alpha \quad ; \quad B'_\mu = B_\mu - \frac{1}{g_B}\partial_\mu\alpha \tag{5}$$

and
$$g_A A'_\mu - g_B B'_\mu = g_A A_\mu - g_B B_\mu \tag{6}$$

for (2),

$$A'_\mu = A_\mu - \frac{g}{g_A}\partial_\mu\alpha \quad ; \quad B'_\mu = B_\mu - \frac{g}{g_B}\partial_\mu\alpha \tag{7}$$

and
$$\frac{g_A}{g} A'_\mu - \frac{g_B}{g} B'_\mu = \frac{g_A}{g} A_\mu - \frac{g_B}{g} B_\mu \tag{8}$$

The following covariant tensors are obtained from (5) and (7),

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{9}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{10}$$

$$G_{\mu\nu} = \partial_\mu A_\nu - c_1 \partial_\nu B_\mu \tag{11}$$

$$H_{\mu\nu} = \partial_\mu B_\nu - c_2 \partial_\nu A_\mu \tag{12}$$

Where c_1 and c_2 are constants depending on g, g_A and g_B. They depend on (5) or (7). Gauge invariant Lagrangians will be build up through combinations of the above tensors. If two tensors X and Y transform covariantly the addition and product will also remain

$$X \pm Y \rightarrow U(X \pm Y)U^{-1}$$

$$XY \rightarrow U(XY)U^{-1} \tag{13}$$

As example consider the cases,

$$\mathcal{L}_A = a_1 A_{\mu\nu}A^{\mu\nu} + a_2 A_{\mu\nu}B^{\mu\nu} + a_3 A_{\mu\nu}G^{\mu\nu} + a_4 A_{\mu\nu}H^{\mu\nu} \tag{14}$$

$$\mathcal{L}_B = b_1 B_{\mu\nu}A^{\mu\nu} + b_2 B_{\mu\nu}B^{\mu\nu} + b_3 B_{\mu\nu}G^{\mu\nu} + b_4 B_{\mu\nu}H^{\mu\nu} \tag{15}$$

$$\mathcal{L}_G = g_1 G_{\mu\nu}A^{\mu\nu} + g_2 G_{\mu\nu}B^{\mu\nu} + g_3 G_{\mu\nu}G^{\mu\nu} + g_4 G_{\mu\nu}H^{\mu\nu} \tag{16}$$

$$\mathcal{L}_H = h_1 H_{\mu\nu}A^{\mu\nu} + h_2 H_{\mu\nu}B^{\mu\nu} + h_3 H_{\mu\nu}G^{\mu\nu} + h_4 H_{\mu\nu}H^{\mu\nu} \tag{17}$$

Consider (14). The motion equation with respect to A_μ is

$$A_1^A \partial_\mu A^{\mu\nu} + A_2^A \partial_\mu B^{\mu\nu} = j^\nu$$

where

$$A_1^A = 4a_1 + 2(a_3+a_4)$$

$$A_2^A = 2a_2 + (a_3+a_4)$$

$$j^\mu = \bar{\psi}\gamma^\mu\psi \tag{18}$$

the variation in relation to B_μ gives,

$$A_1^B \partial_\mu A^{\mu\nu} = j^\nu$$

where $$A_1^B = -2a_2 + a_3 - a_4 \tag{19}$$

the current conservation for (18) and (19) is

$$\partial_\nu j^\nu = 0 \tag{20}$$

In Appendix A is shown that for (15), (16) and (17) the gauge fields are also involved in the current conservation.

The corresponding canonical momenta to (14) are

$$\pi_\mu(A) = (4a_1+a_3+a_4)\, A_{o\mu} + 2a_2 B_{o\mu} + 2a_3 G_{o\mu} + 2a_4 H_{o\mu} \tag{21}$$

$$\pi_\mu(B) = (2a_2+a_3+a_4)\, A_{o\mu} \tag{22}$$

Observe that (21) has four degrees of freedom. If $a_3 = a_4$ one degree is reduced. It shows that depending on relations between the constants more than one possibility for the particles spectrum appear. In a further work it is intended to study with more detail the possibilities. If the mass term is included, the fields A_μ and B_ν become just current fields. Then the physical fields are C_μ and D_ν. The massive field C_μ will be given by

$$C_\mu = A_\mu - B_\mu \tag{23}$$

and the gauge field D_μ by

$$D_\mu = A_\mu + B_\mu \tag{24}$$

A phenomenological application could be to associate the photon and Z^0 to the same e^+e^- matter multiplet. The properties of the associated current should be the same for both cases.

b) Two matter fields and two gauge fields in the same group.

Define different matter fields ψ and χ associated to the gauge fields A_μ and B_ν respectively. The interaction part is given by

$$\mathcal{L}_{MI} = j^\mu_\psi A_\mu + j^\mu_\chi B_\mu \tag{25}$$

where $j^\mu_\psi = \bar{\psi}\gamma^\mu\psi$. Similarly j^μ_χ. Consider now these fields rotating under the same group,

$$\psi \to e^{i\alpha(x)}\psi(x) \quad \text{and} \quad \chi \to e^{i\alpha(x)}\chi \tag{26}$$

(26) gives propagation for matter terms and for gauge fields transformations as (5) or (7). The basic difference is about the current conservation. For (14) it gives,

$$\partial_\mu [j^\mu_\psi + j^\mu_\chi] = 0 \tag{27}$$

The result appears to be consistent with the imposition of two matter fields in the same group. Due to it, both contribute to the charge conservation as in (27). In the cases

(15), (16), (17) the same discussion as before follows.

Another possibility would be to associate two matter fields to the same gauge field.

2. NON ABELIAN CASE

a) One matter field and two gauge fields in the same group.

Consider the field transformation for SU(N), where $U(x) = e^{iw^a(x)t_a}$. From (3) and (4), two non-abelian covariant derivatives must be also introduced. It yields for A^a_μ

$$(t^a A_{\mu a})' = U(t^a A_{\mu a})U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1} \qquad (28)$$

and the same transformation for the gauge field B^a_ν. These transformations allows to write a basis as in Fig. 1. It is made by the covariant derivatives and the difference between the

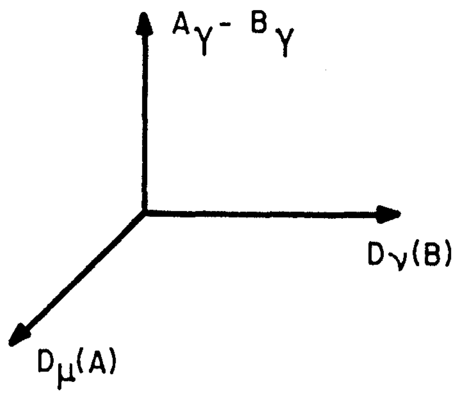

Fig. 1
A Covariant basis

Any tensor written in terms of these coordinates will transform covariantly. For instance, from (30),(44) and (53) different types of Lagrangians are generated.

gauge fields

$$(D_\mu(A),\ D_\nu(B),\ A_\gamma - B_\gamma) \tag{29}$$

Notation is $t^a X_a \equiv X$. Combinations of the covariant coordinates (29) yield covariant strength tensors. QCD would be reobtained just working in one of the axis. In order to characterize the Lagrangian there is still space for others variations. It is in terms of how to take the trace. One gets the following possibilities in the adjoint representation of SU(N)

a) $\mathrm{tr}\, t^a t^b = N\,\delta^{ab}$

b) $\mathrm{tr}\, t^a t^b t^c = \frac{i}{2}\, N\, c^{abc}$

c) $\mathrm{tr}\, t^a t^b t^c t^d = \delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} + \frac{N}{4}\,(d_{abe}d_{cde} - d_{ace}d_{dbe} + d_{ade}d_{bce})$

Combinations of the covariant basis with a), b) and c) will generate different kinds of gauge invariant Lagrangians. This fact we denominate as an extended gauge symmetry. It is selected two qualitative cases to be studied.

Case I: Based on the covariant derivatives

(1) $G^a_{\mu\nu}$ tensor. Define

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu B^a_\mu - ig\, c^a{}_{bc}\, B^b_\mu A^c_\nu \tag{30}$$

(30) is not antisymmetric. Considering the infinitesimal rotation for (28),

$$\delta A^a_\mu = -c^a{}_{bc}\,\omega^b A^c_\mu - \frac{1}{g}\,\partial_\mu \omega^a \tag{31}$$

$$\delta B^a_\mu = -c^a{}_{bc}\,\omega^b B^c_\mu - \frac{1}{g}\,\partial_\mu \omega^a \tag{32}$$

Using

$$f_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu B^a_\mu \tag{33}$$

and replacing (32) in (33) gives

$$\delta f^a_{\mu\nu} = c^a{}_{bc}[\omega^b f^c_{\mu\nu} + \partial_\mu \omega^b . A^c_\nu - \partial_\nu \omega^b . B^c_\mu] \tag{34}$$

The second term in (30) transform like [1]

$$g c^a{}_{bc}\,\delta[B^b_\mu A^c_\nu] = c^a{}_{e\delta} c_{sbm}\omega^e B^b_\mu A^m_\nu + c^a{}_{bc}[-(\partial_\mu \omega^b)A^c_\nu + \partial_\nu \omega^b . B^c_\mu] \tag{35}$$

Adding (34) and (35)

$$\delta G^a_{\mu\nu} = c^a{}_{bc}\,\omega^b G^c_{\mu\nu} \tag{36}$$

that gives

$$t_a G^a_{\mu\nu} \rightarrow U t_a G^a_{\mu\nu} U^{-1} \tag{37}$$

(ii) $H^a_{\mu\nu}$ tensor. Define,

$$H^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu A^a_\mu - ig\, c^a{}_{bc} A^b_\mu B^c_\nu \tag{38}$$

Similarly to (7),

$$t_a H^a_{\mu\nu} \to U t_a H^a_{\mu\nu} U^{-1} \tag{39}$$

$H_{\mu\nu}$ is not antisymmetric either. It obeys the relation

$$H^a_{\mu\nu} = -G^a_{\nu\mu} \tag{40}$$

Another way to obtain the relations (37) and (39) is by defining the following covariant derivatives,

$$D_\mu(A) = \partial_\mu + ig_A A_\mu \quad ; \quad D_\nu(B) = \partial_\nu + ig_B B_\nu \tag{41}$$

giving

$$[D_\mu(A)\, , D_\nu(B)] = g_A \partial_\nu A_\mu - g_B \partial_\mu B_\nu + ig_A g_B [A_\mu\, , B_\nu] \tag{42}$$

$$[D_\mu(B)\, , D_\nu(A)] = -g_A \partial_\mu A_\nu + g_B \partial_\nu B_\mu + ig_A g_B [B_\mu, A_\nu] \tag{43}$$

(iii) Similarly to QCD define

$$A^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - ig_A c^a{}_{bc} A^b_\mu A^c_\nu \tag{44}$$

$$B^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu - ig_B c^a{}_{bc} B^b_\mu B^c_\nu \tag{45}$$

Considering (32)

$$A_{\mu\nu} \rightarrow U\,A_{\mu\nu}\,U^{-1}$$

$$B_{\mu\nu} \rightarrow U\,B_{\mu\nu}\,U^{-1} \qquad (46)$$

(iv) Define the tensors

$$F^a_{\mu}(A) = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - ig\,c^a{}_{bc}\,B^b_\mu B^c_\nu \qquad (47)$$

$$F^a_{\mu\nu}(B) = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu - ig\,c^a{}_{bc}\,A^b_\mu A^c_\nu \qquad (48)$$

yielding,

$$F_{\mu\nu}(A) + F_{\mu\nu}(B) \rightarrow U({}^A F_{\mu\nu} + {}^B F_{\mu\nu})U^{-1} \qquad (49)$$

(v) Consider (23), (24) and define the relation

$$c^a{}_{bc}A^b_\mu A^c_\nu = c^a{}_{bc}B^b_\mu B^c_\nu + \Lambda^a_{\mu\nu} \qquad (50)$$

where $\Lambda^a_{\mu\nu}$ is something that transform like

$$\Lambda_{\mu\nu} \rightarrow U\,\Lambda_\mu\,U^{-1} \qquad (51)$$

Observe that the antisymmetric matrices whose coefficients are numbers can satisfy (51) directly. Thus,

$$F_{\mu\nu}(A) \rightarrow U\,F_{\mu\nu}(A)\,U^{-1}$$

$$F_{\mu\nu}(B) \rightarrow U\,F_{\mu\nu}(B)\,U^{-1} \qquad (52)$$

The gauge symmetry can also be generated through other basis in (28).

Case II: Based in term of covariant derivative and in the difference between fields

VII) $C_{\mu\nu}$ tensor. Define

$$C_{\mu\nu} = [D_\mu(A) , A_\nu - B_\nu] \tag{53}$$

giving

$$C^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu B^a_\mu - gc^a{}_{bc} A^b_\mu (A^c_\nu - B^c_\nu) \tag{54}$$

Similarly for $D_\mu(B)$.

A further work will explore the variations with items b) and c).

3. GAUGE INVARIANT MASS TERM

A problem for the gauge formalism is how to obtain fields with mass preserving the gauge invariance. The gauge symmetry for just one field does not yield a massive field. The transformations (5) allows us to obtain a covariant expression by subtracting the fields

$$A_\mu - B_\mu \rightarrow U(A_\mu - B_\mu) U^{-1} \tag{55}$$

that allows a gauge invariant mass term $m^2(A^a_\mu - B^a_\mu)^2$. (55) could represent a mass term with colour indices. It would be a NxN matrice (where N is the dimension of the adjoint representation) such that

$$[U,m^2] = 0 \qquad (56)$$

(56) yields a mass term written in terms of Casemir operators. This fact can be used to define a gauge boson mass depending on the matrices of the colour group.

The presence of a mass term yields that the physical fields will be given by D_μ and C_ν, while A_μ and B_ν become current fields. C_μ is not a proper gauge field. However it participate in properties of gauge theories. It can contribute in the gauge fixing term, Slavnov-Taylor identities and to interact with ghosts. (54) yields a general covariant tensor involving a massive field

$$C^a_{\mu\nu} = p\,\partial_\mu C^a_\nu - q\,\partial_\nu C^a_\mu + C^a{}_{bc}[g_1 X^b_\mu C^c_\nu + g_2 Y^b_\mu C^c_\nu] \qquad (57)$$

where p, q, g_1 and g_2 are constants. X_μ and Y_ν are any gauge fields. Observe that C_μ will not have Yang-Mills corrections if the fields X_μ and Y_ν do not propagate. It means that the massive field longitudinal part will not participate for the antisymmetric case.

PART II: A COLOUR MODEL

4. THE TWELVE COLORUFUL STONES

In 1964 the number three started to be introduced through quarks [2]. At that time quarks were viewed mostly as algebraic entities, serving as the building blocks of the observed internal symmetry. Later it was understood that the number three for different quarks does not have a fundamental meaning in strong interaction physics itself. The number three aspect has returned, however, when each quark was assumed to occur in three different types [3]. In 1972 the idea took shape to interpret the colour quantum number as a hidden variable. Thus a particle just made by colour will show consequences. A Lagrangian based only on such variable will provoke measurable numbers.

Our belief is that colour concept would be not restricted to quarks. It would appear as a nature implicit observable. For leptons to depend also on colour a more primitive structure must be identified. Nevertheless, among the varieties for that, it becomes necessary to make a choice. Therefore in order to guide us for this basic structure it is necessary to identify a principle. Following the number three, it is chosen for that the $SU(3)_c$ symmetry. The next assumption is to consider two different matter fields in the same group. They are chosen to have spin zero (yang stones-ϕ^i) and spin half (yin stones-ψ^i). The yang stones will have three colours and the respective anticolours. The yin stones will have the same colours and anti-colours. The total give twelve different stones as in Fig. 2. Observe that it is not an $SU(3)_c \otimes SU(3)_c$.

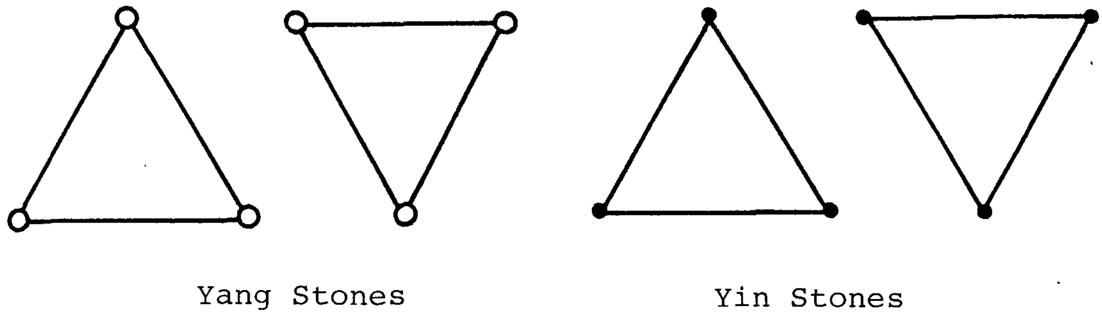

Fig. 2. Twelve yang-yin stones in triplets and antitriplets.

Another principle is that fundamental fields are associated with different gauge fields. Physically this means that these two fundamental fields can not interact directly, but via gauge boson. Terms like $g\bar{\psi}\psi\phi$ would be forbidden. Matter interaction will be given by

$$\mathcal{L}_{MI} = i\psi_i \not{D}_{ij}(A)\,\psi_j + |D_\mu(B)\phi^i|^2 \tag{59}$$

The Lagrangians obtained from Fig. 2 are, in principle, available to the gauge part.

5. CONFINEMENT AND ASYMPTOTIC FREEDOM

We know from perturbative QCD that the three gluon vertex is responsible for asymptotic freedom [4]. Thus it is expected that all non-abelian Lagrangians obtained from the extended gauge symmetry will reproduce such property. Basically they will yield two types of three gauge boson vertex. They can include the gauge bosons as in Fig. 3 or with different nature as in Fig. 4. In terms of stones a consequence of asymptotic freedom is that quarks and leptons would also interact weakly for short distances.

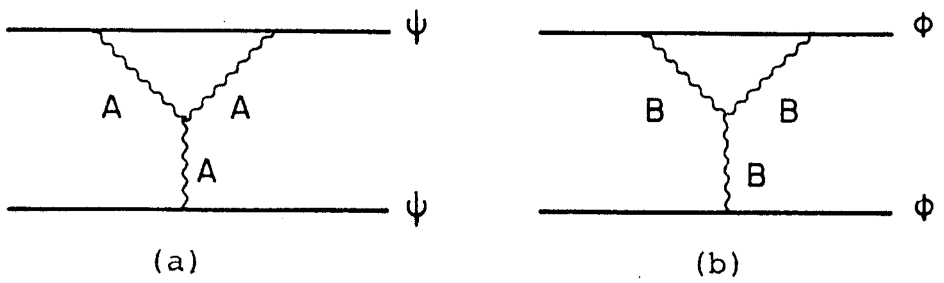

Fig. 3. Three gauge bosons graphs. They can be generated by (44), (45).

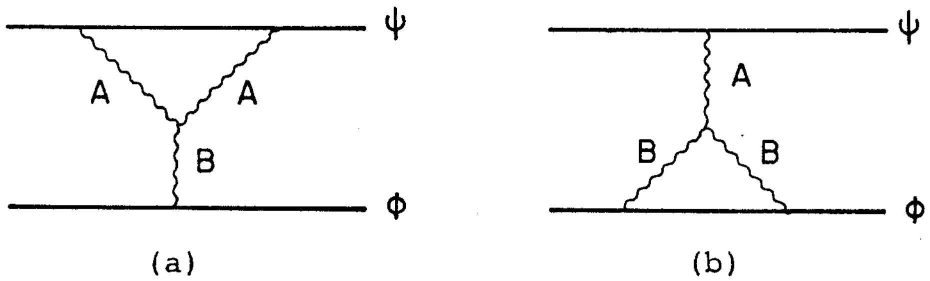

Fig. 4. Three gauge bosons graphs. They can be generated by (30),(38), (47), (48).

Confinement here will be understood in a naive approach. It is the static case where the interacting potential is given by the Fourier transform of the propagator [5]. The potential will depend on the momentum space propagator expression. There are three kinds of propagators as in Fig. 5. A mixing propagator as in Fig. 5(c) emerges from the study involving some Lagrangian with two gauge fields in the same group. Dimension analysis limits propagator expression to $\frac{1}{k^2}$, therefore for a non-confining potential mass term inclusion is crucial. It can produce a confining expression as $\frac{m^2}{k^4}$. Considering two gauge fields, a massive expression like $\frac{f(k^2,m^2)}{k^4-m^4k^2}$ can be obtained. It will not confine in an absolute form. For this, it is necessary to introduce three gauge fields in the same group. Then the Fourier transform of expression like $\frac{g(k_1^2,m_1^2,m_2^2)}{a_0k^4+(a_1m_1^2+a_2m_2^2)k^2+h(m_1^2,m_2^2)}$ will give linear potentials (a_0,a_1,a_3 are constants that will depend on the Lagrangian studied).

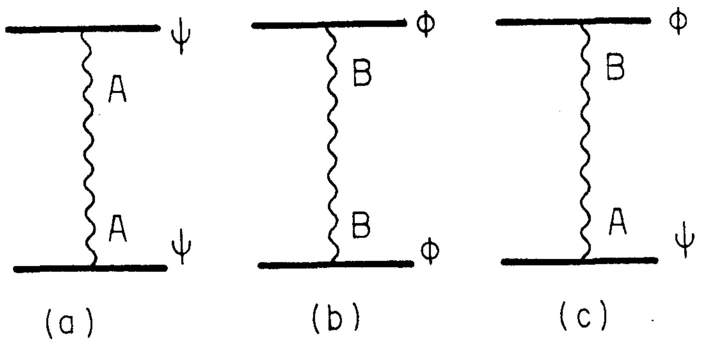

Fig. 5. Bosonic (Yang) and fermionic (Yin) fields interacting in tree level.

Consider a case where only the mixing propagator yields a linear potential. This means that bound states made by yin--yin and yang-yang stones would be broken. It would justify only the existence of fermionic quarks and leptons. We symbolically would interpret it as a cake recipe. In order to prepare it we need basically wheat and butter. After, some fruits can be added. An accelarator can separate these fruits, but not the yang butter from the yin wheat.

Observe that for this approach terms like $\frac{1}{m^2}[\partial^\mu A_{\mu\nu}]^2$ will have meaning. There is a freedom due to confinement situation in introducing another variables like the mass inverse. In conclusion it is discussed that the most important aspect is understand how to build up a bridge between confinement and non-confinement regions. An insight should be to look for the inverse of the phenomenological terms like mass. However in the conclusion it is also emphasized the possibility of defining the colourful world without considering phenomenological entities like mass.

In order to make a first opinion we prefer to start by calculating what we consider as the most intuitive Lagrangian involving two gauge fields. It is

$$\mathcal{L} = \mathcal{L}_G + i\bar{\psi}\not{D}(A)\psi + |D_\mu(B)\phi|^2$$

where

$$\mathcal{L}_G = -\frac{1}{4}F^{a\,2}_{\mu\nu}(A) - \frac{1}{4}F^{a\,2}_{\mu\nu}(B) + \frac{1}{2\alpha}[\partial_\mu(A^\mu_a + B^\mu_a)]^2 \qquad (60)$$

In Appendix B beta function is calculated up to one loop. It was not considered the mixed propagator coming from gauge fixing term. The asymptotic freedom is obtained and its inclusion only would improve it. It is interesting to note the beside gauge invariance being broken, the one loop explicit calculation shows that the longitudinal part gets trivially renormalized. This result works as counter-example to gauge theories. We observe that this result is valid only for the three gluon vertice given in (60). Another feature of Yang-Mills theories is that magnetic charge can exist. This fact is explored for (60) in Appendix B.

6. CONCLUSION

This work's effort is to enlarge gauge symmetry and the colour concept. It is a remarkable fact that all the interactions presently regarded are based on gauge invariance. However it is necessary to be careful about how to interpret relations between forces of nature and gauge theories. In our opinion, gauge theories are just a servomechanism. They realize interactions with gauge boson emissions, such as photons and gluons. Nevertheless the evolution for the massive case has not been natural. An engineering as Higgs mechanism becomes necessary. The doubt about it is due to the requirement of an extra particle - the Higgs. Based on this attitude, physics has grown. The approach here is to invest in the enlargement of the gauge concept by introducing more than one gauge field in the same group. The numbers of fields associated to a group is not necessarily the same as the generators of the group. This means that the abstraction of group theory should have the rôle of a rhythm. Thus, in principle, it would not matter how many fields are involved in the group transformations. A discussion relating the current fields (A_μ, B_ν) and physical fields (C_μ, D_ν) will be presented in a further work.

In the order of simplicity, we have started by extending gauge symmetry to cases involving two fields in the same group. The first part of the project is only to extend it. Relations between this gauge symmetry, renormalizability and unitarity will be developed in a further work.

The importance of a massive gauge boson for strong in teractions is increasing. Besides generating the possibility of having a fourth power propagator in momentum space, it also appears as a condition to preserve the perturbative approach. QCD is being mediated through a massless gluon. The Yang Mills aspect of the theory has brought the asymptotic freedom property from ultraviolet divergence [4], but there is the infrared aspect to be understood [6]. Considering the failure of the factorization theorems in quark-gluon scattering, an interrogation for perturbative QCD validity arises [7]. Thus, in order to preserve the perturbative meaning of asymptotic freedom, either the impulse approximation must be revised with the presence of soft gluons interacting with the spectator or the gluons must be massive. Under this second aspect (57) appears as an attempt. It leads to a massive Yang Mills theory.

Colour is a new parameter that the physics of our age has revealed. The present experimental results are giving it the status of an implicit observable. We would emphasize that despite this new feature, colour is not a metaphysical entity. The moment to be lived is in how to interpret colour. We identify three different spaces where the concept of colour could be developed. The first case would be to view colour measurement as a technological problem. Depending on the energy scale colour would be a deconfined generalized charge. Then it would be necessary to discuss more explicitly the relation between QCD and colour. QCD is based on the $SU(3)_c$ symmetry. Therefore under this symmetry it is not possible to build up a thought measurement in order to separate blue from red. It will appear a three coloured rainbow. A second approach would be to consider colour just as a non-definitive theoretical concept valid for a time. It would be closer to isospin than to a Grassmann variable. Nevertheless there is still room for a third interpretation. It is to observe that results are revealing colour not just as a pure index to be painted, but as an implicit experimental parameter. This third approach is the one we shall develop below.

Colour is being revealed as an implicit observable which influences decay rates, such as for $\pi^0 \to 2\gamma$. It can add new sentences to discuss the meaning of an elementary particle. Gauge and colour concepts are enough to generate a dynamics. Therefore particles made only by colour will carry packets of energy and momentum. They identify the most primitive dynamics for a big-bang. However these particles have a different content. They appear not be detectable. In order to understand this fact, we think that there is room for two kinds of sensibility:

i) Colour may be considered as a confined generalized charge.

ii) Colour may be assumed to be an implicit observable.

Observe the first case. Any model which conceals colour becomes valid. For instance, it could be satisfied by imposing boundary conditions or by obtaining a linear potential. At high temperatures the deconfinement could appear. Consider a model where to each quark family is associated different gauge bosons,

$$\begin{pmatrix} u \\ d_c \end{pmatrix} \to A^i_\mu \quad ; \quad \begin{pmatrix} c \\ s_c \end{pmatrix} \to B^i_\mu \quad ; \quad \begin{pmatrix} t \\ b \end{pmatrix} \to D^i_\mu$$

Redefining these fields we get two massive fields. They can have different masses and their propagators will have the form on (page 18). The Fourier transform gives a linear potential for the static case. In a further work this model will be presented. We observe that colours will not be differentiated through these models. It is similar for the deconfinement case. The thermal Wilson loop is written depending on the trace and the consequent potential is colourless. Our main criticism is that confinement should not be understood only by an algebra showing a confining expression. The importance is in the interpretation of the context that emerges. Then it is necessary to answer

What aspect is behind a confined charge?

We think that the qualitative answer for this question would depend on the region being analysed. Linear potential models give a more mechanical aspect for the confinement context. This approach would be more consistent with nuclear physics than with high energy. However colour has emerged on a fundamental level. This would be a clue to manipulate it as an entity with its own character. Particle physics should be based on the fundamental structure sophism. Colour interpretation should get its meaning in terms of its contribution for this sophism (ultimate constituents of matter). Therefore we prefer to develop the description ii).

The colour overture will be played through the second sensibility. The tendency is to characterize colour as a confined and maybe implicit parameter. For this purpose linear potential models can work. We are only going to assume now its implicit aspect. Intuition guides us to understand colour as a source of explicit parameters. In order to manifest this direction the concept of noumenon will be invoked. Thus two kinds of observables contribute to nature. The first observable is implicit and associated to the noumenal region, the other to the phenomenal region. The latter is defined through its explicit measurability. These two regions are independent as in Fig. 6. A principle is that the noumenon can not be understood without phenomena and similarly phenomena can not be understood without being related to the noumenon. Physics will appear to make contact (a bridge) between these two regions.

Colour is the only entity which characterize the noumenon. Its unmeasurability will define the meaning of the noumenon. We are excluding other philosophical meanings of "noumenon". However it is an interesting exercise to compare, as observables, colour and thought. Our insight is that if through the progress of physics the requirement for colour's presence increases, it will show a necessity for the noumenal region to be manifested. In order to work consistently in this approach the method should be first to learn how to manipulate colour by itself. Intuition is the only hope of reaching the noumenonal region. In the beginning colour should not be mixed with other phenomenological entities. Criteria to make this precise will appear through

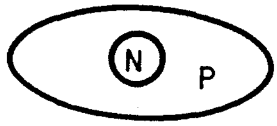

Fig. 6

Nature Observables

Nature contains as observables phenomenal and noumenal aspects. The principle is that the phenomenal meaning can not be understood without the noumenal presence. On other hand the noumenon needs phenomena to reveal itself. The rôle of physics is to construct the bridge between these regions.

the consistency of a bridge with phenomena made by physics. The principle is to work with the two regions independently as in Fig. 6. The present behaviour is to mix these regions. A vice of this mixing lies in the attitude to explain confinement. The approach is to explain an implicit region through parameters observed in the explicit (phenomenal) region. Another example is in the big-bang models. There a chemistry involving colour and electric charge is used. Are we sure of the arbitrariness in using different physical parameters to generate dynamics? Recapitulating, our attitude is to make an effort to analyse the two regions independently, after developing a model for the noumenal region, and then to build a bridge between the implicit and explicit regions. Under this approach the rôle of physics would be to construct the bridge.

The twelve colourful stones are a model for the implicit region based only on colour. Following the above principle, intuitions of the noumenon must come from phenomena. Observing that the explicit region has a fermion-boson structure motivates us to define the stones nature also in two families. The basic case is to consider spin half fermions and spin zero bosons. Another axiom is to consider three colours at the fundamental level through $SU(3)_c$. These two structures yield twelve colourful stones. A basic model in particle physics will have two trends. They are in the type of particles that it generates, and in the way it reveals the path to unity. A test for the model's adequacy is if it is able to realize both, each depending on the other. Take the SU(5) case. The quintet could be used as a fundamental representation at high temperatures.

Then particles generated by these building block structures would start to cool. Another interpretation consists of making the model suffer a spontaneous symmetry breaking at some scale of energy. The importance is to observe whether the two processes commute. This means to ask if cooling structures generated at high temperature gives the same quarks and leptons as obtained from $SU(3)_c \otimes SU(2) \otimes U(1)$. Thus analysis of the stones must be carried out in terms of building blocks and in the unity context. The construction of the bridge must organize them. However the $SU(3)_c$ symmetry can not be broken.

Following the unity road we have to find at which stop these stones get in. It is reasonable to consider that at present, physics has arrived at the nature flux through leptons, quarks and light. This leads to twenty five building blocks. Considering the antiparticles, it becomes natural to ask the question,

Is the concept of unity consistent with forty nine structures?

In the experimental season of the last decade quarks and leptons where dressed as spin half point-like objects. Although we are not in an age for preon experimental results, there emerges a need for an attitude. At present the only guide is intuition. The question to be or not to be will depend on the road that is chosen. We begin with the insight that colour would not just belong to quarks but would be a general property of nature. In this atmosphere it becomes necessary to have a new colourful fashion for quarks and leptons. However this parade can not be random. There thus appears the next question, What is the principle to guide the composite model engineering?

In the literature there are different kinds of preon models. Our approach will be based on the number three numerology. This means that unity construction is realized through the existence of three colours. Following this principle, we have to learn how to generate particles and their properties without breaking this symmetry. Observe that the three colours here are not defined in terms of quarks. The model adopts different matter fields in the same group $SU(3)_c$. It gives twelve stones. These stones would play like a noumenonic piano. The simplest sounds are generated with a colourful and a colourless structure. We identify them as quarks and leptons. Observe that through such a piano, the presence of a bridge crossing from the noumenon to the phenomena region will start to be developed. For instance, fermionic quarks will given by $\chi_i = f_{ijk}\phi^j\psi^k$ where f_{ijk} is a colourful constant. The Young tableaux show quarks appearing as triplets and sextets. They will be observables in both regions. Fermionic leptons will be given by $\eta = \phi^i\psi_i$. This lepton based only on colour would be the common origin for the electron and its neutrino.

In the future other stages for the bridge must be generated. Relations as in (56) would be an insight for that. A further aspect is to understand how to connect colour with phenomenological parameters as electric charge and mass. Any group that commutes with $SU(3)_c$ can be used. This relation would provide a mechanism to differentiate quarks. The twelve stones model has classified quarks in the same representation. However they will be made by different colourful stones interaction. If there is a dependence between colour and charge, the colourful interaction in different bound states means that quarks will appear differentiated through electric charge (for example). Another way to connect these regions is by making an engineering. It introduces the inverse of each phenomenological entity. These inverse parameters, such as mass inverse, would produce other features to be studied.

As building blocks there is a philosophical difference between the standard quark model and the twelve stones. It is in the procedures for colour treatment. While the first is based just on the concept of opposites (colour - anticolour triplets), the other theoretically includes the concept of complement (fermion-boson). This generalization was a motivation to express the noumenal region in terms of bosons and fermions instead of just fermions. Because of the presence of the complement concept we would like to justify the words yang and yin. Summarizing, the twelve colourful stones model is identified as being a noumenal theory, based on the number three numerology and realized with the concepts of opposites and complements. Its experimental evidence is that quarks and leptons appear as the minimum structures obtained, that they do have bosonic and fermionic structures, and that quarks can appear in triplets. The next question in the path under consideration is,

Do the twelve colourful stones represent a unity?

A method to look for unity is by searching for the ultimate constituent of matter. However colour confinement brings experimental obstacles to knowing if the subdividing process will continue. In the noumenon method the unity concept will not require measurability. Observe that up to now the model is being developed just in terms of colour. It still is necessary to be more explicit about the space-time. Supersymmetry will come in for that. Perhaps connecting these yang-yin stones would be the most natural place for supersymmetry. It would not appear in the phenomenological region but as an approach developed at the noumenon. It would be like having an unbroken supersymmetry at energies higher than 1 TEV. Leaving the road, we would say that supersymmetry brings more untouchability and unity to the symbolism of the number three as in Fig. 7.

The phenomenal region characterizes what could be identified as Galileo's attitude. There investigation is developed through trial and error explicitly. However such behaviour can not be blindly repeated with the colour parameter. Perhaps there is a different cosmogony. In order to localize the situation we observe the following general question,

..., STARS,..., ATOMS,..., QUARKS/LEPTONS, YANG STONES/YIN STONES, —SUPERSYMMETRY→ △ THREE COLOURS

Fig. 7

A Unity Road

We consider that is impossible to understand how Nature reveals its unity. However to look for a unity path is real work. In our approach $SU(3)_c$ symmetry is the basis for nature's flux. In order to understand it a bridge between the noumenon and the phenomena must be developed. For this, it is expected that colour generates electric charge and mass without breaking $SU(3)_c$.

Does the colour parameter represent an epistemological cut to galilean physics?

The necessity of the concept of noumenon is that which will define this cut. A phenomenon invoked by itself would lose meaning. If this happens, nature would transcend the aspect of status quo. It would appear with a flavour of consequence. Then, we would have to redefine our perspective towards nature. It will exist as a manifestation of a bridge. It is the place where the experience noumenon-phenomena takes place.

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Appendix A: TWO GAUGE FIELDS IN THE SAME GROUP

Defining more than one gauge field in the same group different possibilities to build up gauge invariant Lagrangians are generated.

Abelian case

We intend to analyse (15), (16) and (17). Calculating motion equations with respect to A_μ yields for each case respectively,

$$B_2^A \partial_\mu B^{\mu\nu} = j^\nu \tag{A1}$$

$$G_1^A \partial_\mu A^{\mu\nu} + G_2^A \partial_\mu B^{\mu\nu} + G_3^A \partial_\mu G^{\mu\nu} + G_4^A \partial_\mu H^{\mu\nu} = j^\nu \tag{A2}$$

$$H_1^A \partial_\mu A^{\mu\nu} + H_2^A \partial_\mu B^{\mu\nu} + H_3^A \partial_\mu G^{\mu\nu} + H_4^A \partial_\mu H^{\mu\nu} = j^\nu \tag{A3}$$

where

$$B_2^A = 2b_1 + b_3 + b_4$$

$$G_1^A = 2g_1 \ , \ G_2^A = g_1 + g_2 \ , \ G_3^A = 2g_3 \ , \ G_4^A = 2g_4$$

$$H_1^A = 2h_1 \ , \ H_2^A = h_1 + h_2 \ , \ H_3^A = 2h_4 \ , \ H_4^A = 2h_3 \tag{A4}$$

Similarly, the variation with respect to B_μ yields respectively,

$$B_1^B\,\partial_\mu A^{\mu\nu} + B_2^B\,\partial_\mu B^{\mu\nu} = j^\nu \tag{A5}$$

$$G_1^B\,\partial_\mu A^{\mu\nu} + G_4^B\,\partial_\mu H^{\mu\nu} = j^\nu \tag{A6}$$

$$H_1^B\,\partial_\mu A^{\mu\nu} + H_2^B\,\partial_\mu B^{\mu\nu} + H_3^B\,\partial_\mu G^{\mu\nu} + H_4^B\,\partial_\mu H^{\mu\nu} = j^\nu \tag{A7}$$

where

$$B_1^B = 2b_1 + b_3 + b_4 \quad , \quad B_2^B = 4b_2 + b_3 + 2b_4$$

$$G_1^B = -g_1 + g_2 \quad , \quad G_4^B = 2g_3 - 2g_4$$

$$H_1^B = a_1 + a_2 \ , \ H_2^B = 2a_2 \ , \ H_3^B = 2a_3 \ , \ H_4^B = 2a_4 \tag{A8}$$

Depending on relation between the constants, the local current conservation will be not expressed by $\partial_\nu j^\nu = 0$. Consider a case where the notion equations are

$$a\,\partial_\mu G^{\mu\nu} + b\,\partial_\mu H^{\mu\nu} = j^\nu \tag{A9}$$

it yields,

$$\partial_\nu [j^\nu - (a-b)\,\Box\,(A^\nu - B^\nu)] = 0 \tag{A10}$$

(A10) shows that for $a \neq b$ the gauge fields also contribute to the charge conservation. In the case of pure gauge fields,

$$\Box\,\partial_\mu (A^\mu - B^\mu) = 0 \tag{A11}$$

it shows that $A_\mu \neq B_\mu$. They differ by polynomials.

The canonical momenta corresponding to (15) are

$$\pi_\mu(A) = (2b_1 + b_3 + b_4)\ B_{o\mu} \qquad \text{(A12)}$$

$$\pi_\mu(B) = 2b_1\ A_{o\mu} + (4b_2 + b_3 + b_4)\ B_{o\mu} + 2b_3\ G_{o\mu} + 2b_4\ H_{o\mu} \qquad \text{(A13)}$$

For (16),

$$\pi_\mu(A) = g_1\ A_{o\mu} + g_2\ B_{o\mu} + (2g_1 + 2g_3)\ G_{o\mu} + 2g_4\ H_{o\mu} \qquad \text{(A14)}$$

$$\pi_\mu(B) = g_1\ A_{o\mu} + g_2\ B_{o\mu} + (2g_2 + 2g_4)\ G_{o\mu} + 2g_3\ H_{o\mu} \qquad \text{(A15)}$$

For (17),

$$\pi_\mu(A) = h_2\ B_{o\mu} + 2h_4\ G_{o\mu} + (2h_1 + h_3)\ H_{o\mu} \qquad \text{(A16)}$$

$$\pi_\mu(B) = h_1\ A_{o\mu} + h_2\ B_{o\mu} + (h_2 + 2h_4)\ H_{o\mu} + 2h_3\ G_{o\mu} \qquad \text{(A17)}$$

Non-Abelian case

Consider the (14)-(17) non abelian version. The variation of the fields A^a_μ and B^a_ν gives respectively.

For (14),

$$(18)^a = ig\ c^a{}_{bc}\,[A^b_\mu\,(4a_1 A^{\mu\nu c} + 2a_2.B^{\mu\nu c} + 2a_3 G^{\mu\nu c} + 2a_4 H^{\mu\nu c}) + B^b_\mu (a_3 + a_4) A^{\mu\nu c}] \qquad \text{(A18)}$$

$$(19)^a = ig\, c^a{}_{bc}[(a_4-a_3)\, A^b_\mu + 2a_2\, B^b_\mu]\, A^{\mu\nu c} \tag{A19}$$

For (15),

$$(A1)^a = ig\, c^a{}_{bc}[2b_1 A^b_\mu + (b_3+b_4)\, B^b_\mu]\, B^{\mu\nu c} \tag{A20}$$

$$(A5)^a = ig\, c^a{}_{bc}[2b_1 B^b_\mu A^{\mu\nu c} + ((b_3+b_4)A^b_\mu + 4b_2 B^b_\mu)B^{\mu\nu c} + + 2b_3 B^b_\mu G^{\mu\nu c} + 2b_4 B^b_\mu H^{\mu\nu c}] \tag{A21}$$

For (16),

$$(A2)^a = ig\, c^a{}_{bc}[g_1 B^b_\mu A^{\mu\nu c} + g_2 B^b_\mu B^{\mu\nu c} + (2g_1 A^b_\mu + 2g_3 B^b_\mu)G^{\mu\nu c}] \tag{A22}$$

$$(A6)^a = ig\, c^a{}_{bc}[-g_1 A^b_\mu A^{\mu\nu c} + g_2 A^b_\mu B^{\mu\nu c} + (-2g_4 A^b_\mu + 2g_2 B^b_\mu)G^{\mu\nu c} + - 2g_3\, A^b_\mu\, H^{\mu\nu c} \tag{A23}$$

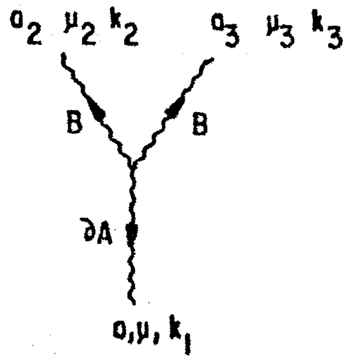
For (17),

$$(A3)^a = ig\, c^a{}_{bc}[h_1 B^b_\mu A^{\mu\nu c} + h_2 B^b_\mu B^{\mu\nu c} + (2h_1+2h_4)\, A^b_\mu H^{\mu\nu c}] \tag{A24}$$

$$(A7)^a = ig\, c^a{}_{bc}[h_1 A^b_\mu A^{\mu\nu c} + h_2 A^b_\mu B^{\mu\nu c} - 2h_3 A^b_\mu G^{\mu\nu c} - 2h_4 A^b_\mu H^{\mu\nu c}] \tag{A25}$$

APPENDIX B

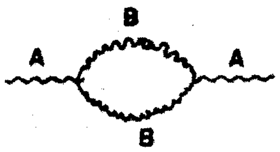
Here we study the Lagrangian (60) in terms of asymptotic freedom and magnetic charge. The calculations are similar to QCD and scalar QCD cases. A difference appears in the three gauge vertice expression. It gives,



$$\mathcal{L}_I = g\, C_{a_1 a_2 a_3}\, g_{\mu_1 \mu_3} (\partial_{\mu_2} A^{a_1}_{\mu_1})\, B^{a_2}_{\mu_2}\, B^{a_3}_{\mu_3}$$

$$i\,\Gamma^{\mu_1\mu_2\mu_3}_{a_1 a_2 a_3} = -g\, C_{a_1 a_2 a_3} [g^{\mu_1\mu_2} (k_1)_{\mu_3} - g^{\mu_1\mu_3} (k_1)_{\mu_2}] \tag{B1}$$

The gauge fields contribution to the polarization tensor is given by



$$= g^2 C_2(G)\, \delta_{ab} (g_{\mu\nu} p^2 - p_\mu p_\nu) [2-(1-\alpha)(3-\frac{n}{2}) + (1-\alpha)^2 (1-\frac{n}{4})] I_{11,0} \tag{B2}$$

where $$I_{11,0} = \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 (k+p)^2}$$

Observe that ghosts are note necessary to be introduced, as it is in QCD, as the longitudinal part of the polarization tensor disappears. As an example, we write down the renormalization constant for one loop correction to the A^a_μ field,

$$Z^A_3 = 1 + g^2 \delta_{ab} \{c_2(G) [\frac{1}{\varepsilon} (2+2\alpha) + 1 + L - \alpha(1-L) + \frac{1}{2} (1-\alpha)^2 + \sigma(\varepsilon)$$

$$- T(R) [\frac{1}{\varepsilon} \frac{8}{3} - \frac{4}{9} + \frac{4}{3} L + \sigma(\varepsilon)]\} \tag{B3}$$

where $$L = -\ell n(-\frac{p^2}{\mu^2}) + \ell n\, 4\pi + 2-\gamma \Big|_{p^2=-\mu^2}$$

Calculating others graphs yields for the beta function,

$$\beta(g_R) = -\frac{g_R^3}{(4\pi)^2}\left[\frac{11}{3}\, c_2(G) - T(R)\right] \tag{B4}$$

that is negative for twelve stones.

We consider now the electric and magnetic fields. The antisymmetric tensors define the electric and magnetic fields by

$$F^{ka}(A) = E^{ka}(A)$$

$$F^{ija}(A) = -\varepsilon^{ijk}\mathcal{B}^{ka}(A) \tag{B5}$$

We can also write

$$\vec{E}^a(A) = -\vec{\nabla} A^a - \frac{\partial \vec{A}^a}{\partial t} - g\, c^a{}_{bc}\, \vec{B}^b\, B^{oc}$$

$$\vec{\mathcal{B}}^a(A) = \nabla \times \vec{A}^a + \frac{1}{2}\, g\, c^a{}_{bc}\, \vec{B}^b \wedge \vec{B}^c \tag{B6}$$

Physically (B6) gives a magnetic charge density,

$$\rho^{aM}(A) = \frac{1}{2}\, g\, c^a{}_{bc}\, \nabla.(\vec{B}^b \wedge \vec{B}^c)$$

Similarly for $\vec{E}^a(B)$ and $\vec{\mathcal{B}}^a(B)$ the magnetic sources $\rho^{aM}(A)$ and $\rho^{aM}(B)$ have an interesting aspect for (60). They can be interpreted as a shower of the field A_μ in the dynamics made by the propagation of the other B_μ. This makes the two gauge field concept to appear in terms that each field influences the other through a magnetic source.

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