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HIGH ENERGY NEUTRON REACTIONS AND THE NUCLEAR OPTICAL MODEL*

L. C. Gomes and J. Leite Lopes
Centro Brasileiro de Pesquisas Físicas
and
Faculdade Nacional de Filosofia
Rio de Janeiro, D. F.

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Introduction

Total cross sections of neutron induced nuclear reactions have been measured for many elements and for different ranges of energy. The measures at about 90 Mev[1] were satisfactorily described by the optical model proposed by Fernbach, Serber and Taylor[2].

In this model, a nucleus is represented by a partially transparent sphere endowed with an index of refraction and an absorption coeficient. This is equivalent to the assumption that the incident neutron is subjected by the nucleus to an average complex potential, the real part of which, U_0, determines the refractive index of nuclear matter whereas the imaginary part, U_1, is responsible for the partial

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neutron absorption. For 90 Mev neutrons, the experimental data were successfully described[2] by the following values of the constants: $U_0 = 30.8$ Mev, $U_1 = 9.5$ Mev, with a nuclear radius $R = r_0 A^{1/3}$, $r_0 = 1.37 \times 10^{-13}$ cm.

This model was, however, not so successful in accounting for the measurements at 153 Mev[3], 270 Mev[4] and 280 Mev[5]. At the first of thes energies, the nuclear potential U_0 required by the model would be 6 Mev. At 270 Mev, not even for $U_0 = 0$ are the experimental points brou into agreement with the curve predicted by the optical model. These results did not support the extrapolated hope that U_0 would be about the same as indicated by the low energy data.

More recently, an extension of the optical model to the descrip tion of low energy neutron induced reactions[6] was proposed by Feshbac Porter and Weisskopf[7]. The behaviour of the total cross sections as a function of the energy, up to 3 Mev, indicates that the neutron wave is not completely absorbed by the nucleus - a suggestion that the latter could be conveniently described by a complex potential. The constants required by experiment are in this case[8]:

$$U_0 = 42 \text{ Mev}, \quad U_1 = 2.1 \text{ Mev}, \quad r_0 = 1.45 \times 10^{-13} \text{cm}.$$

The knowledge of U_0, U_1 and r_0 for different energies is of interest. It may give some information to the question of velocity-dependent nuclear forces and of the distribution of nucleons in nuclei at such energies.

A re-examination of the description of the high energy data by the optical model is reported in this note. In this energy region, some approximation method should be used for the calculation of the cross sections involved. Our results are based on the use of the Born approximation. The computation of the elastic scattering cross section in this approximation is well known. In paragraph 1 we show that the Born method can be extended to the reaction cross section, which enables one to evaluate the total cross section in this approximation. In paragraph 2, the formula is compared with experiment; the angular distribution of the elastically scattered neutrons is also given.

1. Reaction and total cross sections in Born approximation.

The exact expressions for the elastic scattering cross section and the reaction cross section σ_r are respectively:

$$\sigma_s = \frac{4\pi}{k^2} \sum_{\ell} (2\ell + 1) \, e^{-2\eta_1^{(\ell)}} \left[\sin^2\eta_0^{(\ell)} + \sinh^2\eta_1^{(\ell)} \right]$$

$$\sigma_r = \frac{\pi}{k^2} \sum_{\ell} (2\ell + 1)(1 - e^{-4\eta_1^{(\ell)}})$$

$\eta_0^{(\ell)} + i\,\eta_1^{(\ell)}$ is the ℓ-th complex phase shift corresponding to a complex potential $V_o(r) + iV_1(r)$ in a single particle wave equation.

In Born's approximation, both parts $\eta_0^{(\ell)}$ and $\eta_1^{(\ell)}$ of the phase-shift are given by:

$$\eta_{o,1}^{(\ell)} = - \frac{2mk}{\hbar^2} \int_0^\infty V_{o,1}(r) \left[f_\ell(r) \right]^2 r^2 \, dr$$

$$f_\ell(r) = \left(\frac{\pi}{2kr} \right)^{1/2} J_{\ell + \frac{1}{2}}(kr) \qquad (1)$$

and are to be inserted in the Born scattering cross section:

$$\sigma_s^B = \frac{4\pi}{k^2} \sum_{\ell} (2\ell + 1)(\eta_o^{(\ell)2} + \eta_1^{(\ell)2}) \qquad (2)$$

Since the phase-shift occurs quadratically in (2), the reaction cross section should have the following form, in Born's approximation:

$$\sigma_r^B = \frac{4\pi}{k^2}\sum_\ell (2\ell + 1)(\eta_1^{(\ell)} - 2\,\eta_1^{(\ell)^2}) \qquad (3)$$

We write the potential in the follosing way:

$$V_o(r) + i\,V_1(r) = (U_o + iU_1)\,\Phi(r)\ , \qquad (4)$$

where U_o and U_1 do not depend on r. The phase shifts are, from (1) and (4):

$$\eta_{o,1}^{(\ell)} = -\frac{2mk}{\hbar^2}\,U_{o,1}\,\zeta_\ell\ ;\quad \zeta_\ell = \int_o^\infty \Phi(r)\left[f_\ell(r)\right]^4 r^2\,dr \qquad (5)$$

The total cross section is then:

$$\sigma_{tot} = \sigma_s + \sigma_r = \frac{4\pi}{k^2}\sum_\ell (2\ell + 1)\eta_1^{(\ell)} + \frac{U_o^2 - U_1^2}{U_o^2 + U_1^2}\,\sigma_s \qquad (6)$$

where:

$$\sigma_s = 2\pi\int_o^\pi \left[f(\Theta)\right]^2 \sin\Theta\, d\Theta$$

$$f(\Theta) = -\frac{2m}{\hbar^2}\int_o^\infty (U_o + iU_1)\,\Phi(r)\,\frac{\sin Kr}{Kr}\,r^2\,dr\ , \qquad (7)$$

$$K = 2\,k\,\sin\frac{\Theta}{2}$$

The sum which appears in the right-hand side of (6) can be easily computed by means of (5) and of the following identity:

$$\sum_{\ell}(2\ell+1)\left[f_{\ell}(r)\right]^{2} = 1$$

which is the limit of:

$$\frac{\sin Kr}{Kr} = \sum_{\ell}(2\ell+1)\left[f_{\ell}(r)\right]^{2} P_{\ell}(\cos\Theta)$$

for $\Theta \longrightarrow 0$. One obtains:

$$\sum_{\ell}(2\ell+1)\eta_{1}^{(\ell)} = -\frac{2mkU_1}{\hbar^2}\int_0^{\infty}\phi\, r^2\, dr$$

Therefore (6) is:

$$\sigma_{tot} = -\frac{8\pi m}{\hbar^2 k} U_1 \int_0^{\infty}\phi(r)r^2\, dr + \frac{U_o^2 - U_1^2}{U_o^2 + U_1^2}\,\sigma_s$$

An attractive square well potential is defined by:

$$\phi(r) = \begin{cases} -1, & \text{for } r < R \\ 0\ , & \text{for } r > R, \end{cases}$$

where $R = r_o A^{1/3}$ is the nuclear radius. It gives rise to the following cross sections:

$$\sigma_s = \frac{2\pi m^2}{\hbar^4 k^2}(U_o^2 + U_1^2)\, R^4\ ,$$

$$\sigma_r = \frac{8\pi m U_1}{3\hbar^2 k} R^3 - \frac{4\pi m^2}{\hbar^4 k^2} U_1^2 R^4\ ,$$

$$\sigma_{tot} = \frac{8\pi m U_1}{3\hbar^2 k} R^3 + \frac{2\pi m^2}{\hbar^4 k^2}(U_o^2 - U_1^2)\, R^4$$

The dependence of the total cross section with A has the following f m:

$$\sigma_{tot} = \alpha A + \beta A^{4/3}$$

where

$$\alpha = \frac{8\pi m}{3\hbar^2 k} U_1 r^3_o , \quad \beta = \frac{2\pi m^2}{\hbar^4 k^2} (U_o^2 - U_1^2) r_o^4$$

2. Comparison with experiment.

We see that the dependence of $\frac{\sigma_{tot}}{A}$ with $A^{1/3}$ is linear. This is in agreement with the experimental results, which are plotted in Fig. 1 for 90 Mev and 410 Mev and in Fig. 2 for 156, 270 and 280 Mev. The slopes of the straight lines are negative and this means that, at least in the Born approximation, the absorptive part of the potential U_1 is larger than the real part U_o.

TABLE I

	neutron energy E = 90 Mev		E = 156 Mev		E = 275 Mev		E = 410 Mev	
r_o	U_o	U_1	U_o	U_1	U_o	U_1	U_o	U_1
1.3	25.0Mev	29.3Mev	12.4Mev	20.8Mev	15.8Mev	24.4Mev	25.2Mev	29.7Mev
1.4	19.4	23.4	9.7	16.7	11.2	19.5	19.5	23.8
1.5	15.2	19.1	6.0	13.6	7.7	15.9	11.8	19.4

In Table I are the values of U_o and U_1 for the different incoming neutron energies and a few values of r_o (in 10^{-13} cm unit). The potentials change with the excitation energy; this is visualized in Fig. 5. Whereas the value of U_o required by the treatment of Serber et al., would be zero for 270 Mev neutrons[4], in the presente approximation the corresponding value is about 16 Mev for $r_o = 1.3 \times 10^{-13}$ cm.

These values of U_o and U_1, however, do not fit well the angular distribution of the elastically scattered neutrons. Fig. 3 reproduces the curve as predicted by Born's approximation at 90 Mev. It is about three times as large as the experimental points[9] which fit the partial wave analysis[10] for Aluminum. The curve predicted by the WKB approximation[2] is also shown in Fig. 3 and gives results of the opposite size as compared to Born's approximation. Fig. 4 shows the angular distribution in Born's approximation for the higher energies. Experimental measurements of this distribution at such energies are not known to us. Another quantity which is not reproduced by Born's approximation in the present model is the ratio of reaction to total cross section*. This ratio is about one half and independent of the nucleus, as indicated experimentally[4]. This is not reproduced by the present treatment; the ratio σ_r / σ_{tot} is smaller than 0.5 and is not independent of A. This is essentially the reason why we did not add a spin-orbit coupling to the complex potential, as suggested previously by Fermi[11] in treating the high energy polarization of protons. This additional potential would increase only the elastic cross section and would decrease the ratio of reaction to total cross sections still more (assuming the absorption to be spin independent).

The results show that the Born approximation, as extended to encompass the inelastic processes, is not a good tool to decide about the validity of the optical model at the energies investigated. Indeed, the phase shift for $\ell = 0$ and at 90 Mev, as computed by formula (1) with the potentials of Table I is about 1.4 .

A preliminary calculation which employs the first 14 terms of the Faxen-Holtsmark series, the phase-shifts being still evaluated by the integral (1), gives a complex potential equal to (19.5 + i 17) Mev, if the total cross section is adjusted for Al and Pb.

The authors had several stimulating conversations with L. Marquez on the subject of this paper.

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* We are indebted to Dr. L. Marquez for calling our attention to this question.

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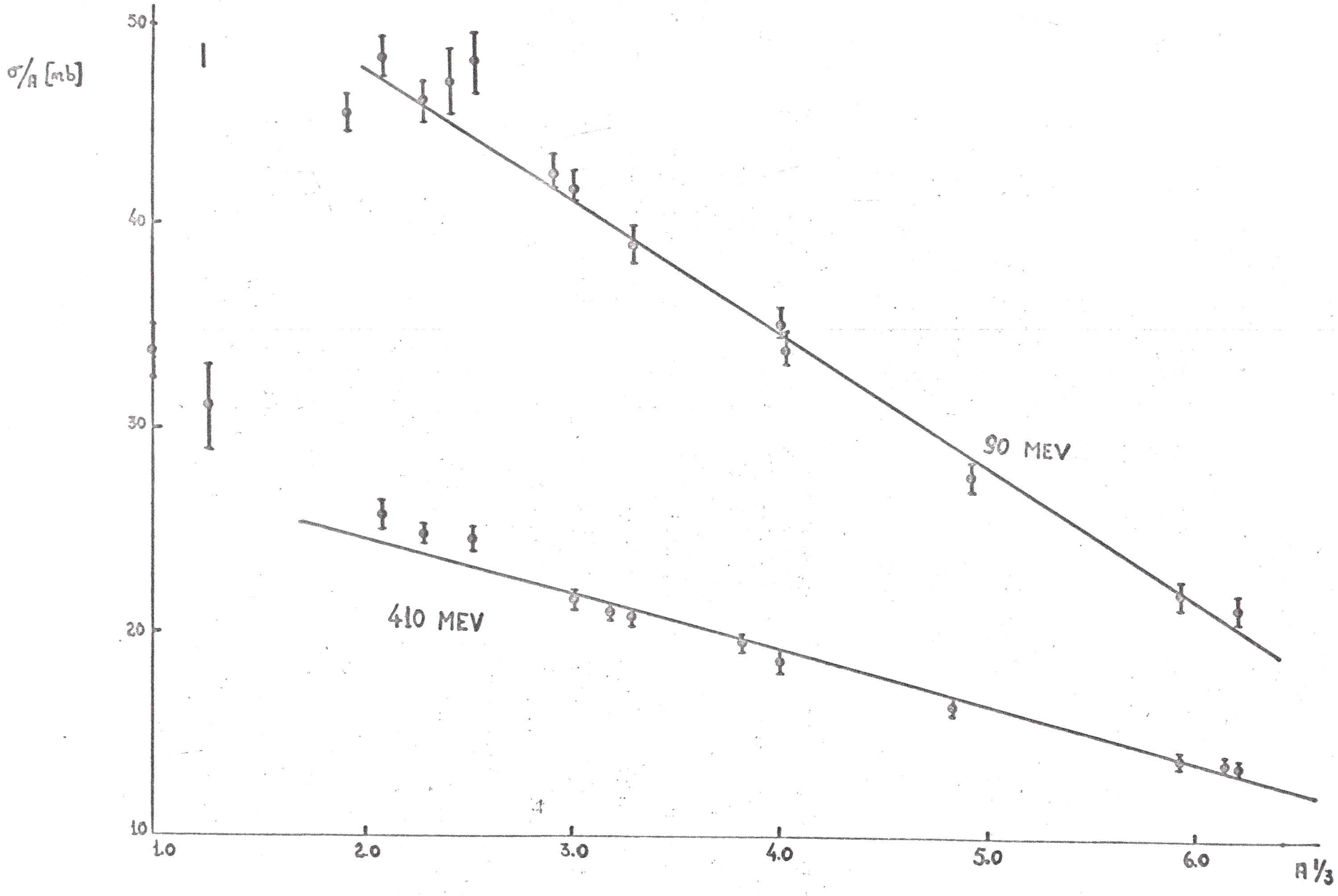


Fig. 1

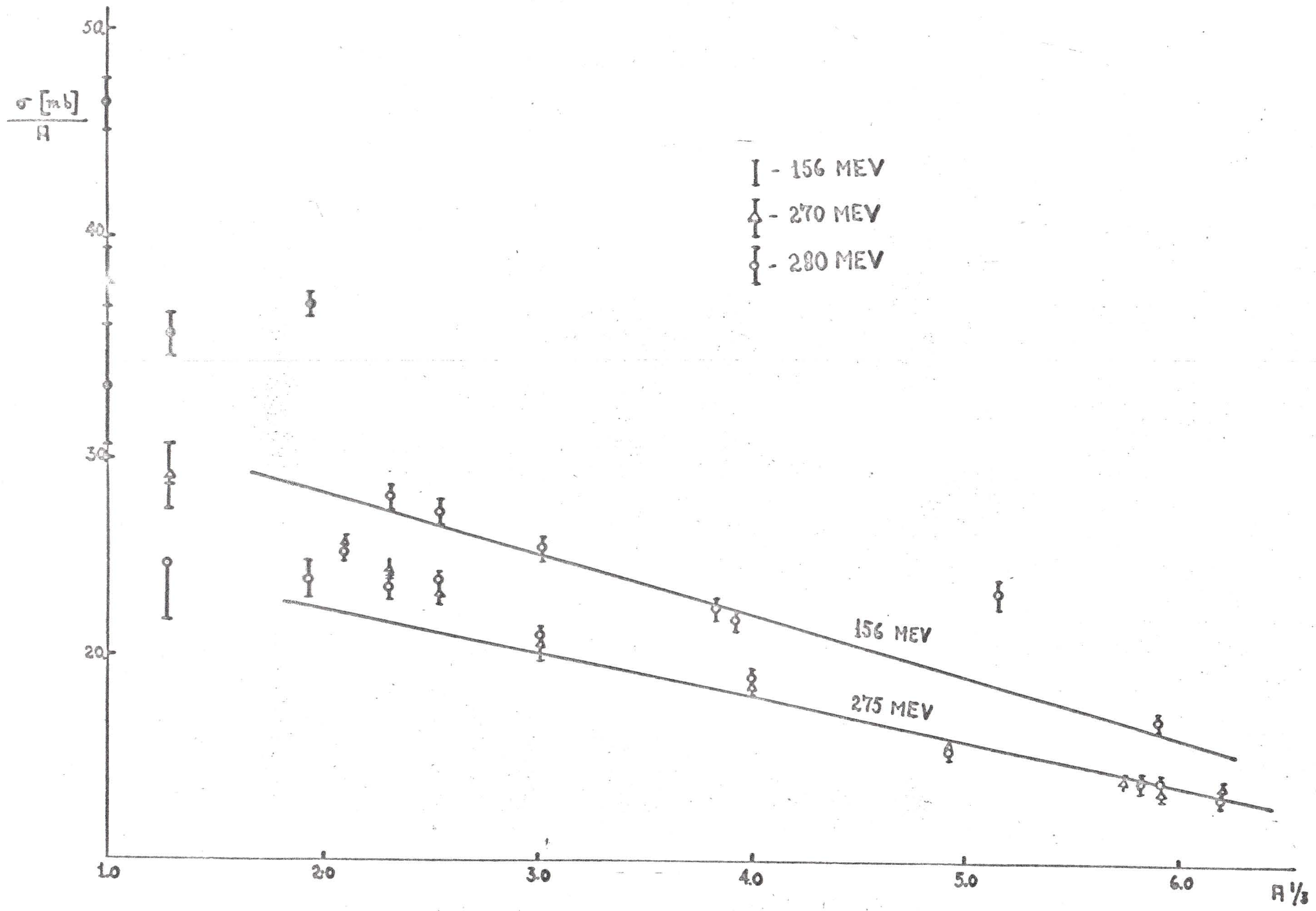
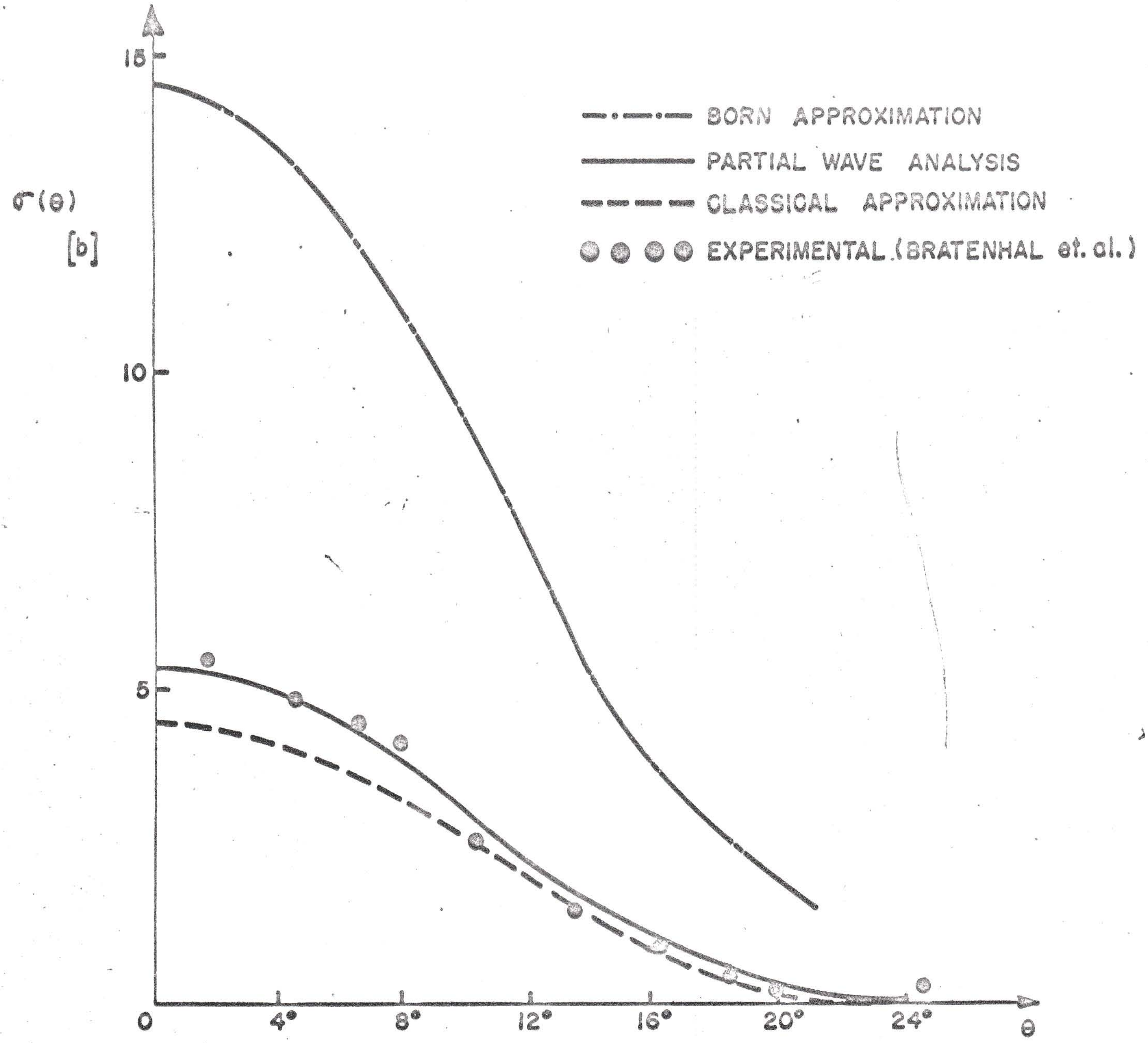

50
40
30
20
σ [mb] / A
- 156 MEV
- 270 MEV
- 280 MEV
156 MEV
275 MEV
1.0
2.0
3.0
4.0
5.0
6.0
A 1/3

Fig. 2



BORN APPROXIMATION
PARTIAL WAVE ANALYSIS
CLASSICAL APPROXIMATION
EXPERIMENTAL (BRATENHAL et. al.)
σ(θ)
[b]
15
10
5
0
4°
8°
12°
16°
20°
24°
θ

FIG. 3

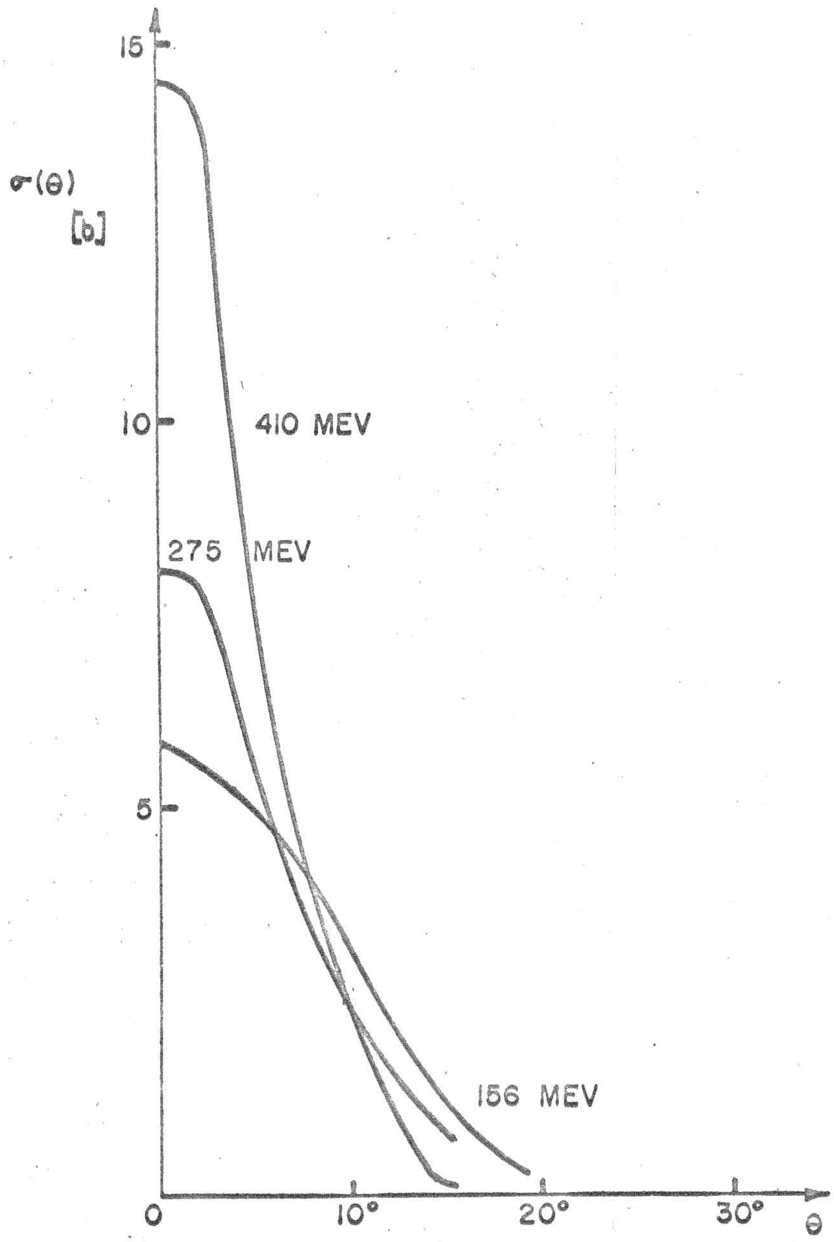


FIG. 4

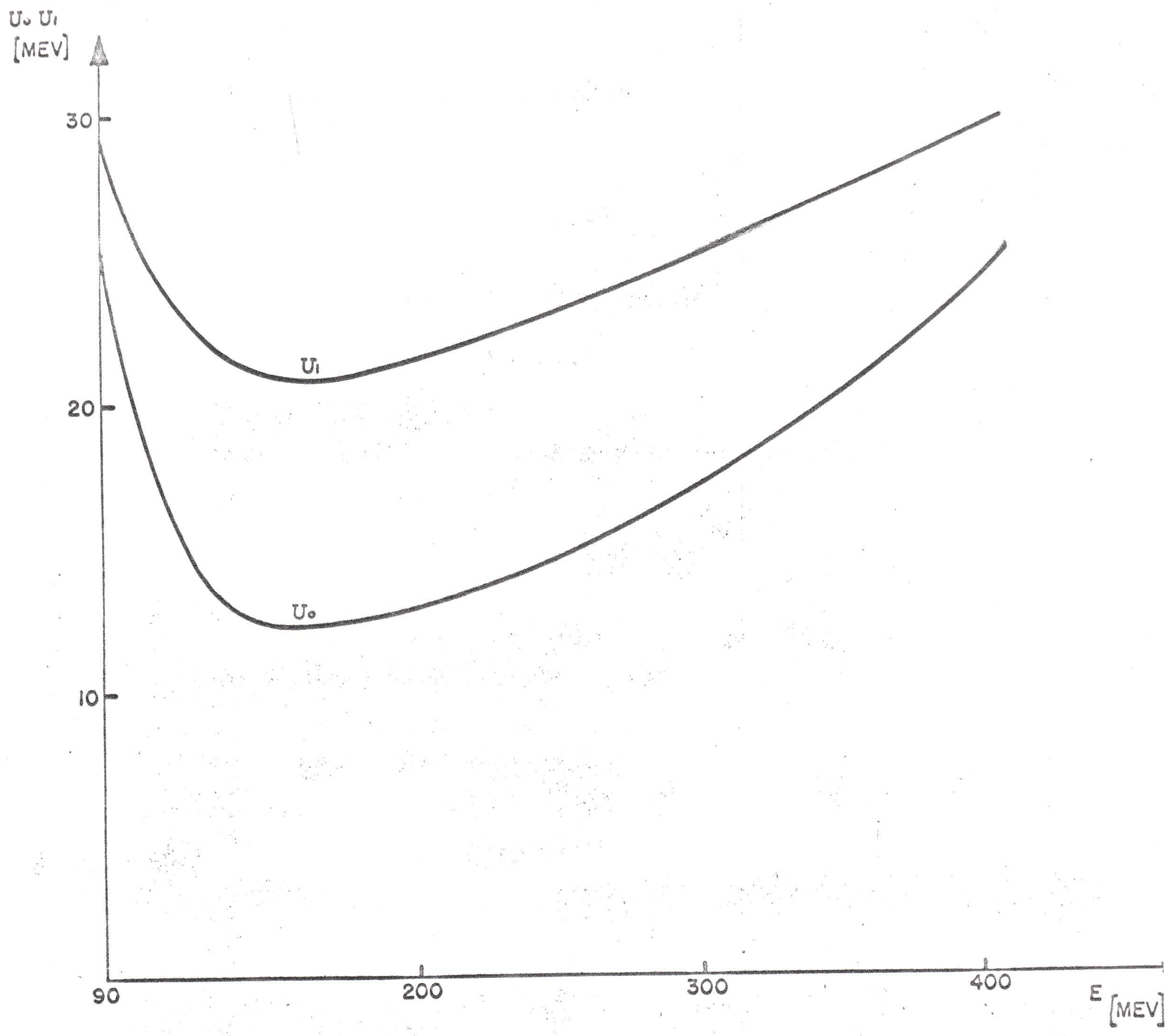


FIG. 5