CNPq

CBPF - CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Notas de Física

CBPF-NF-021/92

GENERALIZED ARCHIE'S LAW - APPLICATION TO PETROLEUM RESERVOIRS

by

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ABSTRACT - The experimental formation factor (F) vs. porosity (ϕ) as well as resistivity index (I) vs. water saturation (S_w) results typically present, in double-logarithmic representation, bendings which we now interpret as crossovers between different fractal-like regimes. We consistently propose for the ratio (rock resistivity)/(brine resistivity) $\equiv R_t/R_w = F(\phi)I(S_w)$, where the functional form of $F(\phi)$ and of $I(S_w)$ is a simple one and the same. This proposal yields $F \sim \phi^{-m}$ and $F \sim B_F\phi^{-m'}$ in the $\phi \to 1$ and $\phi \to 0$ limits respectively, as well as $I \sim S_w^{-n}$ and $I \sim B_I S_w^{-n'}$ in the $S_w \to 1$ and $S_w \to 0$ limits respectively (m, m', B_F, n, n' and B_I are rock parameters); hence Archie's law $R_t/R_w \sim \phi^{-m} S_w^{-n}$ is recovered in the $(\phi, S_w) \to (1, 1)$ limit. The agreement between theory and experiments is quite satisfactory <u>for the entire range of (ϕ, S_w)</u>. The use of the present generalized Archie's law into electrical resistivity logs improves the precision of the evaluation of the hydrocarbon reserves.

Half a century ago, G. E. Archie empirically proposed power-law relationships between the (DC or low frequency AC) electrical resistivity of a fluid-bearing rock, its porosity and its water saturation; see, for instance, Ref. [1] for a review on the subject. This so called Archie's law is since then currently used for evaluating the oil content of petroleum reservoirs. Consequently its practical importance in oil exploration and production can hardly be overestimated.

To be more precise, let us restate Archie's law. We consider an insulating porous rock sample characterized by its porosity ϕ ≡ (volume of the connected pores)/(total volume) $\in [0,1]$ and assume that the pore space is saturated with brine (say NaCl diluted in water). Under quite general conditions, the rock resistivity R_o is expected to be proportional to the brine resistivity R_w, i.e.

$$R_o = F \cdot R_w \qquad (1)$$

where the formation factor F is expected to depend on ϕ. Archie assumed that:

$$F(\phi) = 1/\phi^m \qquad (2)$$

where the porosity exponent m is a real positive number. Let us next consider a more general situation, namely, that in which the pore space is not necessarily 100% water-saturated, but might also contain a non-conductive fluid (petroleum); it is assumed that the brine-petroleum mixture fully occupies the pore space. This situation is characterized by the water saturation $S_w \in$

[0,1]; $S_w = 1$ (brine saturated) corresponds to the particular case considered in Eqs. (1) and (2), whereas $S_w = 0$ corresponds to the particular case where the entire porous volume is occupied by oil (hence the rock sample resistivity diverges). As before, it is quite reasonable to expect that the partially water-saturated rock resistivity R_t is proportional to the fully water-saturated rock resistivity R_o, i.e.,

$$R_t = I \cdot R_o \tag{3}$$

where the resistivity index I is expected to depend on S_w. Archie assumed that:

$$I(S_w) = 1/S_w^{\ n} \tag{4}$$

where the saturation exponent n is a real positive number. Putting together Eqs. (1-4) immediately yields Archie's law, namely

$$R_t = \frac{R_w}{\phi^{\ m}\ S_w^{\ n}} \tag{5}$$

The typical evaluation of the oil content essentially proceeds as follows. Eq. (5) implies:

$$S_w = \left[\frac{R_w}{\phi^m\ R_t} \right]^{1/n} \tag{6}$$

Formation factor *vs.* porosity laboratory experiments performed on a conveniently chosen set of rock samples from a given petroleum well provide m. Resistivity index *vs.* water saturation laboratory experiments performed on a few samples from the previous set yield an average value for n. Furthermore, R_w as a function of the well depth is obtained, for instance, by measuring the resistivity of the brine extracted from the rock. In addition to that, electrical and acoustic/radioactive logs measured in the particular petroleum well respectively yield (after some standard corrections) R_t and ϕ as functions of the well depth. These data replaced into Eq. (6) provide the depth dependence of S_w, or equivalently of the petroleum content $(1 - S_w)$.

This procedure would be extremely satisfactory if Eqs. (2) and (4) were always verified experimentally. It happens that quite frequently they are not. Indeed, violations of Eq. (2) are verified in the experimental data presented in Ref. [2] as well as in those indicated in Fig. 1; in fact, a corrected version of Eq. (2), namely $F(\phi) = a/\phi^m$ with $a \neq 1$, is quite frequently used (this expression tends to cover the typical data but yields a nonsense limit $F(1) = a \neq 1$). In what concerns Eq. (4), it is sometimes satisfied (see Fig. 2) sometimes not (see Refs. [3 - 5]). In the present paper we propose fractal-based heuristic generalizations of Eq. (2) and (4) (and consequently of Archie's law) which enable a satisfactory agreement between theory and experiments such as those reported in Refs. [2 - 5] as well as those exhibited herein (Figs. 1 and 2).

It has been convincingly argued (Ref. [6]) that Archie's law is underlain by the fractal nature of the porous rock; consistently, the m and n exponents should be closely related to the relevant fractal dimensionalities associated with the rock. Moreover, if the length scale is small enough, a micropore structure might become relevant (see Refs. [1] and [5] as well as Fig. 3). It is consequently very natural to assume that crossovers might occur in both Eqs. (2) and (4). For example, if $S_w << 1$, the electrical conduction will occur through a brine layer which might now be extremely close to the rock internal walls, and therefore the micropore fractality will be the relevant one. If its fractal dimensionalities are different from those of the pore structure, a different value is expected to emerge for the saturation exponent. More precisely, while we expect the value n for $S_w \lesssim 1$, the value n' ($n' \gtrless n$) might appear for $S_w << 1$. Analogous considerations should hold for F (ϕ), the porosity exponent being m for $\phi \lesssim 1$ and m' ($m' \gtrless m$) for $\phi << 1$.

Let us now propose mathematical equations consistent with the just mentionned standpoint. Eqs. (1) and (3) imply

$$R_t(\phi,S_w) = F(\phi)\cdot I(S_w)\cdot R_w \qquad (7)$$

Since the geological process of the porous rock formation is in principle independent from the various physico-chemical phenomena involved in the electrical conduction through the saturating water, it seems a reasonable first approximation to preserve, through the present generalization, the factorisation we observe in Eq. (7). So, our proposal will restrict itself to generalize Eqs. (2) and (4). Let us work out F(ϕ) (the treatment of I (S_w) will be completely analogous). We wish

$$F \sim \begin{cases} 1/\phi^m & \text{if } \phi \to 1 \quad (m > 0) \qquad (8.a) \\ B_F/\phi^{m'} & \text{if } \phi \to 0 \quad (m' > 0,\ B_F > 0) \qquad (8.b) \end{cases}$$

If we define

$$x \equiv \ln \phi \tag{9.a}$$

$$y \equiv \ln F \tag{9.b}$$

what we wish is equivalent to

$$y \sim \begin{cases} -mx & \text{if } x \to 0 \quad (10.a) \\ \ln B_F - m'x & \text{if } x \to -\infty \quad (10.b) \end{cases}$$

We propose

$$y = f\,(x;\, m,\, m',\, B_F) \tag{11}$$

where

$$f\,(x;\, m,\, m',\, B_F) \equiv \frac{x\left[\, m - \dfrac{m'(m - m')}{\ln B_F}\, x \,\right]}{\dfrac{m - m'}{\ln B_F}\, x - 1} \tag{12}$$

We straightforwardly verify that Eq. (12) satisfies the asymptotic Eqs. (10); see Fig. 4. The crossover point x_C is given by

$$x_C = \frac{\ln B_F}{m' - m} \tag{13}$$

Since $x_C \le 0$, we have $B_F > 1$, $B_F = 1$ and $B_F < 1$ if $m' < m$, $m' = m$ and $m' > m$ respectively. Eq. (2) is recovered for arbitrary ϕ, as the particular cases for which ($m' = m$; $B_F = 1$) or ($m' > m$; $B_F \to 0$) or ($m' < m$; $B_F \to \infty$).

Eq. (11) has an explicit inverse which is given by

$$x = f^{-1}(y;\, m,\, m',\, B_F) \equiv$$

$$\equiv \left[m - \frac{m - m'}{\ln B_F}\, y - \sqrt{\left(m - \frac{m - m'}{\ln B_F}\, y\right)^2 + 4\, \frac{m'(m - m')}{\ln B_F}\, y}\, \right] \cdot$$

$$\cdot \left[2\, m'\, (m - m') \,/\, \ln B_F\right]^{-1} \quad (14)$$

For $I(S_w)$ we consistently propose

$$\ln I = f\, (\ln S_w;\, n,\, n',\, B_I) \qquad (15)$$

with $B_I > 0$ and $n \gtrless n' > 0$. By putting together Eqs. (7), (11), (12) and (15) we obtain the following generalized Archie's law

$$R_t = \exp\,[f\,(\ln \phi;\, m,\, m',\, B_F)]\, \exp\,[f\,(\ln S_w;\, n,\, n',\, B_I)]\, R_w \qquad (16)$$

This equation implies

$$S_w = \exp\left\{ f^{-1}\left(\ln \left[\frac{R_t / R_w}{\exp\,(f\,(\ln \phi;\, m,\, m',\, B_F))} \right];\, n,\, n',\, B_I \right) \right\} \qquad (17)$$

which generalizes Eq. (6), thus closing the procedure for evaluating the oil content of a given reservoir.

In order to exhibit the applicability of the present proposal we have run the data appearing in Refs. [3, 5] and have obtained, through use of standard nonlinear fitting procedures, the following results:

	n	n′	ln B_I
Ref. [3] (water-wet)	1.90	4.60	-6.00
Ref. [3] (oil-wet)	10.75	2.75	7.50
Ref. [5] (textured beds)	2.71	0.66	2.37
Ref. [5] (smooth beds)	2.98	0.35	10.11

Let us conclude by saying that, since the theory-experiment agreement we can see in Figs. 1 and 2 (as well as in our treatment of the data of Ref. [5]) is quite satisfactory, the present proposal should, for most cases, enable oil evaluations quite more reliable than those which essentially use either the original ($a = 1$) or the corrected ($a \neq 1$) Archie's law without allowing for fractal crossovers. In those few cases where the crossover knee is nearly a cusp (see, for instance, Ref. [3]) the present proposal must be further improved.

We acknowledge computational assistance from S. A. Cannas.

Two of us (C.T. and E.M.F.C.) are pleased to acknowledge warm hospitality received at PETROBRAS/CENPES.

Caption for Figures

Fig. 1 - Formation factor F versus porosity ϕ obtained from rock samples of Brazilian wells 1(a), 2(b) and 3(c).

Fig. 2 - Resistivity index I *versus* water saturation S_w obtained from rock samples of Brazilian well 3. Data were obtained with drainage through porous plates method.

Fig. 3 - Samples from Brazilian well 3: (a) Thin section photomicrograph (plain light); (b) Scanning Electron Microscopy photomicrograph. Brine and petroleum are contained in pore (P) and micropore (MP). The micropores are those associated with the clay coatings around the grains (G).

Fig. 4- Schematic graph of $y \equiv \log F$ *versus* $x \equiv \log \phi$ (generalized Archie's law).

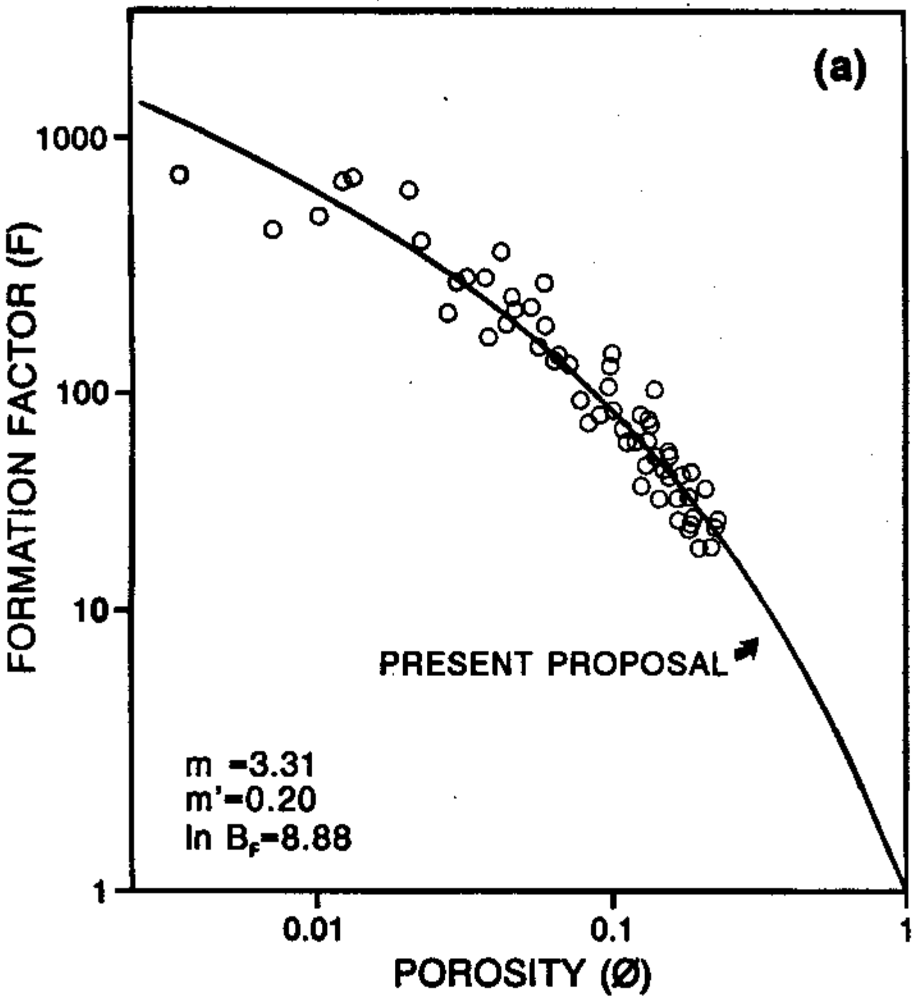
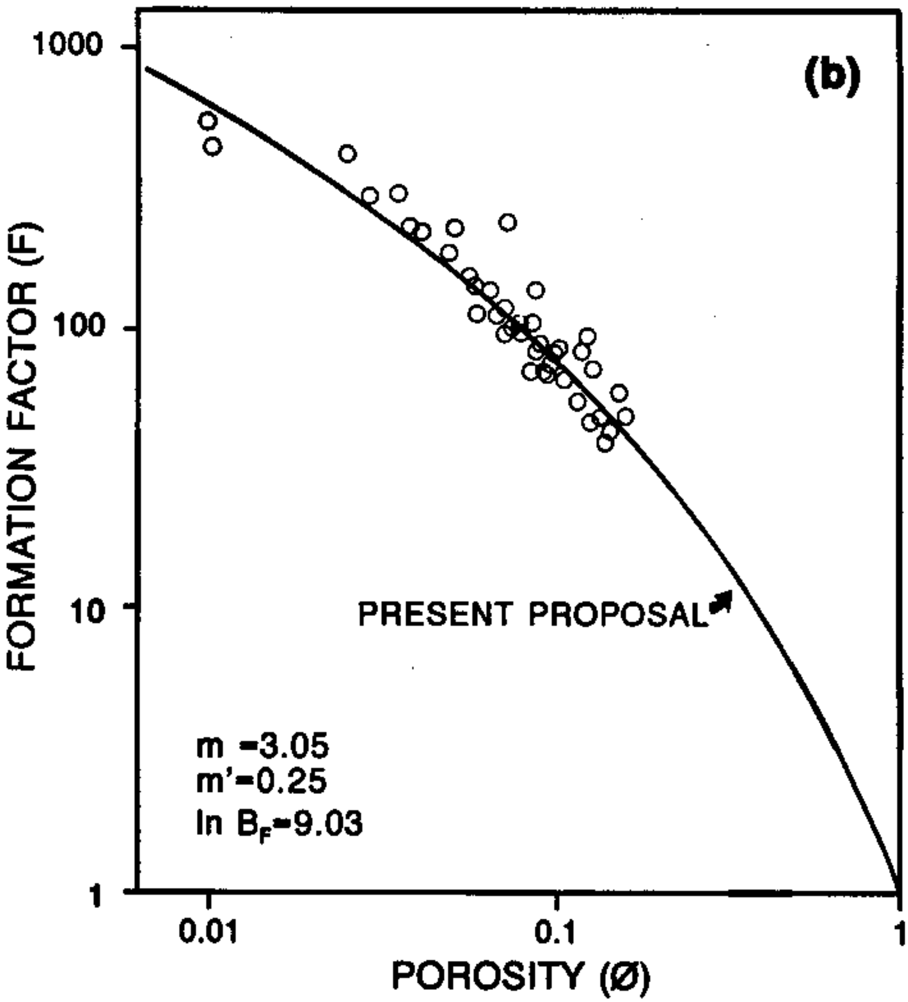

(a)
1000
100
10
1
FORMATION FACTOR (F)
0.01
0.1
1
POROSITY (Ø)
PRESENT PROPOSAL
m =3.31
m'=0.20
ln B_F=8.88

Fig. 1



(b)
1000
100
10
1
0.01
0.1
1
FORMATION FACTOR (F)
POROSITY (Ø)
PRESENT PROPOSAL
m =3.05
m'=0.25
ln B$_F$=9.03

Fig. 1

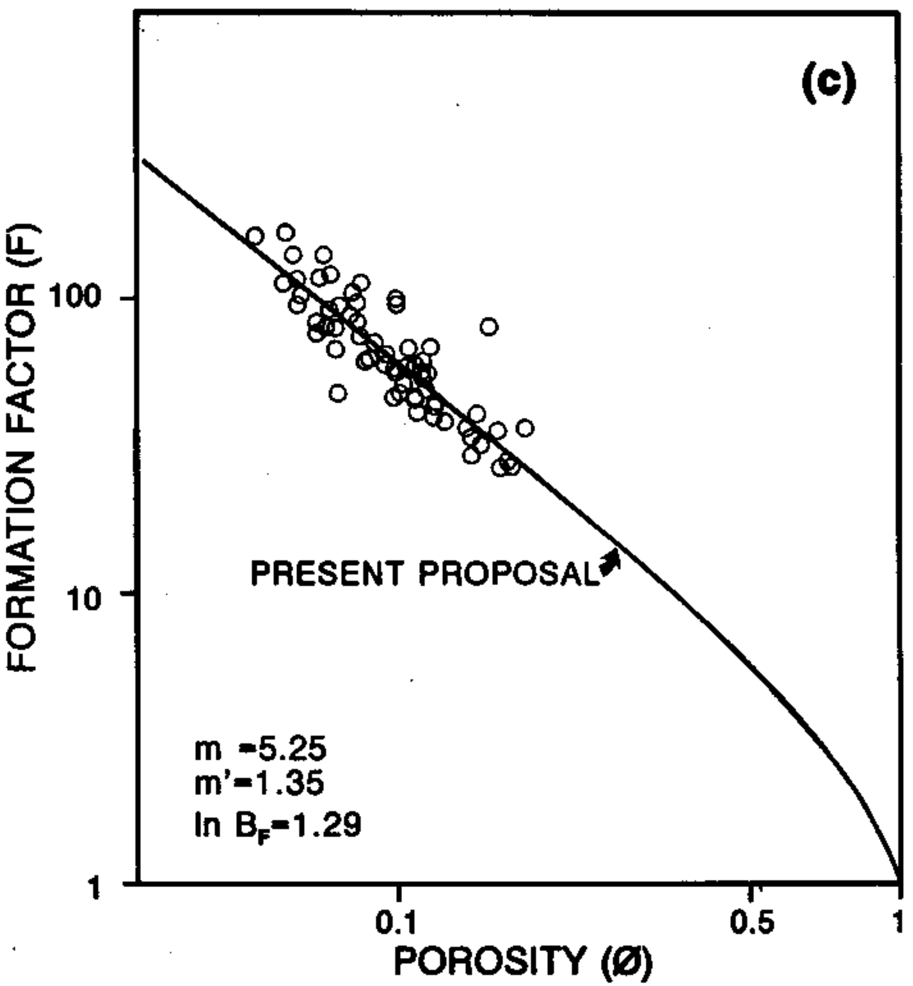

(c)
FORMATION FACTOR (F)
100
10
1
PRESENT PROPOSAL
m =5.25
m'=1.35
ln B_F=1.29
0.1
0.5
1
POROSITY (Ø)

Fig. 1

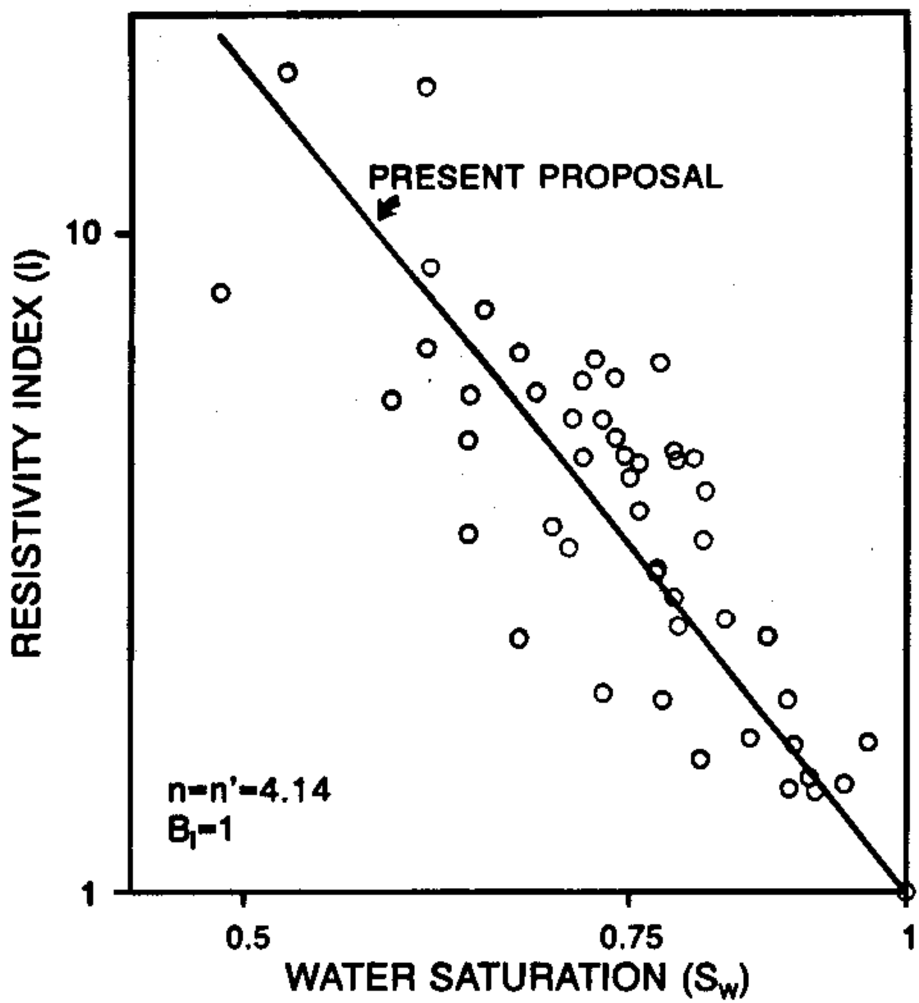
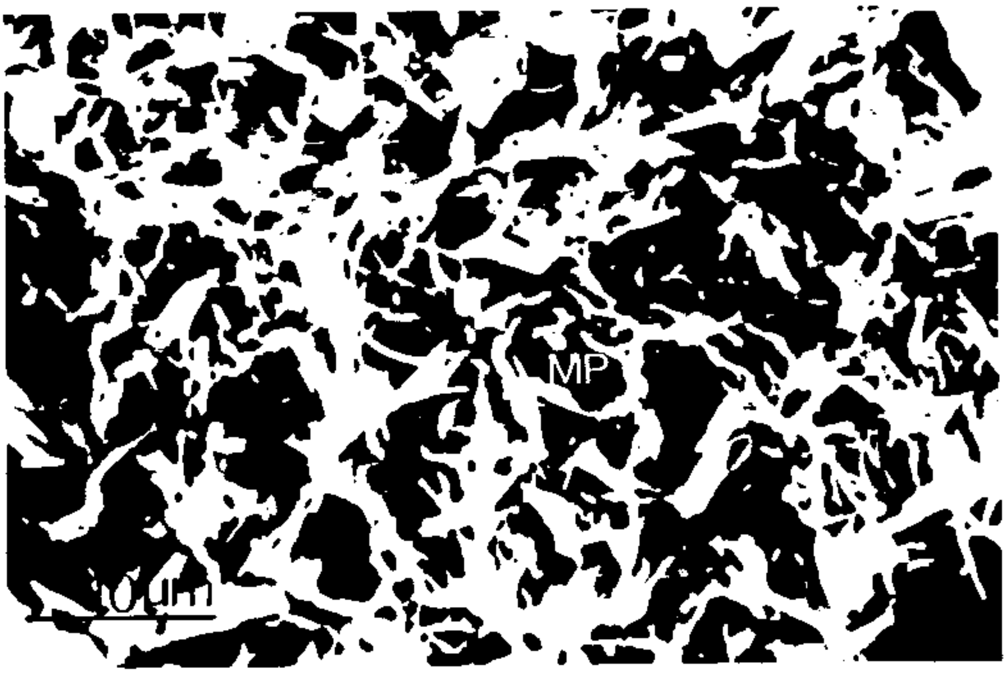

PRESENT PROPOSAL
10
1
RESISTIVITY INDEX (I)
n=n'=4.14
B_I=1
0.5
0.75
1
WATER SATURATION (S_w)

Fig.2

(a)



(b)

Fig.3

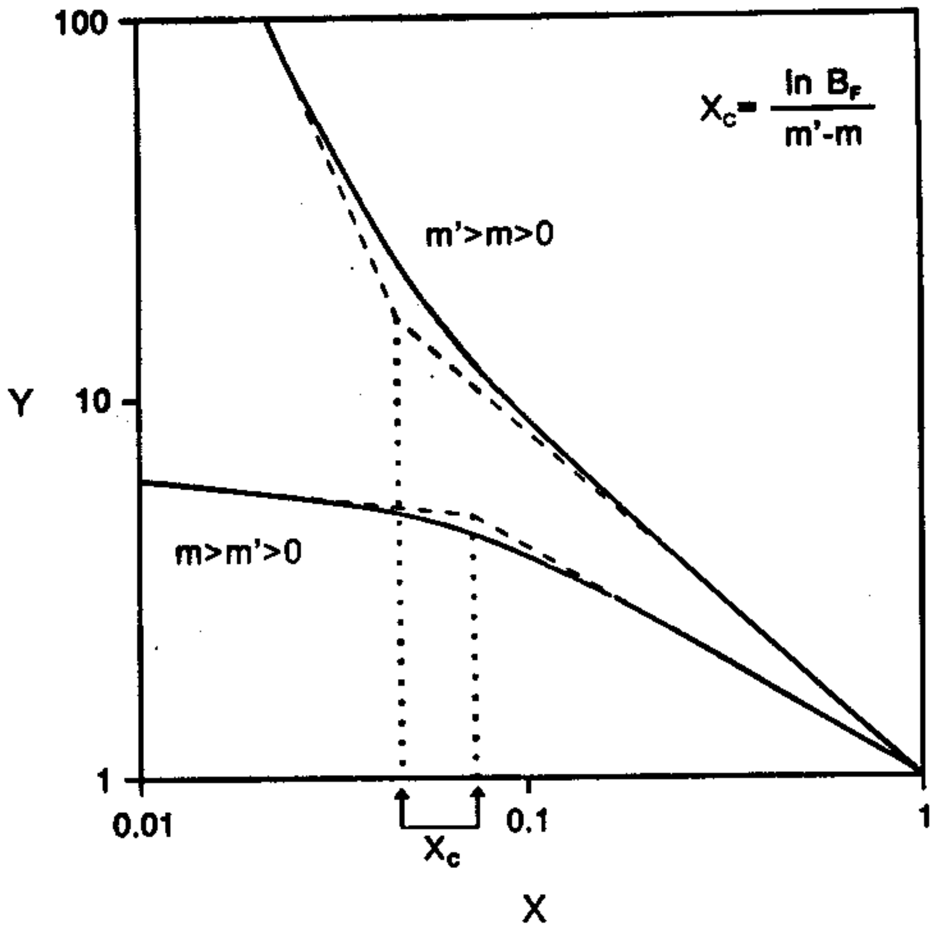


Fig.4

References

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