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FULLY ANISOTROPIC TRIANGULAR LATTICE QUENCHED BOND-RANDOM POTTS FERROMAGNET: ALMOST EXACT CRITICAL FRONTIER *

by

Constantino TSALLIS

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - BRASIL

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ABSTRACT

Through convenient duality and star-triangle-type generalized transformations, we obtain a (presumably excellent for $1 \leqslant q \leqslant 4$) approximate phase diagram for the fully anisotropic triangular lattice quenched bond-random q-state Potts ferromagnet. Several **exact** particular results are recovered ; a small error is however present in one of the limiting slopes.

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C. Tsallis

Centro Brasileiro de Pesquisas Físicas/CNPq
Rua Dr. Xavier Sigaud 150, Rio de Janeiro, Brazil

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Through convenient duality and star-triangle-type generalized transformations, we obtain a (presumably excellent for $1 \lesssim q \lesssim 4$) approximate phase diagram for the fully anisotropic triangular lattice quenched bond-random q-state Potts ferromagnet. Several exact particular results are recovered; a small error is however present in one of the limiting slopes.

I. INTRODUCTION

A certain amount of effort is presently being devoted to the study of random models, in particular the quenched bond-random q-state Potts ferromagnet (characterized by the Hamiltonian $\mathcal{H} = -q\sum_{i,j} J_{ij}\,\delta_{\sigma i,\sigma j}$, where $\sigma_i = 1,2,\ldots,q$ for all sites and $J_{ij} \geq 0$) in regular lattices (see the excellent recent review by Wu [1] and references therein). We are herein concerned with the phase diagram (or critical frontier, CF) associated with a quite complex model, namely that of the fully anisotropic homogeneous triangular lattice (only first-neighbour interactions are taken into account). To be more explicit we shall assume that the J_{ij} along the three crystalline axes of the triangular lattice are respectively given by independent distribution laws $P_k(J_{ij})$ (k=1,2,3). To achieve a proposal for the still unknown associated CF we shall follow along the (conjectural) lines we have recently established [2] for the fully anisotropic square lattice.

II. FORMALISM AND CONJECTURE

With a bond whose coupling constant is J_k, let us associate a convenient variable (hereafter referred as <u>transmissivity</u>)

$$t_k \equiv \frac{1-e^{-qJ_k/k_BT}}{1+(q-1)e^{-qJ_k/k_BT}} \in [0,1] \qquad (1)$$

The equivalent transmissivities t_s and t_p of the elementary series and parallel arrays of two bonds (associated with t_1 and t_2)

are respectively given[2,3] by $t_s = t_1 t_2$ and $t_p^D = t_1^D t_2^D$ where $t_k^D \equiv (1-t_k)/[1+(q-1)t_k]$, $(k = 1,2,p)$ (D stands for dual). Furthermore the transmissivities t_Δ and t_{YD} associated with the three-rooted graphs indicated in Figs. 1.a and 1.b are respectively given[3] by $t_\Delta = [t_1t_2 + t_2t_3 + t_3t_1) + (q-3)t_1t_2t_3]/[1+(q-1)t_1t_2t_3]$ and $t_{YD} = t_1^D t_2^D t_3^D$. The laws $\{P_k(J_{ij})\}$ provide, through definition (1), the t-variable laws (noted $Q_k(t)$) and the t^D-variable ones (noted $Q_k^D(t^D)$). Finally if we decorate the graph of Fig. 1.a (Fig.1.b) with $\{Q_k(t)\}(\{Q_k^D(t^D)\})$ in the places of $\{t_k\}(\{t_k^D\})$ we obtain an equivalent distribution $Q_\Delta(t)(Q_{YD}(t^D))$ given by

$$Q_\Delta(t) = \iiint \left[\prod_{k=1}^{3} dt_k \; Q_k \; (t_k)\right] \delta(t - t_\Delta(t_1,t_2,t_3)) \tag{2}$$

$$Q_{YD}(t^D) = \iiint \left[\prod_{k=1}^{3} dt_k^D \; Q_k^D(t_k^D)\right] \delta(t^D - t_{YD}(t_1^D,t_2^D,t_3^D)) \tag{3}$$

Let us now introduce the convenient variable [2,4,5] $s \equiv \{\ell n[1+(q-1)t]\}/\ell nq \in [0,1]$, which coincides with t in the limit $q\to 1$, and satisfies a remarkable property, namely $s^D(t) \equiv s(t^D) = 1 - s(t)$; this fact stands at the center of the conjectural proposal we shall immediately present. A good approximation of the CF we are looking for is given by

$$\langle s \rangle_{Q_\Delta} \equiv \int_0^1 dt\, s(t) Q_\Delta(t) = \int_0^1 dt\, s(t) Q_{YD}(t) \equiv \langle s \rangle_{Q_{YD}} \tag{4}$$

We are aware of the fact that stated like this, the above equation seems to be completely unjustified; let us however say that the set of particular situations (too lengthy to be recalled herein) analyzed in Ref. [2-5] very strongly suggests Eq. (4). In what follows we shall restrict ourselves to exhibit how this equation recovers a considerable amount of exact particular results.

III. PARTICULAR CASES

We shall consider the bond-dilute situation (J_{ij} either vanishes or takes an unique finite value, eventually different for each direction), which corresponds to $Q_k(t) = (1-p_k)\delta(t) + p_k\delta(t-t_k)$ ($k = 1,2,3$) and for which a certain amount of particular exact results are already available.

(i) The anisotropic pure Potts model: ($p_k = 1, \forall k$)

Eq. (4) recovers the exact result [1] which can be re-written as follows:

$$\frac{t_1 t_2 + t_2 t_3 + t_3 t_1 + (q-3) t_1 t_2 t_3}{1+(q-1)t_1 t_2 t_3} = \frac{1-t_1}{1+(q-1)t_1} \frac{1-t_2}{1+(q-1)t_2} \frac{1-t_3}{1+(q-1)t_3} \quad (5)$$

(we recall that the phase transition is a second order one only if $q \leq 4$).

(ii) The anisotropic pure percolation model: ($T=0$, hence $t_k = 1$, $\forall k$)

The associated CF is commonly believed to be one and the same for all q, namely that of the bond percolation problem, and is given [6] by $p_1 p_2 p_3 - p_1 - p_2 - p_3 + 1 = 0$

Eq. (4) exactly recovers this relation. The same result can be recovered through Eq. (4) by considering arbitrary $\{t_k\}$ but $q \to 1$ (Kasteleyn and Fortuin isomorphism [1]); in this case we obtain the following (exact) result: $\prod_{k=1}^{3} p_k t_k - \sum_{k=1}^{3} p_k t_k + 1 = 0$.

(iii) The isotropic bond-dilute almost pure percolation model: ($p_k = p \gtrsim p_c$ and $t_k = t_o \lesssim 1$, $\forall$ k)

Eq. (4) recovers the exact [7] (or believed so) derivative $dt_o/dp|_{p=p_c} = -q\ell nq/p_c(q-1)$ ($q \leq 4$) where p_c satisfies $p_c^3 - 3p_c + 1 = 0$.

iv) The isotropic bond-dilute almost pure Potts model: ($p_k = p \lesssim 1$ and $t_k = t_o \gtrsim t_c$, $\forall$ k)

The value t_c satisfies Eq. (5) for $t_1 = t_2 = t_3$. The following derivatives [7] are believed to be exact:

$$- \frac{dt_o}{dp}\bigg|_{p=1} = \begin{cases} 0.3473\ldots & \text{for } q=1 \;; \\ 0.3028\ldots & \text{for } q=2 \;; \\ 0.2720\ldots & \text{for } q=3 \;; \\ 0.2494\ldots & \text{for } q=4 \;. \end{cases} \tag{6}$$

Eq. (4) provides instead

$$- \frac{dt_o}{dp}\bigg|_{p=1} = \begin{cases} 0.3473\ldots & (0\% \text{ error}) & \text{for } q=1 \;; \\ 0.3040\ldots & (0.40\% \text{ error}) & \text{for } q=2 \;; \\ 0.2750\ldots & (1.11\% \text{ error}) & \text{for } q=3 \;; \\ 0.2539\ldots & (1.80\% \text{ error}) & \text{for } q=4 \;. \end{cases} \tag{7}$$

IV. CONCLUSION

On conjectural grounds, we have proposed an equation (namely Eq. (4)) for the still unknown critical frontier of a quite general and complex bond-random q-state Potts ferromagnet in the anisotropic triangular lattice. Although the proposal is not exact (small errors have been detected for $q > 1$ in the asymptotic slope when we approach the pure Potts model in the isotropic bond-dilute problem), we believe it is an excellent approximation (at least for $1 \leq q \leq 4$) as it provides the <u>exact</u> results in the anisotropic pure Potts and pure bond percolation limits, as well as in the low temperature neighbourhood of the isotropic bond-dilute problem; furthermore the present proposal fully preserves the $q \to 1$ Kasteleyn and Fortuin isomorphism[1] . In a forthcoming paper we shall give more details (as well as supplementary checks) on the present conjecture and we shall perform a natural transformation in order to cover the honeycomb lattice as well.

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CAPTION FOR FIGURE

Fig. 1 – Triangle (a) and star (b) three-rooted graphs; o (●) denotes terminal (internal) sites (see [3,5]).

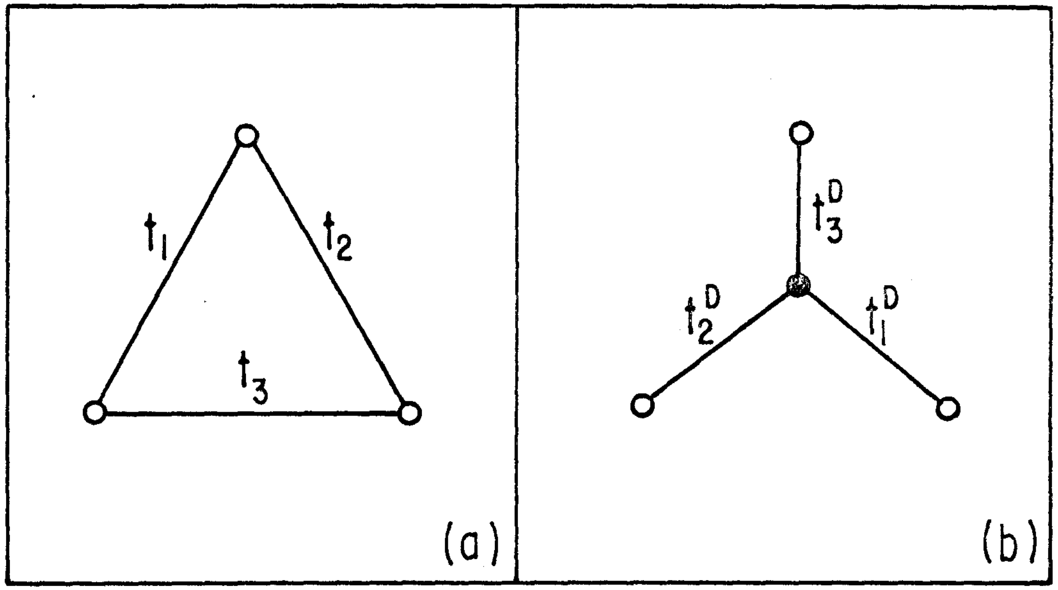

FIG.1