REMARKS ON THE POSSIBILITY OF NONEXISTENCE OF THE AHARONOV-BOHM EFFECT (ESAB EFFECT)

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ABSTRACT - A recent proposal made by Bocchieri and Loinger about the purely mathematical origin of the Ehrenberg-Siday-Aharonov-Bohm effect (ESAB effect) is examined. Inconsistencies are shown that invalidate the conclusions of Bocchieri and Loinger, also recently criticized by Klein.

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As is well known, the Ehrenberg-Siday-Aharonov-Bohm[1] (ESAB) effects should illustrate the rôle of electromagnetic potentials, rather than fields, in quantum mechanics. Electrons moving in a field free region have its diffraction or interference patterns disturbed by a non zero potential appearing in that region. The potential manifests through the value of its line integral on a closed contour, which in turn is equal to the value of the flux of the magnetic field enclosed by the contour, by elementary Gauss-theorem.

A similar reasoning applies for the case of a plane rotator when we compare the result for its energy levels with that obtained with an infinite straight solenoid placed concentric with the rotator.

It was soon recognized by many phyisicists that the ESAB effects did not contradict any previous evidence, and as was stated by H.J.Lipkin in those times[*], they merely joined classical electrodynamics with the fact of electrons obeying the wave Schrödinger equation instead of newtonian equations of motion. However, a number of physicists tried along 20 years to reformulate the relevant theory keeping the predominant rôle of the fields as is the case in the classical theory[2].

Recently, P. Bocchieri and A. Loinger[3] (B.L.) advanced arguments favouring the point of view that ESAB effects have a purely mathematical origin, and there is room enough in the arbitrariness admissible in the currently

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(*) Quoted by Prof. Guido Beck at "1st Brasilian Meeting on Elementary Particles and Field Theory" accepted theory to eliminate these effects completely. Therefore, their experimental tests [4] deserved some caution. A short time ago U.Klein[5] showed in a short note that part of B.L. arguments have no support from a physical point of view. Though we share his conclusion partially, his arguments focused in a definite piece of B.L.'s reasoning, but missed others.

In the present note we intend to show that B. L. arguments are possibly not true from a mathematical and physical standpoint in a systematic way. We profit in passing to coment shortly Klein's work. We develop our reasoning in two steps, dealing first with some general features of the problem at hand (generally well known) and then going to elaborate on those points of B.L.discourse that can be shown to be wrong (or, at least, ill founded). Everywhere in what follows we consider time independent situations, phenomena and solutions.

To begin with, let us recall the meaning of gauge invariance in quantum mechanics. Given a physical situation with a magnetic field and/or vector potential $\vec{A}(\vec{r})$ present and a corresponding solution to the Schrödinger equation $\Psi(\vec{r})$, we are able to sum a gradient term to the vector potential without physical consequences

$$\vec{A}(\vec{r}) \longrightarrow \vec{A}'(\vec{r}) = \vec{A}(\vec{r}) + \text{grad}\,\chi(\vec{r}) \qquad (1)$$

provided the phase of the wave function for any state is adequately modified

$$\Psi(\vec{r}) \longrightarrow \Psi'(\vec{r}) = \exp\left(\frac{ie}{\hbar c}\,\chi(\vec{r})\right)\,\psi(\vec{r}) \tag{2}$$

Something similar happens whenever we choose to modify the (linear) momentum operator representation in the coordinate basis[6]. Let $\Psi(\vec{r})$ the wave function corresponding to a given situation, and have us for the momentum operator:

$$\vec{p} = -\,i\hbar\,\mathrm{grad} \tag{3}$$

This representation is not unique, and can be altered without any physical consequence

$$\vec{p} \longrightarrow \vec{p}\,' = \vec{p} + \mathrm{grad}\,F(\vec{r}) \tag{4}$$

provided we multiply the wave functions by a common phase factor

$$\Psi(\vec{r}) \longrightarrow \Psi'(\vec{r}) = \exp\left(-\,\frac{i}{\hbar}\,F(\vec{r})\right)\,\Psi(\vec{r}) \tag{5}$$

Looking at both couples of equations, (1) and (2), and (4) and (5), it appears as possible to start in a given physical problem with $\vec{p}, \vec{A}(\vec{r})$ and $\Psi(\vec{r})$, and, through a simultaneous change of gauge and of momentum operator representation in the coordinate basis, end with new $\vec{p}\,'$ and $\vec{A}'(\vec{r})$ but the same $\Psi(\vec{r})$. In other terms, it seems possible to absorb the change due to a gauge transformation with a change in the momentum operator representation and viceversa, in the wave function. By absorb we mean that no change is made at the end of the process. This is essentially the content of the work of Bocchieri and Loinger [3].

To be more precise, let us consider three (in principle) diffrent situations. Consider first one with neither magnetic field nor magnetic vector potential present ($\vec{A}(\vec{r}) \equiv 0$). Be $\Psi(\vec{r})$ the solution to the Schrödinger equation; the problem at hand may be the double slit interference device, or the plane rotator, etc. We shall label it as case I.

Let us now consider the same situation, but this time with a "pure gauge" potential in some region of space

$$\vec{A}(\vec{r}) = \text{grad}\ \chi(\vec{r}) \tag{6}$$

This will be our case II. We may now have an ESAB effect provided the line integral of $\vec{A}(r)$ in (6) is not zero on a set of closed contours within the region considered. This is related easily with the presence somewhere else of a confined megnetic field whose potential has to be connected to (6) through continuity conditions on some boundary. Nonetheless, as stated by B.L. we can "gauge" away in all cases the potential and recover the preceding situation (case I), with no ESAB effect. This was in fact used by B.L., who noticed that the potential used by Aharonov-Bohn [1] outside a straight, infinite, circular solenoid with a constant, uniform field $\vec{B}$ inside is

$$\vec{A}(\vec{r}) = \frac{\Phi}{2\pi}\,\frac{1}{r}\,\hat{e}_\phi = \text{grad}\left(\frac{\Phi}{2\pi}\,\phi\right) = \text{grad}\,\chi(\vec{r}) \qquad (7)$$

where ϕ is the azimuth in cylindrical coordinates (with the z-axes in the center of the solenoid), $\hat{e}_\phi$ its corresponding unit vector and Φ is the flux of the field inside the solenoid.

Another point raised by B.L. and mentioned before is that we may profit from the apparent freedom displayed in eqs.(4) and (5) for the choice of momentum representation in the coordinate basis and modify the phase of the wave function such as to compensate (absorb) the change induced by the the ("pure gauge") magnetic potential. Moreover, coming back to the situation of our case I, we would be able, with no magnetic field and/or potential present, to produce an ESAB effect, from a purely mathematical arbitrariness taking in Eq. (4) $F(\vec{r}) = -\chi(\vec{r}) = -\frac{\Phi}{2\pi}\,\phi$. This will constitute our case II.

The important matter is to notice that all three cases presented above are trivially equivalent (no ESAB effect) if χ and/or F are true potential functions in a precise mathematical sense, i.e., if χ and/or F are single valued functions (or continuous functions for the axially symmetric cases with the definition of the azimuth as $0 \leq \phi < 2\pi$). If it were not so, we arrive at physical consequences (shown below) that seems to be unnoticed to Bocchieri and Loinger.

It is simpler to consider case III first. Assume that starting from a pure quantum mechanical situation (as in case I), with no magnetic fields or vector potentials present, a change in the momentum operator representation in the coordinate basis has been performed (Eq.(4)). But Dirac shows[6] that in order to accomplish this the function F should be single valued. This is the very condition for the commutativity of the components of the mechanical canonical momentum be respected:

$$[p'_r \, , \, p'_s] = \frac{\partial^2 F}{\partial q_r \ \partial q_s} - \frac{\partial^2 F}{\partial q_s \ \partial q_r} = 0 \tag{8}$$

Stokes theorem requires then F to be a potential function (in the mathematical sense). Otherwise we give up the simultaneous measuremente of all momentum components and the simple linear transformation properties between coordinate and momentum bases.

Next, we argue about the possibility already mentioned in case II of eliminating the ESAB effect via a "gauge" transformation to fall back in case I. Bocchieri and Loinger proposed , for the case of an infinite, straight, circular solenoid of radius r_0, to eliminate ESAB effects using as a magnetic potential vector,

$$\vec{A}_{BL} = \frac{\Phi}{2\pi} \left(\frac{r}{r_0^2} - \frac{1}{r} \right) \hat{e}_\phi \quad , \quad r \leq r_0 \tag{9}$$

$$= 0 \quad , \quad r < r_0$$

which seems formally related though a "gauge transformation" (7) to the Aharonov-Bohm potential

$$\vec{A}_{BL} = \vec{A}_{AB} - \text{grad} \left(\frac{\Phi}{2\pi} \phi \right) \qquad (10)$$

We can see that this is not the case (i.e., it is not a legitimate gauge transformation) because the function χ in this case is not single valued, and therefore the curl of its gradient cannot be zero everywhere. The two vector potentials, then, do not produce the same magnetic field $\vec{B}$!

We notice that Klein's[(5)] argument regarding B.L. analysis is that the choice between the two potentials above cannot be made only on the ground of its curl, that is, provided they give rise to the same field $\vec{B}$, but it is needed instead to consider whether they are equivalent indeed in every other physical aspect. Since, however, they produce different fields, as we show below, they are not equivalent in a trivial way.

C.G.Bollini and J.J.Giambiagi[(7)] provided the tools needed to arrive at the second of the following expressions:

$$\text{curl } \vec{A}_{BL} = \text{curl } \vec{A}_{AB} - \text{curl} \left(\text{grad} \left(\frac{\Phi}{2\pi} \phi \right) \right) \qquad (11)$$

$$\vec{B}_{BL} = \vec{B}_{AB} - \Phi\, \delta(x)\, \delta(y)\, \hat{e}_z \qquad (12)$$

where $\hat{e}_z$ is the unit vector in the z-direction. The physical picture for $\vec{B}_{BL}$ is that of the usual constant field of a solenoid ($\vec{B}_{AB}$) plus a field concentrated in the axis of the solenoid pointing in the opposite direction. As this second field has the same total flux Φ as the original (first) one, B_{AB}, we see that $\vec{B}_{BL}$ in a rather loose interpretation, corresponds to the field of a solenoid whose lines of force curve to the inside axis at one end ($z=+\infty$) and come back to the other end through the z-axis to open again ($z=-\infty$) into the lines of a solenoid. The lines of force close inside instead of outside. It is no surprise that this field is not able to produce an ESAB effect in the analysis by Bocchieri and Loinger, since the net result depends on the total flux, and this is zero outside the solenoid for $\vec{B}_{BL}$ (and $\vec{A}_{BL}$).

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