STANDSTILL ELECTRIC CHARGE GENERATES MAGNETOSTATIC FIELD UNDER BORN-INFELD ELECTRODYNAMICS

S.O. Vellozo[1,3*], José A. Helayël-Neto[1,2] [†],A.W. Smith[1] [‡]
and L. P. G. De Assis[1,2¶]

[1]*Centro Brasileiro de Pesquisas Físicas – CBPF,*
Rua Dr. Xavier Sigaud 150, 22290-180, Rio de Janeiro, RJ, Brasil
[2]*Grupo de Física Teórica José Leite Lopes,*
P.O. Box 91933, 25685-970, Petrópolis, RJ, Brasil
[3]*Centro Tecnologico do Exército – CTEx*
Av. das Americas 28705, 230020-470, Rio de Janeiro, RJ, Brasil

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Abstract

The Abelian Born-Infeld classical non-linear electrodynamic has been used to investigate the electric and magnetostatic fields generated by a point-like electrical charge at rest in an inertial frame. The results show a rich internal structure for the charge. Analytical solutions have also been found. Such findings have been interpreted in terms of vacuum polarization and magnetic-like charges produced by the very high strengths of the electric field considered. Apparently non-linearity is to be accounted for the emergence of an anomalous magnetostatic field suggesting a possible connection to that created by a magnetic dipole composed of two mognetic charges with opposite signals. Consistently in situations where the Born-Infeld field strength parameter is free to become infinite, Maxwell's regime takes over, the magnetic sector vanishes and the electric field assumes a Coulomb behavior with no trace of a magnetic component. The connection to other monopole solutions, like Dirac's, t' Hooft's or Poliakov's types, are also discussed. Finally some speculative remarks are presented in an attempt to explain such fields.

[*]E-mail: vellozo@cbpf.br
[†]E-mail: helayel@cbpf.br
[‡]E-mail: awsmith@cbpf.br
[§]E-mail: lpgassis@cbpf.br
[¶]L.P.G. De Assis is leaving CBPF to Universidade Federal Rural do Rio de Janeiro

1   INTRODUCTION

This work investigates, under a classical approach and exploring the non-linear properties of the Abelian B-I Theory, the configuration of the fields generated by a single electric charge at rest. The main motivation for this paper came from the question if a pure electic point-like charge at rest in an inertial frame generates some kind of magnetostatic field. The challenge resides on find classical solutions from B-I magnetic sector without speculative additional assumptions. Only a few suitable restrictions have been imposed. The results can be listed as following:

a) There is analytical and real stable magnetostatic dipole-like solution generated by intense electric field strength;

b) It is possible to define a single magnetic charge in terms of the electric point-like charge;

c) The findings is consistent concerning to the Maxwell linear theory. In other words, it vanishes when the B-I parameter b is free to become infinite.

d) The magnetic charge intensity calculated is close Dirac's prediction.

Born-Infeld (B-I) non-linear classical electrodynamics [1, 3] represents an advanced theory to explain the structure and the finite energy of the electron. It emerges in the more broad context of the M-Theory, where the Superstrings Theories are enclosed. Recent revival of nonlinear electrodynamics has been verified, mainly due to the fact that these theories appear as effective theories at different levels of string/M-theory, in particular in Dp-branes and super-symmetric extensions, and non-Abelian generalizations. B-I Lagrangian describes the electromagnetic fields that live on the world-volume of D-branes and T-duality gives direct evidence that it governs the dynamics of the electromagnetic fields on D-branes[2, 12]. B-I Lagrangian density is one of the general non-derivative Lagrangians which depend only on the two algebric Maxwell invariants. Among others its most attractive properties B-I Lagrangian is one of the simplest non-polynomials that preserve gauge and Lorentz invariance, the vacuum is characterized by $f_{\mu\nu} = 0$ and the energy density is positive. The field strength $f_{\mu\nu}$ is finite everywhere and is characterized by its length r_o. B-I Theory is the only non linear electrodynamic theory ensuring the absence of bi-refringence, this is, the vacuum light speed is always c.

The organization of this paper is as follow: in Section 2 one accounts for the exposition of the problem and the formulation of the main assumptions and constraints. In Section 3 one gets the angular and radial differential equations and solves then. Section 4 sets up the connection of the angular moment and magnetic charge. Section 5 fixes the single magnetic charge and Section 6 makes some final considerations.

1.1   CLASSICAL BORN-INFELD EQUATIONS IN MINKOVSKI SPACE-TIME

In these two sub-sections one exposes the standard Abelian B-I theory embeded in flat space. The signature of the metric tensor and the first assumption is established. The constitutive relations are suitable constructed in order to make sure the integrability of the system.

1.1.1   THE ELECTRIC CHARGE AT REST

The B-I non-linear electrodynamics action [1] is defined, in Minkovski space-time, as:

$$S = \int d^4x b^2 \left[1 - \sqrt{-\det\left(\eta_{\mu\nu} + \frac{f_{\mu\nu}}{b}\right)} \right]$$

The metric tensor $\eta_{\mu\nu}$ has signature (1,-1,-1,-1) and $f_{\mu\nu}$ is the electromagnetic tensor. The parameter b, like the speed of light in Einstein's relativity theory, is the maximum field strength allowed by the B-I Theory and has a large value (about 10^{15} esu). Setting its value to infinite leads to Maxwell's linear electrodynamics. That means that there is no limit to the field strength in Maxwell linear electrodynamics. Enclosed in the action integral is the Born-Infeld Lagrangian and evaluating its determinant yields:

$$L = b^2 \left[1 - \sqrt{1 - \frac{E^2 - B^2}{b^2} - \left(\frac{\overrightarrow{E} \cdot \overrightarrow{B}}{b^2} \right)^2} \right]$$

In addition the electric induction $\overrightarrow{D}$ and the magnetic field $\overrightarrow{H}$, as derived from the canonical relation, are:

$$\overrightarrow{D} = \frac{\partial L}{\partial \overrightarrow{E}} = \frac{\overrightarrow{E} + \left(\frac{\overrightarrow{E} \cdot \overrightarrow{B}}{b^2} \right) \overrightarrow{B}}{\sqrt{1 - \frac{E^2 - B^2}{b^2} - \left(\frac{\overrightarrow{E} \cdot \overrightarrow{B}}{b^2} \right)^2}} \qquad (1)$$

$$\overrightarrow{H} = \frac{\partial L}{\partial \overrightarrow{B}} = \frac{\overrightarrow{B} - \left(\frac{\overrightarrow{E} \cdot \overrightarrow{B}}{b^2} \right) \overrightarrow{E}}{\sqrt{1 - \frac{E^2 - B^2}{b^2} - \left(\frac{\overrightarrow{E} \cdot \overrightarrow{B}}{b^2} \right)^2}} \qquad (2)$$

The interaction with other charged particles is introduced by adding a term $j_\mu A^\mu$ to the B-I Lagrangian. The equations of motion are the standard Maxwell equations and the non-linearity is inserted in equations (1) and (2). For a static point-like charge these equations, on macroscopic level, are:

$$\overrightarrow{\nabla} \cdot \overrightarrow{D} = e\delta(\overrightarrow{x}) \qquad \overrightarrow{\nabla} \times \overrightarrow{E} = \overrightarrow{0}$$
$$\overrightarrow{\nabla} \cdot \overrightarrow{B} = 0 \qquad \overrightarrow{\nabla} \times \overrightarrow{H} = \overrightarrow{0}$$

and the solution for the electric induction $\overrightarrow{D}$, by taking $\overrightarrow{B} = \overrightarrow{H} = \overrightarrow{0}$, is well known [1, 3, 4]. It is singular and identical to the Maxwell solution, while the field $\overrightarrow{E}$ remains well defined at all points, even at $r = 0$. Thus:

$$\overrightarrow{D} = \frac{e}{4\pi r^2} \widehat{r} \qquad and \qquad \overrightarrow{E} = \frac{e}{\sqrt{r^4 + r_o^4}} \widehat{r} \qquad (3)$$

$$r_o = \sqrt{\frac{e}{4\pi b}} \qquad \widehat{r} = \overrightarrow{r}/|\overrightarrow{r}|$$

These are the necessary tools, for the present work, to describe that electric charge in more broad way. No change will be done on electric sector and it will be preserved by appropriate assumptions raised further.

1.1.2 THE NON ZERO MAGNETOSTATIC SECTOR

What are the consequences of not setting $\overrightarrow{B}$ and $\overrightarrow{H}$ equal to zero? What kind of fields could the theory provide? If real and finite solutions do exist then what originates those fields? In order to try to answer such questions one must consider each component of the constitutive equations (1) and (2) at a time. By assuming that the magnetic sector has only radial and polar components that are functions of the radial distance r from the point-like charge, and the polar angle θ, yields the following relations for $\overrightarrow{B}$ and $\overrightarrow{H}$:

$$\overrightarrow{H}(r,\theta) = H_r(r,\theta)\widehat{r} + H_\theta(r,\theta)\widehat{\theta} \qquad (4)$$
$$\overrightarrow{B}(r,\theta) = B_r(r,\theta)\widehat{r} + B_\theta(r,\theta)\widehat{\theta}$$

Where the subscripts refer to the components of the vector. So the problem is axially symmetric. The introduction of φ dependence will violate the $\overrightarrow{\nabla} \times \overrightarrow{H} = \overrightarrow{0}$.

Then the constitutive relation (1) and (2) for each component is written leading to an algebric non-linear system of equations relating all electric and magnetic components.

$$E_r = D_r \frac{R}{\left(1 + \frac{B_r^2}{b^2}\right)} \qquad E_\theta \approx 0 \tag{5}$$

$$B_r = H_r \frac{R}{\left(1 - \frac{E_r^2}{b^2}\right)} \qquad B_\theta = H_\theta R \tag{6}$$

with

$$R = \sqrt{\left(1 + \frac{B_r^2}{b^2}\right)\left(1 - \frac{E_r^2}{b^2}\right) + \frac{B_\theta^2}{b^2}}$$

The magnetostatic field components (B_r and B_θ) are then required to satisfy the following constraints $B_r \ll b$ and $B_\theta \ll b$ at all points, while preserving the major feature of the theory, that is, its nonlinearity. This is the first assumption introduced and is necessary in order to ensure that the system remains integrable. The aforementioned set of equations is then reduced to:

$$R = \sqrt{\left(1 + \frac{B_r^2}{b^2}\right)\left(1 - \frac{E_r^2}{b^2}\right) + \frac{B_\theta^2}{b^2}} \approx \sqrt{1 - \frac{E_r^2(r)}{b^2}}$$

$$E_r(r) = \frac{D_r(r)}{\sqrt{1 + \frac{D_r^2(r)}{b^2}}} \tag{7}$$

$$B_r(r, \theta) = H_r(r, \theta)\sqrt{1 + \frac{D_r^2(r)}{b^2}} \tag{8}$$

$$B_\theta(r, \theta) = \frac{H_\theta(r, \theta)}{\sqrt{1 + \frac{D_r^2(r)}{b^2}}} \tag{9}$$

That action keeps the B-I original behavior of the electric sector while still preserving the connection between the magnetic and electric sectors. At this point one concludes that, under the first assumption, equations (7,8,9) tell that the electric sector shall not be affected by the induced magnetic sector. However each one depends on $D_r(r)$, a subtle message from the electric to the magnetic sector.

2 SOLUTION TO THE BORN-INFELD EQUATION

This section is devoted to construct and solve the main differential equation. One more basic assumption is necessary.in order to turn the system integrable.

The field $\overrightarrow{H}(r, \theta)$ is to satisfy $\overrightarrow{\nabla} \times \overrightarrow{H}(r, \theta) = \overrightarrow{0}$ and the field $\overrightarrow{B}(r, \theta)$ must be such that $\overrightarrow{\nabla} \cdot \overrightarrow{B}(r, \theta) = 0$. So one has two PDE for the components of the fields.

$$\frac{1}{r}\left[\partial_r(rH_\theta(r, \theta)) - \partial_\theta H_r(r, \theta)\right] = 0$$

$$\frac{1}{r^2}\partial_r[r^2 B_r(r, \theta)] + \frac{1}{r\sin(\theta)}\partial_\theta[\sin(\theta)B_\theta(r, \theta)] = 0$$

Now one assumes that the variables can be separable. This is the second assumption of this paper. The magnetic field components can be written as:

$$H_r(r,\theta) = h_r(r)G(\theta) \qquad H_\theta(r,\theta) = h_\theta(r)J(\theta) \tag{10}$$

The magnetic induction components are then determined by the magnetic field and become:

$$B_r(r,\theta) = b_r(r)G(\theta) = h_r(r)\sqrt{1 + \frac{D_r^2(r)}{b^2}}G(\theta) \tag{11}$$

$$B_\theta(r,\theta) = b_\theta(r)J(\theta) = \frac{h_\theta(r)}{\sqrt{1 + \frac{D_r^2(r)}{b^2}}}J(\theta) \tag{12}$$

Where $h_r(r)$, $h_\theta(r)$, $G(\theta)$ and $J(\theta)$ are unknown functions to be determined and $b_r(r) = h_r(r)\left(1 + \frac{D_r^2(r)}{b^2}\right)^{1/2}$ as well $b_\theta(r) = h_\theta(r)\left(1 + \frac{D_r^2(r)}{b^2}\right)^{-1/2}$.

Then substituting $H_r(r,\theta)$ and $H_\theta(r,\theta)$ on $\overrightarrow{\nabla} \times \overrightarrow{H}(r,\theta) = \overrightarrow{0}$, yields a set of two differential equations where λ is a constant.

$$\frac{1}{h_r(r)}\frac{d[rh_\theta(r)]}{dr} = \frac{1}{J(\theta)}\frac{dG(\theta)}{d\theta} = \lambda \tag{13}$$

Likewise, the substitution of $B_r(r,\theta)$ and $B_\theta(r,\theta)$ in $\overrightarrow{\nabla} \cdot \overrightarrow{B}(r,\theta) = 0$ leads to:

$$\frac{1}{rb_\theta(r)}\frac{d}{dr}\left[r^2 b_r(r)\right] = -\frac{1}{\sin(\theta)G(\theta)}\frac{d(\sin(\theta)J(\theta))}{d\theta} = \varsigma \tag{14}$$

Where ς is another constant. That can be resumed in a system of four differential equations further.

$$\frac{d[rh_\theta(r)]}{dr} = \lambda h_r(r) \tag{15}$$

$$\frac{dG(\theta)}{d\theta} = \lambda J(\theta) \tag{16}$$

$$\frac{d}{dr}\left[r^2 b_r(r)\right] = \varsigma rb_\theta(r) \tag{17}$$

$$\frac{d(\sin(\theta)J(\theta))}{d\theta} = -\varsigma \sin(\theta)G(\theta) \tag{18}$$

Considering the angular equations (16) and (18) one arrives on a second order differential equation for $G(\theta)$.

$$\frac{d}{d\theta}\left\{\sin(\theta)\frac{d(G(\theta))}{d\theta}\right\} + \lambda\varsigma \sin(\theta)G(\theta) = 0 \tag{19}$$

which has the general angular solution given in terms of the Legendre functions of the first kind $P_n(\cos(\theta))$ and Legendre function of second kind $Q_n(\cos(\theta))$. The second has has the undesirable feature of not be always real.

$$G_n(\theta) = C_1 P_n(\cos(\theta)) \qquad n = \frac{\sqrt{1 + 4\lambda\varsigma}}{2} - \frac{1}{2}$$

Doing the same action on radial equations (15) and (17), with the support of the relation between $b_i(r)$ and $h_i(r)$, the result is a differential equation for $\Psi(r) = rh_\theta(r)$.

$$\frac{d}{dr}\left\{\sqrt{r^4+r_o^4}\frac{d\Psi\left(r\right)}{dr}\right\}-\frac{\lambda_\varsigma\Psi(r)r^2}{\sqrt{r^4+r_o^4}}=0 \tag{20}$$

The general solution is the associated Legendre functions of the first and second kind $P_n^{1/4}(z)$ and $Q_n^{1/4}(z)$, with $z=\sqrt{\left(\frac{r}{r_o}\right)^4+1}$ and n exactly the same for the angular sector. Once again the second one takes complex values and the solution can be written as:

$$\Psi_n(r)=\overline{m}\sqrt{r}P_n^{\frac{1}{4}}\left(z\right)\qquad z=\sqrt{\left(\frac{r}{r_o}\right)^4+1} \tag{21}$$

Acceptable solutions for (19) require that n must be a natural number, otherwise the field lines will not be closed. As result λ_ς assumes special integers $(0,2,6,12,...)$. Looking for solutions (21) the component $h_\theta(r)$ is calculated and from (12) the component $b_\theta(r)$ are obtained. However, among this infinite set only two solutions will give physical meaning. For $\lambda_\varsigma\geq 6$ the component $h_\theta(r)$ grows as the radial distance become large. For $\lambda_\varsigma=0$ one gets no angular dependence, $P_0(\cos(\theta))=1$, and the field will be entirely radial like a monopole field. In this situation there is no angular moment and no way to express the magnetic charge in terms of electric charge. On the other side the first assumption is violated when $r\approx 0$ because equation (17) tells that $b_r(r)$ grows without limit. Taking $\lambda_\varsigma=2$ the situation is different, $G_1(\theta)=C_1\cos(\theta)$, and equation (16) gives the second angular function such that, $J_1(\theta)=-C_1\sin(\theta)/\lambda$. The solution (21) will be[11]:

$$\Psi(r)=\overline{m}\sqrt{r}P_{\frac{1}{4}}^{\frac{1}{4}}\left(z\right)$$

and the constant $\overline{m}$ will be closely related to the magnetic dipole moment for suitable choice of the constants λ and ς. It is then possible to represent the magnetostatic field polar component, $b_\theta(r)$, like this:

$$b_\theta(r)=\frac{h_\theta(r)}{\sqrt{1+\frac{D_r^2(r)}{b^2}}}=\frac{r}{\sqrt{r^4+r_o^4}}\Psi(r)$$

The differential equation (17) allows $b_r(r)$ calculation.

Each component, polar and radial, is finite everywhere and the Figure 1 shows the field strength of the total magnetostatic field, in arbitrary units, as a function of the radial distance of the electric point-like charge.

Each one has a maximum near the r_o and vanishes as $r\to 0$ or $r\to\infty$. An accurate mathematical analysis shows that the asymptotic behaviour is the well known r^{-3}, for $r\gg r_o$, indicating the connection of $\overline{m}$ with some magnetic dipole moment, and proportional to r for $r\approx 0$, indicating some induced magnetic charge distribution. Also, the nonlinearity effect can be viewed near r_o where polar and radial field intensity change its relative magnitude. Out of that range the components recover their linearity. To see this one takes $h_\theta(r)$ as:

$$h_\theta(r)=\frac{\overline{m}r_o^2f(z)}{r^3}\qquad f(z)=z^2\left[\sqrt{z}P_{\frac{1}{4}}^{\frac{1}{4}}\left(\sqrt{1+z^4}\right)-kz\right]$$

The function $f(z)$, called here as "$form\ fuction$" can be showed in Figure 2. The asymptotic behavior is constant indicating that closely connection of $\overline{m}$ with the magnetic dipole moment.
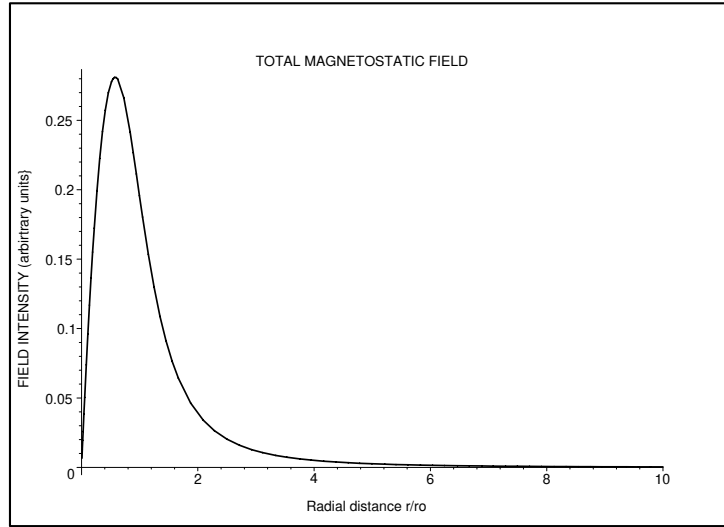
Figure 1

Tangent field lines can be evaluated equating the ratio of field components to the slop of the curve in polar coodinates.

$$\frac{B_r(r,\theta)}{B_\theta(r,\theta)} = \frac{dr}{rd\theta} \tag{22}$$

Integrating the last equation for two different assymptotic distances from the electric charge one arrives to the following equations in polar coordinates:

$$r = k_1 \sin^2(\theta) \qquad r \gg r_o$$
$$r = k_2 \sin^{2/3}(\theta) \qquad r \approx 0$$

Figure 3 shows the tangent closed lines for the total magnetic induction. It's a tipical magnetic dipole pattern. The constants k_1 and k_2 are set up for different range of radial distance. For small loop the entire curve is near correct. The other has some restrictions for polar angles near north and southern pole.

The interpretation carried out was as two magnetic charge of oposite signal distributed over each hemisphere. The vacuum so polarized by the intense electric field behaves like a material surrounding the electric charge. These induced magnetic field can produce angular moment when coupling with the electric field.

3   THE ANGULAR MOMENT AND THE CONNECTION WITH THE MAGNETIC CHARGE

In this section the proposed induced magnetic charge, interpreted in last section, is written as a combination of fundamental parameters like the electric charge e and the B-I maximum field strength b. In order to understand what that solution means, the angular moment of those fields must be calculated
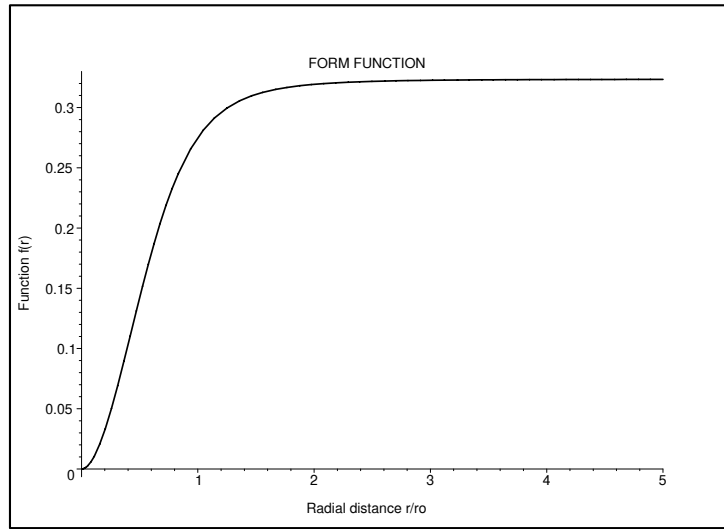
Figure 2

due to its great importance [5, 6, 7]. From the definition $\overrightarrow{L}$ can be evaluated and its interpretation follows immediately.

$$\overrightarrow{L} = \int \overrightarrow{x} \times \left(\overrightarrow{D} \times \overrightarrow{B} \right) d^3 \overrightarrow{x} \tag{23}$$

Using the properties of symmetry of the above integral yields the angular moment, that is completely aligned to the polar z axis.

$$L_z = 2\pi\alpha\mu_0 \overline{m} r_o e \tag{24}$$

$$\alpha = \int\limits_0^\infty dz \frac{f(z)}{\sqrt{1 + z^4}} \approx 0.4003$$

and μ_0 is the magnetic permeability

The result (24) suggests that an interpretation be given in terms of a magnetic charge. It is known that the classical angular moment [7] for a system consisting of an electric monopole and a magnetic monopole is ge, the parameter g being the strength of the magnetic charge. If both results are compared, it is then possible to explain such magnetic charge as the result of a non-linearity effect or of the vacuum polarization of the B-I theory.

$$g_{eff} \rightarrow 2\pi\alpha\mu_0 \overline{m} r_o \tag{25}$$

That is consistent with the rigorously linear behavior found when the maximum field strength is allowed to become infinite. In that case the contribution to the magnetic sector vanishes and that of the magnetic charge completely disappear. Based on the findings presented, a proposal for a model of the structure of the electric charge could consist of an electric monopole, of strength e, and two distributed magnetic charge g, of opposite signs, in each hemisphere, yielding a total null magnetic charge when seen at a great distance, and null net divergence, like a magnet with its closed field lines.
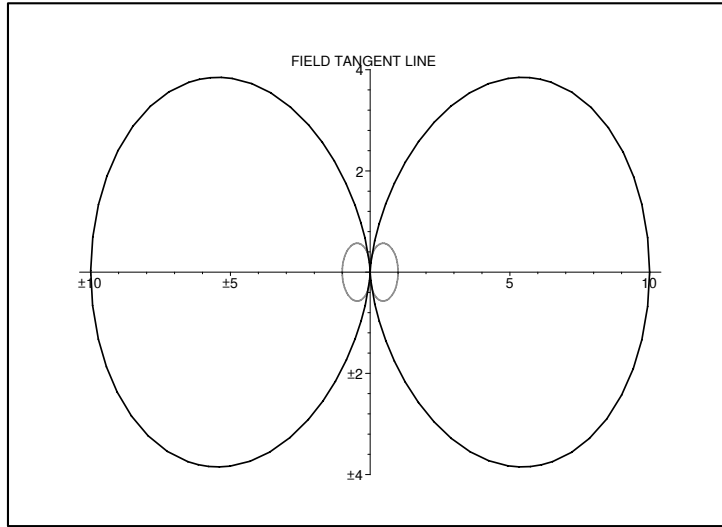
FIELD TANGENT LINE

Figure 3

4 FIELD AS A SOURCE OF FIELD

Take advantage of the result of the previous section and of the separation of variables, one speculates, in this section, on the possibility of get a single magnetic charge. There is another way to interpret the result of the previous section. Looking for the divergence of the vector $\overrightarrow{B}(r,\theta)$ and removing the angular solution covering it, one gets an equation with $-\frac{2}{r}b_\theta(r)$ as a source of the pure radial field. That claims for a divergence of some radial field, $\overrightarrow{\nabla}_r \cdot \overrightarrow{B}_r$. So one has the transverse field component as a source of a pure radial field given by a spread magnetic density $\rho_m(r)$.

$$\frac{1}{r^2}\frac{d[r^2 b_r(r)]}{dr} + \frac{2}{r}b_\theta(r) = 0 \tag{26}$$

$$\overrightarrow{\nabla} \cdot [b_r(r)\widehat{r}] = \rho_m(r) \qquad \rho_m(r) = -\frac{2}{r}b_\theta(r) \tag{27}$$

The total magnetic charge strength , g, is obtained integrating $\rho_m(r)$ over the entire space.

$$g' = \int d^3\overrightarrow{x}\left[\frac{2}{r}b_\theta(r)\right] = 8\pi\alpha\mu_0\overline{m}r_o \tag{28}$$

It differs from (25) only in magnitude, it is four times bigger than g_{eff}. Our interpretation is that the symmetry cancels all three directions contributions to the angular moment. When the symmetry is broken this cancellation desapairs. Extracting it from the z angular moment component one looks only for a fraction of the total. The remained is hidden by the symmetric components cancellation on interacting with the electric charge to produce the angular moment. Moreover the shadowing between both magnetic charges reduce the effective individual magnetic charge ntensity given $g' > g_{eff}$.

5 NUMERICAL CALCULATIONS

This section plays the calculation of the magnetic charge in SI units. For this task it is necessary to recover the SI units of the objects in angular momentum. The constant $\overline{m}r_o^2\alpha$ can be given roughly

as $er_o\alpha c$, where e is the elementary charge, r_o is the B-I radius parameter, c is the speed of ligth and α is that integral involving the Form Function defined previously. We interpret αc as the velocity of the spining charge e around the circle of radius r_o.

Inserting all in (25) we calculated, with $\mu_0 = 4\pi \times 10^{-7} n/A^2$, the intensity of magnetic charge in consideration, about $60e$. This is less than Dirac quantum prevision that is 68.5 e. A more accurate result only a quantum version of that theory could give us.

6   FINAL CONSIDERATIONS

We showed that Classical Abelian Born-Infeld electrodynamics can predicts the existence of real and well-behaved magnetostatic fields solutions associated with electric charges at rest. Definitely it is a non-linear effect simply ruled out by Maxwell's electrodynamics. Although Born-Infeld non-linear electrodynamics has not yet been experimentally confirmed, it was originally conceived to describe the electron properties based only on the structure of the field. The findings of this work apparently suggest that the fields, rather than resembling Dirac's or t'Hooft's magnetic monopoles [9, 10], exhibit properties similar to those produced by a magnet as considered from the macroscopic point of view although its more complex structure is only seen at the microscopic level. In addition they indicate that the magnetostatic solutions ensued from the breaking of the radial symmetry. That was necessary in order to see the dependence of the magnetic charge on basic parameters such the maximum field strenght b and the electric charge e. Such results seem to suggest the existence of vacuum excitation effects caused by the intense electric field strength around the electric charge. In addition by singling out the angular dependence, the current approach allows the investigation of a pure magnetic radial field generated by a spatially distributed magnetic charge derived from an electric charge. It must be stressed that quantum effects prevail over the classical description when distances in the order of $10^{-15}m$ are considered and the Compton wavelength of the electron, $\hbar/m_e c$, is about $10^{-12}m$. Hence r_o still lies well within the limits of the classical validity, and a quantum Born-Infeld Theory still remains to be developed.

One of the major motivations for the use of the Born-Infeld electromagnetic field theory is to overcome the infinity problem associated with a point-like charge source as in Maxwell's theory. Born's original theory may currently be explained as an attempt to find classical solutions to represent electrically charged states produced by sources that have finite self-energy. The present work proposes an extension to that theory by also seeking magnetically stable solutions derived for purely electrical charges. As expected, all additional anomalous magnetic terms vanish when Maxwell's regime is restored by allowing the maximum field strength to become infinite.

References

[1] M. Born, L. Infeld, **Foudations of the New Field Theory**, Proc. Roy. Soc.A144 (1934) 425.

[2] J. Polchinski,**TASI Lectures on D-branes**, hep-th/0108189 v1, 23 Apr 1997.

[3] I. Bialynicki-Birula, Z. Bialynicki-Birula, **Quantum Electrodynamics**, (Pergamon, Oxford, 1975).

[4] S. V. Ketov, arXiv: **Many Faces of Born-Infeld Theory**, hep-th/0108189 v1, 25 Aug 2001.

[5] I. Adawi, **Magnetic Charges in Especial Relativity**, Am. J. Phys. **59**(5), 410-412, May 1991.

[6] W.B. Zeleny, **Symmetry in Electrodynamics: A Classical Approach to Magnetic Monopole**, Am. J. Phys. **59**(5), 412-415, May 1991.

[7] I. Adawi, **Thomson's Monopole**, Am. J. Phys. **44**(8), 762-765, Aug 1976.

[8] P. Hraskó, **Quasiclassical Quantization of the Magnetic Charge**, Am. J. Phys. **45**(9), 762-765, Sep 1977.

[9] P.A.M. Dirac, **Quantised singularities in the Electromagnetic Field**, Proc. R. Soc. London, A133 (1931) 60.

[10] G. 'tHooft, **Magnetic monopoles in unified gauge theories,** Nucl. Phys. B79 (1974) 276 - 284

[11] M. Abramowitz and I. Stegun, **Handbook of Mathematical Functions** Applied Mathematics Series 55, June (1964).

[12] C.G. Callan Jr, J.M. Maldacena,**Brane Dynamics from the Born-Infeld Action**, Nuclear Phys. B513 (1998) 198-212. [hep-th/9708147].