Possible origin of magnetic fields in very dense stars

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We discuss the bosonization of a relativistic very dense Fermi gas in a magnetic field and the consequent Bose-Einstein condensation of the resulting relativistic vector gas of charged particles. The model may be applied to paired spin-up electrons and ρ or ω mesons. We show that such systems may maintain self-consistently magnetic fields of order $10^{10} - 10^{19}$ Gauss. That pairing could be the origin of large magnetic fields in some white dwarfs and neutron stars. But for fields large enough ($\sim 10^{13}$ for white dwarfs), the system becomes unstable and collapses.

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I. INTRODUCTION

Recent experiments [1] on extremely cold atomic Fermi gases indicate that a bosonization occurs and a Bose-Einstein condensate appears at sufficiently low temperatures. A background magnetic field is present. If we assume that this mechanism is universal, we expect that for superdense objects like white dwarfs and neutron stars, which can be considered extremely degenerate Fermi systems, the same mechanism would operate. This would lead to the arising of very strong magnetic fields as a consequence of condensation in presence of lower magnetic fields.

On the other hand, in the paper [2, 3] it was shown that a relativistic Fermi gas under the influence of a magnetic field B of order of the quantum electrodynamics limit of $m_e^2/e \sim 10^{13}$ G for densities $N \leq 10^{30}$ cm^{-3} becomes unstable and collapses, since the pressure perpendicular to the field vanishes. Physically, the system, infinitely degenerate with regard to the orbit's center quantum number, becomes otherwise one dimensional, all the electrons falling on the Landau ground state $n = 0$. The magnetic "Bohr radius" being of order $\sqrt{\hbar c/eB} = 10^{-2}$Å. Its spectrum looks equivalent to that of a free one-dimensional particle moving along the external field. Because of the spin-B-field coupling the system is unable to exert a transverse pressure under these conditions.

Looking at the problem of bosonization of the Fermi gas from another side, in the paper [4] it has been shown that an electron gas confined to the Landau ground state cannot be in β-decay equilibrium in a neutron star due to an incompatibility among the spin orientation of the particles involved. The suggestion is given in [5] that a bosonization of the electron gas must take place.

II. THE ELECTRON VECTOR PAIRING

We will discuss here the behavior of a gas whose constituent particles are relativistic bosons, which we assume to represent the electron pairs, under such extreme conditions of confinement to the $n = 0$ Landau ground state. As it is known in normal superconductors, scalar pairing condensates occurs in absence of a magnetic field (Cooper pairs). When a magnetic field is applied, the superconductivity (condensate) is destroyed at some critical magnetic field, (the Schafroth critical field). But if the magnetic field increases largely enough to force a significant fraction of the system density to occupy the Landau ground state, it has been suggested that some sort of superconductive behavior reappears, i.e., the condensate reappears as a consequence of the arising of a spin-one vector pairing mechanism [6]. This would lead to a behavior superconductive-ferromagnetic. We shall assume that this happens in the relativistic electron gas placed in a sufficiently strong magnetic field.

Such particles would carry twice the charge of the electron and an effective mass which in principle we take of order twice that of the electron mass (some corrections must be introduced however, due to effects coming from the large density and the magnetic field). Then the system may be treated by following the same formalism used in previous references [2, 3], [7]. As a consequence of condensation the system would behave as ferromagnetic and under the action of an external field H, a magnetization $\mathcal{M}$ arises, leading us to define a microscopic magnetic field $B = H + 4\pi\mathcal{M}$. The interesting point here is that, due to the positive character of $\mathcal{M}$ it may occur that $B \sim 4\pi\mathcal{M}$, or $H \ll 4\pi\mathcal{M}$ i.e., the microscopic magnetic field be produced by self-magnetization [3].

We shall assume that the relativistic paired electron system behaves as a vector particle with energy eigenvalues [7], $\epsilon_0(p_3) = \sqrt{p_3^2c^2 + M^2c^4 - 2eB\hbar c}$, (where we take $M = 2m_e$, m_e being the electron mass) for the Landau ground state $n = 0$, $\epsilon_n(p_3) = \sqrt{p_3^2c^2 + M^2c^4 + 4eB\hbar c(n + \frac{1}{2})}$ for the excited states $n = 1, 2....$We observe that the magnetic field introduces an effective mass for vector bosons $M_0 = \sqrt{M^2 - 2eB\hbar/c^3}$ in the ground state such that as B increases, M_0 decreases. This leads to an effective magnetic moment in the ground state $m = e\hbar/2M_0c$. The magnetic mass is $M_n = \sqrt{M^2 + 2eB\hbar(n+\frac{1}{2})/c^3}$ for the excited states, which increases with B and n.

In [7] we have shown that Bose-Einstein condensation, in the sense of a large population in the Landau ground state having its momentum along the magnetic field equal to zero or very small, occurs for scalar and vector particles in presence of a strong magnetic field. We name $n_0^{\pm} = [exp(\epsilon_0 \mp \mu)\beta - 1]^{-1}$ the density of particles and antiparticles, respectively, in the ground state. We expect then most of the population of particles to be around the ground state, since for low temperatures n_{0p}^- is vanishing small and n_0^+ is a bell-shaped curve with its maximum at $p_3 = 0$. We will define $\mu' = \mu - M_0c^2$ and and recall the procedure followed in [7]. We call $p_0(\gg \sqrt{-2M_0\mu'})$ some characteristic momentum. Taking by symmetry the density of particles minus antiparticles (the latter will vanish as $-\mu' \ll T$) we have in a small neighborhood of $p_3 = 0$,

$$
\begin{aligned}
N_0 &= \frac{2eBT}{2\pi^2\hbar^2 c}\int_0^{p_0}\frac{dp_3}{\sqrt{p_3^2c^2 + M_0^2c^4 \pm 2eB\hbar c} - \mu} - \int_0^{p_0}\frac{dp_3}{\sqrt{p_3^2c^2 + M_0^2c^4 \pm 2eB\hbar c} + \mu} \\
&\simeq \frac{2eBT}{2\pi^2\hbar^2c^2}\int_0^{p_0}\frac{(M_0c^2+\mu)dp_3}{p_3^2c^2 + M_0^2c^4 - \mu^2} - \int_0^{p_0}\frac{(M_0c^2-\mu)dp_3}{p_3^2c^2 + M_\pm^2c^4 - \mu^2} \\
&= \frac{2eBT}{4\pi\hbar^2 c}\frac{2\mu}{\sqrt{M_\pm^2c^4 - \mu^2}} \sim \frac{2eBT}{4\pi\hbar^2 c}\sqrt{\frac{2M_0}{-\mu'}} \qquad (1)
\end{aligned}
$$

where $N = N_0 + \delta N$ and δN is the density in the interval $[p_0, \infty]$. Actually as $\mu' \to 0$, $N_0 \to N$ and δN is very small. We get then

$$
\mu' \simeq -\frac{e^2B^2T^2M_0}{2\pi^2N^2\hbar^4c^2}. \qquad (2)
$$

We observe that μ' is a decreasing function of T (because of the minus sign in front of this expression) and vanishes for $T = 0$, where the "critical" condition $\mu = M_0c^2$ is reached. As shown in [7] in that limit the Bose-Einstein distribution degenerate in a Dirac δ function, which means to have all the system in the ground state $p_3 = 0$. From (1) we may write the thermodynamic potential as

$$
\Omega = \frac{eBT}{2\pi\hbar^2 c}\sqrt{M_0^2c^4 - \mu^2} \qquad (3)
$$

III. THE SELF-MAGNETIZATION

From (3), the magnetization is given approximately by

$$
\mathcal{M} = -\frac{\partial\Omega}{\partial B} = \frac{eN\hbar}{M_0c}. \qquad (4)
$$

One can then state the condition for self-magnetization, by writing the equation $H = B - 4\pi\mathcal{M} = 0$. One has,

$$B = 4\pi\mathcal{M} = 4\pi\frac{eN\hbar}{M_0 c} \qquad (5)$$

Let us assume that $N \sim 10^{30-32}$ cm^{-3}. Then $\mathcal{M} \sim 10^{10-12}$ G and $B \sim 10^{11-13}$ G. The condition for self-magnetization is satisfied. The system becomes a giant magnet, whose stability is determined by the transverse pressure condition $P_\perp = -\Omega - B\mathcal{M}$ [2, 3]. However, the estimation of the value of Ω would depend on the fraction of paired electrons. Let us name N_u, $N_p = N - N_u$ the density of unpaired and paired electrons, respectively. If $N_u \sim N_p$ then the dominating pressure comes from the (unpaired) electron gas contribution, $\Omega \sim NM_0 \sim 10^{24-26}$ erg/cm^3. This interval partially overlaps the interval $B\mathcal{M} \sim 10^{21-25}$ erg/cm^3 leading to the vanishing of $P_\perp$. Thus, one can assert that for the self-magnetized star for fields in the interval $B \sim 10^{11-13}$ G, and densities in the region $N \sim [10^{30} - 10^{32}]$ cm^{-3}, for some specific values of these quantities there appears consitions of instability and the white dwarf collapses (the star may be stable for other values of N, B). But if $N_u \ll N_p$ so that the dominant pressure comes from the paired gas the collapse is unavoidable in any case, since as Ω is positive, its contribution to pressure is negative. We conclude that the stability requires from a Fermion background.

There is another point to be considered when eB approaches to M. As M_0 decreases with increasing B, the magnetization $\mathcal{M}$ increases with B, and would diverge for $M_0 \to 0$. For $eB\hbar/c^3$ close enough to M one expects the main contribution to B be produced by $\mathcal{M}$. We get an equation similar to the one discussed in [2] for the W condensate. Let us write $2eB\hbar/M^2c^3 = x^2$ where $0 \leq x \leq 1$. For $x = 1$, we have the critical field $B_c = M^2c^3/2e\hbar \simeq 8.82 \times 10^{13}$ G. Then we can write $M_0 = M\sqrt{1 - B/B_c}$. We easily get

$$x^2\sqrt{1 - x^2} = \frac{8\pi e^2\hbar^2 N}{M^3 c^4} = A. \qquad (6)$$

By simple inspection we find that it has real solutions only for $A < 2\sqrt{3}/9 = A_1$. This means that $N \leq 10^{32}$ cm^{-3}. By solving the cubic equation (6), we find that for $A \ll 1$, these real solutions are $x_1 = \sqrt{A + A^2/2}$ and $x_2 = \sqrt{1 - A^2}$. The first solution means that B increases with increasing N, (up to the value $B_{max} = 2/3B_c$). In the second solution B decreases for growing N, and its limit for $N \to 0$ being B_c. The last result has only meaning if interpreted as indicating that the expression for the magnetization (4) is incomplete. Actually, it must include the contribution from Landau states other than the ground state, which lead to a diamagnetic response to the field. The decrease in population of the ground state is compensated by increasing the number of particles in Landau states with $n > 0$. Their contribution would compensate the increase of the self-consistent field with increasing N to keep $B < B_c$.

IV. CONCLUSIONS

We conclude that a very dense electron system, as for instance, a white dwarf, in presence of a very strong magnetic field, may bosonize and create conditions of self-magnetization. The possibility of a collapse is highly increased as the density and magnetic field grow. The star is hardly stable at fields of order or greater than B_c: the one-dimensional world created by the so large magnetic field is unstable.

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[1] C.A. Regal, M. Greiner, and D.S. Jin Phys. Rev Lett Vol 92 No. 4 (2004)

[2] M. Chaichian, C. Montonen, S. Masood, A. Pérez Martinez, H. Pérez Rojas, Phys. Rev. Lett. Vol. 84, No. 23 (2000) 5261

[3] A. Pérez Martinez, H. Pérez Rojas, H. J. Mosquera Cuesta, Eur. Phys. J. C 29, 111 (2003); see also report hep-ph/0011399, november (2000).

[4] S. Ghosh, S. Mandal, S. Chakrabarty, astro-ph/0207492.

[5] S. Mandal, S. Chakrabarty,astro-ph/0209462

[6] M. S. Osovski, R.J. Soulen Jr., S.A. Wolf, J.M. Broto, H. Rakoto, J.C. Oussel, G. Coffe, S. Askenazi, P. Pari, I. Bozovic, J.N. Eckstein and G. F. Virshup, Phys. Rev. Lett. **71** (1993) 2315; Yoichi Ando , G.S. Boebinger and A. Passner, Phys. Rev. Lett **75** (1995) 4662,

[7] H. Perez Rojas, Phys. Lett. B 379 (1996) 148

[8] H. Perez Rojas. L. Villegas Lelovski, Brazilean Jour of Physics 30 (2000) 410.