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Comment on Relativistic shape invariant potentials

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Abstract

In this comment we point out numerous errors in the paper of Alhaidari cited in the title.

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In a recent paper Alhaidari [1] treats the problem of formulating a relativistic Dirac type equation which can be reduced to solving a Schrödinger equation for shape invariant potentials for the upper component while the lower component can be found once the upper component has been found. The method used is the same as one used in his earlier paper [2].

The proposed Hamiltonian is (we use the notation of Bjorken and Drell [3])

$$H = \boldsymbol{\alpha} \cdot (\mathbf{p} - i\beta\hat{\mathbf{r}}W(r)) + \beta M + V(r) \tag{1}$$

where $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$. The vector $(V(r), \hat{\mathbf{r}}W(r))$ is interpreted as an external electromagnetic field. Due to the matrix β accompanying W in the Hamiltonian, the interpretation of the vector $(V, \hat{\mathbf{r}}W)$ as an electromagnetic potential is not necessary and in fact plays no role in his calculations. The resulting radial equation

$$[-i\rho_2\frac{d}{dr} + \rho_1(W + \frac{\kappa}{r}) - E + V + M\rho_3]\Phi = 0 \tag{2}$$

where $\Phi = \begin{pmatrix} G_{\ell j}(r) \\ F_{\ell j}(r) \end{pmatrix}$ corresponds to Alhaidari's equation (1) where the quantum numbers ℓ and j are omitted.

The subsequent application of a unitary transformation and the imposition of the constraint (in our notation)

$$W(r) = \frac{1}{S}V(r) - \frac{\kappa}{r} \tag{3}$$

with both V and W nonzero and S a constant can only be satisfied for a chosen value of κ. Otherwise we will have different functions W for different values of κ. This cannot be since the functions $V(r)$ and $W(r)$ appear in the Hamiltonian. Forgetting this Alhaidari writes results for the Dirac-Rosen-Morse and Dirac-Eckart potentials which cannot be correct since the energy levels obtained would be degenerate in ℓ, j, m. In the nonrelativistic Schrödinger equation the radial equation does contain the centrifugal barrier contribution for nonzero values of ℓ.

Even if one interprets the results as corresponding to $\ell = 0$ so that $\kappa = -1$, the unitary transformation is inexplicable since it does not reduce to identity in the nonrelativistic limit.

The subsequent calculations are for the case $V = 0$ treated by Castanõs et al. [4] earlier. The results need to be corrected since $\ell = 0$ means $\kappa = -1$ and not $\kappa = 0$ as stated by Alhaidari. This has the effect of replacing W by $W - \frac{1}{r}$. In fact, in this restricted case one can find suitable values of W for the Morse, Rosen-Morse and Eckart problems, without the need of any unitary transformation.

Finally, even if the $\ell = 0$ case can be adjusted to give a reasonable nonrelativistic limit, the validity of the proposed Dirac equation remains unproved.

References

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