Notas de Física Vol. III - Nº 20

μ - MESON DECAY WITH NON-CONSERVATION OF PARITY *

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Rio de Janeiro, D. F.

(July 5, 1957)

Experiments [1] suggested by Lee and Yang [2] showed that there is a correlation between the μ-meson spin direction and the momentum of the electron emitted in its desintegration. As observed by Lee and Yang such a correlation between an axial and a polar vector implies that there is not parity conservation in this process. Calculations have been done [3] to get in some particular cases the form of the angular distribution and energy spectrum of the electrons given off in the desintegration of polarized μ-mesons. In these calculations the simpler order $(e, \mu)(\nu, \nu)$ has been used for the wave functions of the particles.

With the purpose of showing the results of the most ge-

* This work has been supported by the Conselho Nacional de Pesquisas.

neral interaction (not including derivatives)[4] and of providing data for a further discussion of the Universal Interaction, we present in this work results of calculations done in the charge exchange order $(\nu, \mu)(e, \nu)$ for particular forms of interaction which have the advantage of indicating the expected polarizations of the emitted particles. It can be easily shown that the most general possible interaction is a linear combination of the four forms considered.

We made the hypothesis that the μ-meson emits one electron, one neutrino and one antineutrino

$$\mu \longrightarrow e + \nu + \nu'$$

the particles involved in the process being non-identical. This hypothesis implies that the number of particles is conserved if the μ-meson and the electron of same charge are particles of same nature (that is, both are particles or both are antiparticles). We treat neutrino as a zero mass Dirac particle, with a four component wave function.

SPECTRA AND POLARIZATIONS

It can be shown that the most general parity non-conserving hamiltonian not including derivatives can be written as a sum of four terms:

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3 + \mathcal{H}_4$$

where

$$\mathcal{H}_1 = \sum g_{1i} \left(\Psi_\nu^+ \frac{1+\gamma_5}{2} O_i \Psi_\mu \right) \left(\Psi_e^+ O_i \frac{1-\gamma_5}{2} \Psi_\nu \right) \quad (1)$$

$$\mathcal{H}_2 = \sum g_{2i} \left(\Psi_\nu^+ \frac{1-\gamma_5}{2} O_i \Psi_\mu \right) \left(\Psi_e^+ O_i \frac{1+\gamma_5}{2} \Psi_\nu \right) \quad (2)$$

$$\mathcal{H}_3 = \sum g_{3i} (\Psi_\nu^+ \frac{1+\gamma_5}{2} O_i \Psi_\mu)(\Psi_e^+ O_i \frac{1+\gamma_5}{2} \Psi_\nu) \qquad (3)$$

$$\mathcal{H}_4 = \sum g_{4i} (\Psi_\nu^+ \frac{1-\gamma_5}{2} O_i \Psi_\mu)(\Psi_e^+ O_i \frac{1-\gamma_5}{2} \Psi_\nu) \qquad (4)$$

Here we follow the notation of Yang and Lee, that is: $\gamma_4 = \beta$, $\gamma_k = -i\beta\alpha_k$, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$, the sixteen Dirac matrices being: $O_S = \gamma_4$, $O_V = \gamma_4\gamma_\mu$, $O_T = -i/(2\sqrt{2})\ \gamma_4(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$, $O_A = -i\ \gamma_4\gamma_\mu\gamma_5$, $O_P = \gamma_4\gamma_5$. In this notation we have $\vec{\sigma} = -\gamma_5\ \vec{\alpha}$.

It is easy to see that, since the neutrino mass is taken equal to zero, all the interferences between the parts in which the hamiltonian has been divided give zero contribution to the probability. This results from the fact that $1 \pm \gamma_5$ commutes with the projection operator of a particle of zero mass and $(1+\gamma_5)(1-\gamma_5) = (1-\gamma_5)(1+\gamma_5) = 0$ (note that in our notation $\gamma_5^2 = 1$). In this way the electron distribution in the general case will be the sum of the results obtained for the four cases considered.

The most general electron distribution is shown to be given by the formula

$$W(x, \theta)\, d\Omega\, dx =$$

$$\text{Const.}\ \frac{d\Omega}{4\pi} K x^2\, dx \left\{ (1-\lambda x) + \alpha\cos\theta\,(1-\beta x) \right\} \qquad (5)$$

where x is a dimensionless number given by the energy of the emitted electron divided by the maximum electron energy, θ is the angle between the μ-meson spin direction and the electron momentum, $d\Omega$ is the solid angle element, and K, λ , α , β

are parameters which depend on the interaction constants. The electron mass has been neglected in these calculations. The relation between λ and the frequently used Michel ρ parameter is

$$\lambda = \frac{9 - 8\rho}{3\,(3-2\rho)}$$

The value of the constant in (5) is

$$\text{Const.} = \frac{(m_{\mu} c^2)^5}{\pi^3 . 2^{10} . \hbar^7 . c^6}$$

Since the spectra are additive we will have that $K = \sum K_j$, $K\alpha = \sum K_j \alpha_j$, $K\lambda = \sum K_j \lambda_j$, $K\alpha\beta = \sum K_j \alpha_j \beta_j$ where the index j runs over the four cases considered, and K_j , α_j , λ_j , β_j are the parameters obtained in each of these cases.

In what follows we give the dependence of K_j , ρ_j , α_j , β_j on the interaction constants g_j's and the polarizations of the emitted particles in the four cases considered. The polarization is indicated by the indices R and L referring to right and left hand polarizations according to which the particle has the spin pointing parallel or antiparallel to its momentum. The electron polarization is indicated assuming that its rest energy is negligible compared with its total energy, which is true for relativistic velocities. In the following formulae the number corresponding to the index j will be omitted for simplicity.

Case I - Hamiltonian given by (1). For $O_i = O_V$ or O_A the process is $\mu \rightarrow e_R + \nu_L + \nu'_L$ and for $O_i = O_S$, O_P or O_T it is $\mu \rightarrow e_L + \nu_L + \nu'_L$. The parameters are given by

$$K = |g_S+g_P|^2 + 20\, g_T^2 + 8\, |g_V+g_A|^2 + 2(g_S+g_P)g_T^* + 2(g_S^* + g_P^*)g_T$$

$$\rho = \frac{3}{4}\, \frac{|g_S+g_P|^2+4g_T^2-2(g_S+g_P)g_T^*-2(g_S^*+g_P^*)g_T}{|g_S+g_P|^2+12g_T^2+4|g_V+g_A|^2}$$

$$\alpha = \frac{1}{3}\left(|g_S+g_P|^2+52g_T^2-24|g_V+g_A|^2+10(g_S+g_P)g_T^* + 10(g_S^*+g_P^*)g_T\right)\left(|g_S+g_P|^2+20g_T^2+8|g_A+g_A|^2 + 2(g_S+g_P)g_T^*+2(g_S^*+g_P^*)g_T\right)^{-1}$$

$$\beta = 2\left(|g_S+g_P|^2+28g_T^2-12|g_V+g_A|^2+4(g_S+g_P)g_T^* + 4(g_S^*+g_P^*)g_T\right)\left(|g_S+g_P|^2+52g_T^2-24|g_V+g_A|^2 + 10(g_S+g_P)g_T^*+10(g_S^*+g_P^*)g_T\right)^{-1}$$

and are subject to the conditions $0 \leq \rho \leq 1, -1 \leq \alpha \leq 1, -1 \leq \alpha\beta \leq 1$. If g_T is a complex number, ρ, α and β are functions of three other independent parameters [modulus and real part of $g_T/(g_S + g_P)$ and modulus of $(g_V + g_A)/(g_S + g_P)$]. If g_T is real, ρ, α and β depend on two parameters only and must satisfy the conditions $\alpha \geq -\lambda$ and $\lambda\beta \geq 1 - 2\lambda$. besides the fact that given two of them the third is determined.

Interaction (1) has been treated by Tiomno[3] in the particular case of $g_S = g_P = -g_T$, $g_V = g_A = 0$.

Case II - Hamiltonian given by (2). For $O_i = O_V$ or O_A the process is $\mu \rightarrow e_L + \nu_R + \nu'_R$ and for $O_i = O_S$, O_P or O_T it is $\mu \rightarrow e_R + \nu_R + \nu'_R$. The expressions for the parameters in terms of the interaction constants are the same as those of case I, except that α must be substituted by minus α .

Case III - Hamiltonian given by (3). The decay gives $\mu \longrightarrow e_L + \nu_L + \nu'_R$ for the terms O_A and O_V and $\mu \longrightarrow e_R + \nu_L + \nu'_R$ for O_S and O_P . The tensor term does not contribute to the probability. The parameters are $\rho = 3/4$ $(\lambda = 2/3)$, $\beta = 2$,

$$K = |g_S - g_P|^2 + 4 \, |g_V - g_A|^2$$

$$\alpha = \frac{1}{3} \, \frac{|g_S - g_P|^2 - 4 \, |g_V - g_A|^2}{|g_S - g_P|^2 + 4 \, |g_V + g_A|^2}$$

We see that α is restricted by $-1/3 \leqslant \alpha \leqslant 1/3$.

Case IV - Hamiltonian given by (4). In this case we have $\mu \longrightarrow e_R + \nu_R + \nu'_L$ for $O_i = O_V$ and O_A and $\mu \longrightarrow e_L + \nu_R + \nu'_L$ for $O_i = O_S$ and O_P . The tensor term does not contribute in this case too. The parameters are the same as in case III, except that we must change α into minus α . The particular case of interaction (4) for which $g_S = g_P$ is equivalent to that treated by Lee and Yang [3], if we make $f_V = f_A$ in his notation.

CONCLUSION

As we have always two separate groups of interactions (that is, the group V,A and the group S,P,T) which do not interfere with each other, giving in each of forms (1) — (4) opposite polarizations for the electron, the polarization vector of the electron will be along its momentum, and its intensity will be given by

$$p = \frac{W_R - W_L}{W_R + W_L}$$

where $W_R = \sum W_{R_j}$, W_{R_j} being the desintegration probability for the j-th case taking only the interaction which give right hand polarization to the electron (that is, A,V in case I , S,P,T in case II, S,P in case III, V,A in case IV) and $W_L = \sum W_{L_j}$, W_{L_j} being the probability for the j-th case taking only the interactions which give left hand polarization (S,P,T in case I, V,A in case II, V,A in case III and S,P in case IV).

We may observe that in general the polarization intensity will depend on the electron momentum and on the angle it forms with the spin of the desintegrating μ-meson.

As we have noted, the most general possible spectrum is a combination of the results obtained in the four particular cases, and there is a good many number of hypothesis which can be done to make this combination. However we think it is soon yet to make any consideration about the true form of the interaction, since the experimental material is far from complete.

We are greatly indebted to Professor J. Tiomno for the suggestion and orientation of the present work.

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2. Lee, T.D. and Yang, C.N., Phys. Rev. 104, 254 (1956)

3. Friedman, M.H., Phys. Rev. 106, 387 (1957)

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4. When most of the present work was concluded, Professor Tiomno showed us a preprint of a papaer by T.Kinoshita and A.Sirlin in which the most general interaction in the order (e , μ)(ν , ν) is discussed. Although ours results are in agreement with those reported in that paper, we think that the results in the present ordering are more appropriate to a discussion of problems such as the Universal Interaction and for the analysis of the polarization of the emitted electron.