

# A General Relativistic Effect in Quasi-Spherical Objects as the Possible Origin of Relativistic Jets

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## Abstract

Based on recently reported results, we present arguments that sign changes in proper acceleration of test particles on the symmetry axis and close to the horizon of quasi-spherical objects - related to the quadrupole moment of the source - might be at the origin of relativistic jets of quasars and micro-quasars.

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# 1 Jets and quasi-sphericity

Relativistic jets are highly energetic phenomena which have been observed in many systems including active galaxies, X-ray binaries black holes, X-ray transients, supersoft X-ray sources, etc. (see [1] and references therein), usually associated with the presence of a compact object. Despite the difference in typical scales of jets for different cases, which range from light years for the micro-quasars and million of light years for quasars, they have analogous properties.

So far, the spin and the magnetic field of a compact object have appeared to be the most popular candidates in powering and collimating jets [2]. However, no consensus has been reached until now, concerning the basic mechanism for its origin [3]. Furthermore, the great number of models available (see [2] and references therein) and their complexity, implying a large number of assumptions, indicate that the physics underlying these jets have not quite been understood and the question about its origin has not been answered yet.

In this letter we want to call the attention to a possible explanation to at least some of these jets, which is based on the sign change of radial proper acceleration of test particles along the symmetry axis, close to the horizon of compact objects, and related to the quadrupole moment of the source [4].

Let us present our arguments. First of all, it should be reminded that in the context of General Relativity the only static and asymptotically flat vacuum spacetime possessing a regular horizon is the spherically symmetric Schwarzschild solution [5]. If the field is not particularly intense (the boundary of the source is much larger than the horizon) and the deviation from spherical symmetry is small, then it will be possible to represent the corresponding fields, both inside and outside the source, as a suitable perturbation of the spherically symmetric exact solution. However, as the object becomes more and more compact, such perturbative scheme will eventually fail near the source. Indeed, as it is well known [6], as the boundary surface of the source approaches the horizon, any finite perturbation of the Schwarzschild spacetime, becomes fundamentally different from any Weyl metric, even if the latter is characterized by parameters whose values are arbitrarily close to those corresponding to spherical symmetry. In other words, for strong gravitational fields, no matter how small the multipole moments (higher than monopole) of the source are, there is a bifurcation between the perturbed Schwarzschild metric and all the other Weyl metrics.

Therefore if one wishes to describe the gravitational field of a quasi-spherical source close to the horizon, one must use an exact solution of Einstein equations, rather than a perturbed Schwarzschild, no matter how small the non-sphericity might be. If, for simplicity, one restricts oneself to the family of axially symmetric non-rotating sources, then one has to choose among the Weyl solutions.

However, since there are as many different Weyl solutions as there are different harmonic functions, then the obvious question arises: what is the exact vacuum solution of Einstein equations corresponding to a given static axially symmetric source (an ellipsoid, for instance)? Or more specifically: which one among the Weyl solutions is better entitled to describe small deviations from spherical symmetry?

These questions were partially answered in [7], in which the exact solution is presented

and referred to as the  $M$ - $Q$  solution. And when this solution has small values of  $q = Q/M^3$ , where  $M$  stands for the mass of the source and  $Q$  its quadrupole moment, up to the first order of  $q$ , it may be interpreted as the gravitational field outside a quasi-spherical source. This spacetime represents a quadrupole correction to the monopole (Schwarzschild) solution. However this spacetime is in contrast with other well known solutions of the Weyl family, where all the moments higher than the quadrupole present the same order of magnitude in  $q$ . Therefore for small values of  $q$  these solutions cannot be interpreted as a quadrupole perturbation of spherical symmetry.

In a recent study on the motion of test particles in the  $M$ - $Q$  spacetime [4], it was shown that particles moving along radial geodesics experience an attractive force, as one expects, except when they move along the symmetry axis. In this latter case a repulsive force measured by a locally Minkowskian observer - regarded as a positive radial proper acceleration measured by this observer - appears close to the horizon if  $q < 0$  (which corresponds to an oblate source) and even if it is arbitrarily small (though different from zero). Indeed, first we define the local coordinates  $(X, T)$  associated with a locally Minkowskian observer or alternatively a tetrad field associated with such Minkowskian observer,

$$dX = \sqrt{-g_{rr}}dr, \quad (1)$$

and

$$dT = \sqrt{g_{tt}}dt, \quad (2)$$

with  $r$  denoting the usual spherical radial coordinate. Then if the test particle moves along a radial geodesics outside the symmetry axis for  $R \equiv r/M \rightarrow 2$ , up to the first order in  $q$ , one will obtain the following equation of motion

$$\frac{d^2X}{dT^2} \approx -\frac{1}{2^{3/2}M(R-2)^{1/2}} \left[ 1 - \left( \frac{dX}{dT} \right)^2 \right] + \frac{\mathcal{O}(q)}{(R-2)^{1/2}}. \quad (3)$$

Observe that when  $q = 0$ , (3) becomes the exact spherically symmetric situation. Thus for particles moving along radial geodesics, excluding the symmetry axis, small values of the quadrupole moment introduce small perturbations on the trajectories, and the attractive nature of the gravitational force is preserved at all times.

However, for test particles moving along the symmetry axis, it follows that as  $R \rightarrow 2$ , the leading term in the equation of motion is

$$\frac{d^2X}{dT^2} \approx -\frac{5q}{2^{7/2}M(R-2)^{3/2}} \left[ 1 - \left( \frac{dX}{dT} \right)^2 \right] \exp \left[ -\frac{5q}{8(R-2)} \right], \quad (4)$$

which is a positive quantity if  $q < 0$ , for any value of  $q$  different from zero, diverging at the horizon. It should be emphasized that at  $r = 2M$  there are no locally Minkowskian observers and, as expected, the proper radial acceleration diverges there. We stress too that the change of sign takes place at a *finite* distance from  $2M$ , where such Minkowskian observers do exist.

Therefore, small deviations from spherical symmetry close to the horizon with  $q < 0$  produce a positive radial acceleration of particles moving along the symmetry axis, as

measured by a Lorentzian observer. Thus, under these circumstances, the observer would infer the existence of a *repulsive* gravitational force acting on the particle. This conclusion is valid in all locally Minkowskian frames, including, of course, the instantaneous rest frame where  $dX/dT = 0$ . For example, if a test particle initially at rest with respect to the Minkowskian local frame is placed close to the horizon, on the axis of symmetry as seen by such observer, it will tend to move towards increasing  $X$  values, away from the source.

This repulsive force diverges as the particle approaches the horizon, thereby providing an almost unlimited quantity of energy for outflowing jets. On the other hand, the mere fact that such repulsive force acts exclusively on particles along the symmetry axis on the neighborhood of the horizon, it provides at once the explanation to the observed collimation of jets and, furthermore, it is consistent with the conical geometry of the jet, which were recently proposed to interpret observations of SS 433 using the Chandra High Energy Transmission Grating Spectrometer [8]. In the case of prolate compact objects,  $q > 0$ , there are no repulsive forces, but the attractive gravitational force tends to zero as the particle approaches the horizon, this fact on the other hand, would enhance the effect of any other ejection mechanism near that region.

## 2 Conclusions

Relativistic jets are characterized by:

- The presence of a compact object.
- Very high energies involved.
- A remarkable collimation (with possible conical geometry).

Concluding, in the case of a compact object, when its boundary surface is close to its horizon and assuming only general relativistic effects, small quadrupole corrections to spherical symmetry may produce sign changes of proper acceleration of particles moving along the symmetry axis in the neighborhood of the horizon. This would result in outgoing jets, which are highly collimated and very energetic. This result is independent on the presence, or absence, of any other factors and mechanisms which could enhance the ejection. Parenthetically, besides the magnetized accretion disk [3, 12], powerful jets seem to require an additional energy source.

Particularly interesting, in the context of the model here proposed, is the recently observed fact [9] that the velocity jet structure close to the central core indicates a strong deceleration. The slowing down for increasing distances from the outflow source has also been seen in HH 34 [10] and HH 111 [11]. This effect is fully consistent with our model, since the appearance of positive proper radial accelerations takes place only very close to the horizon and change sign as the particle moves away from the source.

It is worth noting that repulsive forces in the context of General Relativity have been already shown. The best known example is probably the case included in the Reissner-Nordström solution (see for example [13]). Also, these repulsive forces appear in connection with rotating and/or unbounded sources [14] or as spurious phenomena associated

with coordinate effects (see [15] and references therein). However, in our case, the repulsive nature of the force is exhibited by means of a quantity measured by locally Minkowskian observers, and therefore it is not a coordinate effect. Furthermore, the  $M$ - $Q$  spacetime is asymptotically flat and, as stated before, physically meaningful.

Finally, it is worth mentioning that the influence of general relativistic effects on the dynamics of jets, in the context of Kerr spacetime, has been recently presented (see [16] and references therein).

## References

- [1] B A Margon, *Annu. Rev. Astr. Astrophys* **22** 507 (1984); B J Sams, B J A Eckart and R Suyae, *Nature* **382** 47 (1996); I F Mirabel, L F Rodriguez, B Cordier, J Paul and F Lebrun, *Nature* **358** 215 (1992); I F Mirabel and L F Rodriguez, *Nature* **371** 46 (1994); A Konigl, *IAU Colloquium 163*, A. S. P., 121 (1997); E de Gouveia Dal Pino *astro-ph/0406319*
- [2] R D Bladford and M J Rees, *Mon. Not. R. Astron. Soc.* **169**, 395 (1974); R D Bladford and M J Rees *Contemp. Phys.* **16** 1 (1975); R D Bladford, *astro-ph/0110394*; P Meszaros and M J Rees, *Ap. J.* **482** L29 (1997); I F Mirabel, *astro-ph/0005591*; A Hujeirat, M Camenzind and M Livio, *astro-ph/0212570*; A Hujeirat, *astro-ph/0302303*; A R Beresnyak, Ya N Istomin and V I Pariev, *astro-ph/0303125*; D Lynden-Bell, *Mon. Not. R. Astron. Soc.* **341** 1360 (2003)
- [3] M Livio in *Wild Stars in the old West* (A. S. P. conf. ser. Eds. S. Howell et al) **137** c264 (1998)
- [4] L Herrera, *gr-qc/0402052* to appear in *Foun. Phys. Lett.*
- [5] W Israel, *Phys. Rev.* **164** 1776 (1962)
- [6] J Winicour, A Janis and E T Newman, *Phys. Rev.* **176** 1507 (1968); A Janis, E T Newman and J Winicour *Phys. Rev. Lett.* **20** 878 (1968); F I Cooperstock and G Junevicius, *Il Nuovo Cimento* **16B** 387 (1973)
- [7] J L Hernández-Pastora and J Martín, *Gen.Rel.Grav.* **26** 877 (1994)
- [8] H L Marshall, C R Canizares and N S Schulz, *Ap. J.* **564** 941 (2002)
- [9] G Giovannini, *astro-ph/0406116*
- [10] D Devine, B Reipurth, J Bally and S Heathcote *Astron. J.* **114** 2095 (1997)
- [11] B Reipurth, J Bally and D Devine, *Ap. J.* **114** 2708 (1997)
- [12] M Livio, *Nature* **417** 125 (2002)
- [13] A Papapetrou *Lectures on General Relativity* (D. Reidel, Dordrecht-Holland, 1974); C Barbachoux, J Gariel, G Marcilhacy and N O Santos, *Int. J. Mod. Phys. D* **11** 1255 (2002)

- [14] W B Bonnor, *J. Phys. A* **10** 1673 (1977); E L Schuking, *Atti del Convegno sulla Cosmologia* (G Barbera Editor, Florence, 1966); L Herrera and N O Santos *J. Math. Phys.* **39** 3817 (1998); W B Bonnor, *Class. Quantum Grav.* **19** 146 (2002)
- [15] Ch McGruder, *Phys. Rev. D* **25** 3191 (1982)
- [16] C Chicone and B Mashhoon, *astro-ph/0404170*; R K Williams, *astro-ph/0404135*