CBPF-NF-019/83

NON-LINEAR PHOTONS IN THE UNIVERSE
II - THE ANISOTROPIC CASE

by

M.Novello, J.M.Salim and E.Ruckert[1]

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Xavier Sigaud, 150
22290 - Rio de Janeiro- RJ - Brasil

[1]Universidade Federal de Viçosa
36570 - Viçosa, MG - Brasil

NON-LINEAR PHOTONS IN THE UNIVERSE II - THE ANISOTROPIC CASE

M.Novello - J.M.Salim

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

E.Ruckert

Universidade Federal de Viçosa
36570 - Viçosa, MG. - Brasil

Abstract

We examine the consequences of non-minimal coupling of electromagnetic and gravitational fields. An exact solution is presented, which corresponds to a generalization of the anisotropic universe of Kasner and which can be interpreted as a previous era of an isotropic (photon dominated) Friedmann like universe.

Classical configurations of energy distribution corresponding to long and/or short range fields coupled minimally to gravitation belong to the class of entities which generate singular cosmologies.

This is an almost direct consequence of the so-called singularity theorems of classical general relativity.

So, in the search of a non-singular universe we have been led to investigate the properties of non-minimal coupling of electromagnetic and gravitational fields.

Our theory starts with the Lagrangian

$$L = \sqrt{-g} \left[\frac{1}{k} R - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - R W_{\mu} W_{\nu} g^{\mu\nu} \right] \tag{1}$$

in which conventions are as usual (see (1)).

In (1) we have found a new solution of the equations derived from Lagrangian (1) which leads to some fascinating features of the Universe.

Among those, we can quote that this cosmos has no singularity, it has no particle horizon and contains an unique free parameter (the longitudinal electromagnetic field) which allows one to fix the density of lighest compression of the cosmos (which separates a contracting phase from our expanding era).

Here we intend to present a new exact solution of equations derived for (1) which is a sort of generalization of the standard anisotropic Universe found by Kasner.

The equations of motion are:

$$\left[\frac{1}{k} - W^2\right]\left[R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}\right] = - E_{\mu\nu} + \Box\, W^2 g_{\mu\nu} + W^2{}_{,\mu;\nu} + R\, W_\mu W_\nu \tag{2a}$$

$$f^{\mu\nu}{}_{;\nu} = R\, W^\mu \tag{2b}$$

in which $E_{\mu\nu}$ is Maxwell's tensor

$$E_{\mu\nu} \equiv f_{\mu\alpha} f^\alpha_\nu + \frac{1}{4} g_{\mu\nu} f_{\alpha\beta} f^{\alpha\beta} \tag{3}$$

and

$$W^2 \equiv W^\mu W^\nu g_{\mu\nu}.$$

We see from (2b) and taking the trace of (2a) that the non-minimal coupling (represented by the "mass" R of the photon) induces non-linearities in the equation of motion of the electromagnetic field.

Let us now look for a cosmical solution of this set of equations which represents an homogeneous and anisotropic universe.

We set

$$ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)dy^2 - c^2(t)dz^2 \tag{4}$$

and

$$W^\mu = (W(t), 0, 0, 0,)$$

Equation (2a,b) reduce in this case to the set

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\ddot{\gamma}}{\gamma} = 0 \tag{5a}$$

$$\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\left(\frac{\dot{b}}{b} + \frac{\dot{c}}{c} + \frac{\dot{\gamma}}{\gamma}\right) = 0 \tag{5b}$$

$$\frac{\ddot{b}}{b} + \frac{\dot{b}}{b}\left(\frac{\dot{a}}{a} + \frac{\dot{c}}{c} + \frac{\dot{\gamma}}{\gamma}\right) = 0 \tag{5c}$$

$$\frac{\ddot{c}}{c} + \frac{\dot{c}}{c}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{\gamma}}{\gamma}\right) = 0 \tag{5d}$$

$$\frac{\ddot{\gamma}}{\gamma} + \frac{\dot{\gamma}}{\gamma}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}\right) = 0 \tag{5e}$$

in which we have defined $\gamma \equiv 1 - W^2$ setting $a = a_0 t^{p_1}$, $b = b_0 t^{p_2}$, $c = c_0 t^{p_3}$ and $\gamma = \gamma_0 t^{p_4}$ equations (5) reduce to two conditions on the four numbers p_k:

$$p_1 + p_2 + p_3 + p_4 = 1 \tag{6a}$$

$$(p_1)^2 + (p_2)^2 + (p_3)^2 + (p_4)^2 = 1 \tag{6b}$$

which is nothing but a five-dimensional Kasner world, the fifth dimension being associated to the electromagnetic field.

This result should be expected, once we recognize that if $E_{\mu\nu} = 0$ the dynamical equations reduce to the set

$$R_{\mu\nu} = - \frac{\gamma_{,\mu;\nu}}{\gamma} \tag{7a}$$

$$\Box \gamma = 0 \tag{7b}$$

and these equations can be interpreted either as a scalar-tensor theory of gravity or a five dimensional geometrical version of it (see the work or Belinskii and Khalatnikov). It seens worth to remark that (5) are reducible to an autonomous system of differential equations.

The origin of the phase space represents the constant Minkowskii Universe in the absence of the electromagnetic field.

The isotropic limit a = b = c which in the four dimensional Kasner solution is nothing but flat space, can be easily seen here to be equivalent to a Friedmann cosmos with euclidean section filled with a photon gas in the standard regime.

We can then conceive that in the very early cosmos scenario anisotropy should be related to the strong non-minimal coupling of electromagnetic and gravitational fields. Then as evolution proceeds such coupling becomes less effective and the photons behave as in the standard (minimal coupling) way. This brings us to the very important point of the passage from a Kasner era to an isotropic Friedmann-like cosmos and to the possible role of the non-minimal coupling on a previous non-isotropic era of the world. This matter is under investigation.

References

[1] Novello, M. - Salim J.M. - Phys. Rev. D, 20, 377 (1979)

[2] Novello, M. - Heintzmann, H. - "An eternal Universe" to be published

[3] Belinskii, V.A. - Khalatnikov, I.M. - Sov. Phys- JETP, 36, 591 (1973)

[4] Ruckert, E. - M. Sci. Thesis (1981) - unpublished