BACKWARD ELASTIC pd SCATTERING IN THE RESONANCE ENERGY REGION

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ABSTRACT - The cross section for backward elastic pd scattering in the proton kinetic energy interval from 0.4 to 1.0 Gev is calculated from a double triangle diagram with intermediate Δ (1232) excitation. Different approximations for the loop integrals are discussed. The predicted cross sections reproduce quite well the enhancement observed in the experimental data at about 600 Mev.

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1 - INTRODUCTION

It is well known that the experimental data on elastic pd scattering exhibits a backward peak, i. e., a sharp rise of the differential cross section towards 180° [1-8]. Recently a systematic measurement of the energy dependence of the backward cross-section revealed an enhancement in the 600 MeV proton kinetic energy region [9,13]. Roughly two kinds of models have been proposed to reproduce the experimental features: exchange (u-channel) models and triangle (s-channel) models.

The one-nucleon-exchange or Chew Goldberger pick-up process [14] (fig.1) is the simplest mechanism able to produce a backward peak. It fits rather well the low-energy data but its contribution in the GeV region is too small. Kerman and Kislinger [15] pointed out that the exchange of baryonic resonances could become important by increasing the energy and hence the maximum momentum transfer. They considered the N* (1688) exchange in addition to the nucleon exchange. But, in order to fit the data their model must have an energy dependent N* probability: at 1 GeV the fit needs a 1% N* component in the deuteron, whereas at 0.58 GeV 2.5% of N* is needed. Sharma, Bhasin and Mitra [16,17] improved this model by including N*(1470), N*(1750) and N*(1520) exchanges and making relativistic calculations for the vertex. However they did not reproduce the detailed data structure. It would be difficult to obtain complex s-channel behaviour like an enhancement with only these exchanges.

Another attempt to explain the backward pd data was made by Bertocchi and Capella [18] using a Glauber-type model. Even including a very strong and quite unreliable off-mass-shell behaviour for the NN elastic amplitude, the single and double scattering terms (fig.2) do not provide the data description, the cross-sections being one order of magnitude smaller than the observed in experiments.

The success of one-pion-exchange(OPE) model of Yao [19] in $pp \rightarrow d\pi$ reaction (fig.3a) and the underlying idea that at large momentum transfer virtual pion production becomes important led Craigie and Wilkin [20] to propose the triangle mechanism (fig.3b) for large angle pd elastic scattering. Using as input the $pp \rightarrow d\pi$ experimental data they obtained good shapes for the $pd \rightarrow dp$ distributions, but the cross sections were underestimated by a factor two due to a mistake in their calculations [21]. This model was retaken by Barry [22] who related the $pd \rightarrow dp$ and the $pp \rightarrow d\pi$ cross sections by a parameter ξ, determined from the deuteron structure. Actually, in order to account for several simplifications and to fit the experimental data one must use this parameter as an overall normalization factor [22-24]. In the triangle model the 600 MeV enhancement in the backward cross section appears naturally, as a consequence of the $\Delta(1232)$ excitation in the $pp \rightarrow d\pi$ sub-reaction.

Which one of these described mechanism actually takes place is a motive of contention. We think however that the existence of an enhancement at 600 MeV, exactly where the $\Delta(1232)$ resonance can be excited in the sub reaction $\pi N \rightarrow \pi N$ is a clear indication that the OPE mechanism dominates the backward cross section, at these energies. In this paper we propose to calculate more accurately the contribution of the triangle diagram (fig. 3b) in this energy region. To do this we will also take the OPE mechanism for the $pp \to d\pi$ sub-reaction obtaining then the double triangle graph (fig.4), that we will actually calculate. As we will see, in the region where the Δ production dominates the $\pi N \to \pi N$ sub-reaction the loop integrals over the wave functions and poles can be performed analytically and we obtain, without any free parameter, good results for the backward cross section in the 0.4 to 1.0 GeV incident proton kinetic energy range.

In the next section we derive the cross section formula. In section 3 we present our approximations for the loop integrals while section 4 is reserved to our results and conclusions. Kinematics and the integral calculation are given in Appendix A and B..

2 - CALCULATION OF THE CROSS-SECTION

We will calculate the contribution of the double triangle graph (fig. 4) to the backward pd scattering. The corresponding Feynman amplitude, for charged pions, is

$$A = \int d^4 f_1 \, d^4 f_2 \; \bar{u}(p_2) \, \gamma_5 \, i \frac{(\not{M}_1 + m)}{M_1^2 - m^2} \; C \frac{\Gamma_1 \cdot \varepsilon_1}{\sqrt{2}} \; \frac{i(-\not{N}_1 + m)}{N_1^2 - m^2} \; \cdot$$

$$\cdot \frac{i\sqrt{2}\; G\; F\,(k_1^2)}{K_1^2 - \mu^2} \; A_{\pi N} \; \frac{i\sqrt{2}\; G\; F\,(k_2^2)}{K_2^2 - \mu^2} \; \frac{i(-\not{N}_2 + m)}{(N_2^2 - m^2)} \; \cdot$$

$$\cdot\, C \frac{\Gamma_2 \cdot \varepsilon_2}{\sqrt{2}} \; \frac{i(\not{M}_2 + m)}{M_2^2 - m^2} \; \gamma_5 u\,(p_1) \qquad (2.1)$$

where the four-momenta are defined in figure 4 and m and μ are the nucleon and pion masses. We choosed as loop variables the relative momenta of nucleons inside the deuterons:

$$f_i = \frac{1}{2} (N_i - M_i) \qquad i=1,2 \qquad (2.2)$$

The quantities G, C, Γ, ε are respectively the ΠNN and the pdn coupling constants ($G^2/4\pi = 14.5$), the relativistic pdn vertex and the deuteron polarization four-vector. The Dirac amplitude for $\pi N \to \pi N$ scattering is written as $A_{\pi N}$ and $F(k^2)$ is the pion exchange form factor in the $NN \to N\Delta$ sub-reaction.

In our calculations we will neglect the deuteron D-wave. So the pdn vertex is given by [25]

$$\Gamma_\mu = \phi\, (N_i^2, M_i^2)\, \gamma_\mu \qquad (2.3)$$

where the off-mass-shell form factor is normalised by $\phi\, (m^2, m^2) = 1$

Now we must relate the relativistic pdn vertex to the nonrelativistic wave function, which is the only available phenomenological information about this vertex. However the relativistic vertex is a function of f_0 and $|\underset{\sim}{f}|$ (via the invariants N_i^2 and M_i^2) whereas the non-relativistic wave function depends only on $|\underset{\sim}{f}|$. Thus it lacks information about the f_0 dependence of the pdn vertex. To overcome this difficulty one must give a suplementary condition relating f_0 to $|\underset{\sim}{f}|$. Here we adopt the Gross [26] *approximation and keep only the contribution in which particles M_1 and M_2 are on the mass shell, i.e., we replace*

$$\frac{1}{M_i^2 - m^2} \longrightarrow \frac{-i\pi}{M_{i_0}}\, \delta\left(M_{i_0} - \sqrt{m^2 + \underset{\sim}{M}_i^2}\right) \qquad (i=1,2) \qquad (2.4)$$

Consequentely, the integration over f_0 is straighforward and the non-relativistic limit of the vertex function

$\phi(N_i^2, m^2)$ and the wave function $\Psi(\underset{\sim}{f}_i)$ are related by [11]

$$\frac{C\ \phi(N_i^2, m^2)}{N_i^2 - m^2} = \frac{1}{2m}\,(32\,\pi^3 m)^{1/2}\quad \Psi(\underset{\sim}{f}_i) \tag{2.5}$$

To perform analytically the remaining integrals is a considerable task and so we must look for good approximations to the amplitude A. As wave functions decrease very fast with $|\underset{\sim}{f}|$ it is reasonable to substitute the rest of the numerator by its value at $\underset{\sim}{f}_i = 0$. This means that we will do the spin calculations outside the integrals and consider internal nucleons on-mass-shell when doing spin summations. So, we have:

$$M_i = N_i = \frac{d_i}{2} \quad \text{and} \quad M_i^2 = N_i^2 = m^2 \qquad i = 1, 2$$

$$(\not{M}_i + m) = 2m\ \Sigma\ u(M_i)\ \overline{u}(M_i) \tag{2.6}$$

$$(-\not{N}_i + m) = -2m\ \Sigma\ v(N_i)\ \overline{v}(N_i)$$

with the approximations 2.3-6 the amplitude becomes

$$A = (2m)^2 \frac{m}{M_{10} M_{20}} \frac{1}{(2\pi)^3} \sum_{\text{spin}} \overline{u}(p_2)\ \gamma_5\ u(M_1)\overline{u}(M_1)\not{\epsilon}_1 v(N_1)\overline{v}(N_1)\ .$$

$$\cdot A_\Delta\ v(N_2)\ \overline{v}(N_2)\ \not{\epsilon}_2\ u(M_2)\ \overline{u}(M_2)\ \gamma_5\ u(p_1)\ \frac{(\sqrt{2}\ G)^2}{(\sqrt{2})^2}\ .$$

$$\cdot\ F(k_1^2)\ F(k_2^2) \int d^3 f_1\ \ d^3 f_2\ \ \Psi(\underset{\sim}{f}_1)\ \ \Psi(\underset{\sim}{f}_2)\ \frac{1}{M_\Delta^2 - Q^2}\ .$$

$$\cdot\ \frac{1}{K_1^2 - \mu^2}\ \frac{1}{K_2^2 - \mu^2} \tag{2.7}$$

where we considered only the Δ- contribution to the $\Pi N \to \Pi N$ amplitude and factorized it into a spin dependent part A_Δ and the Δ pole with $M_\Delta = m_\Delta + i\Gamma/2$

$$Q^2 = (q + f_1 + f_2)^2 \quad , \quad q = \frac{1}{2}(p_1 + p_2) \tag{2.8}$$

If we write explicitly the spin indices (fig.4) and put

$$\bar{v}(N_1,n_1)\, A_\Delta\, v(N_2,n_2) = A^\Delta_{n_1 n_2}$$

$$\bar{u}(p_2,\lambda_2)\,\gamma_5\, u(M_1,m_1) = g_{\lambda_2 m_1} \tag{2.9}$$

$$\bar{u}(M_1,m_1)\,\frac{\not{\varepsilon}_1}{\sqrt{2}}\, v(N_1,n_1) = - \langle 1,\tau_1 | \tfrac{1}{2},\tfrac{1}{2};\, m_1,n_1\rangle$$

The spin part of the amplitude writes

$$S(\tau_1,\tau_2,\lambda_1,\lambda_2) = \sum_{m_i} \sum_{n_i} g_{\lambda_2 m_1} \langle 1,\tau_1 | \tfrac{1}{2},\tfrac{1}{2}; m_1,n_1\rangle A^\Delta_{n_1 n_2} \cdot$$

$$\cdot \langle 1,\tau_2 | \tfrac{1}{2},\tfrac{1}{2};\, m_2,n_2\rangle \; g_{m_2 \lambda_1} \tag{2.10}$$

The spin summation can be easily done in the lab system ($\underset{\sim}{d}_1=0$) if we choose the direction of the vector $\underset{\sim}{N}_2$ as the quantization and z-axis and the xz plane as the reaction plane. In this case the amplitudes have the symmetry properties

$$A^\Delta_{++} = A^\Delta_{--} \qquad\qquad g_{++} = -g_{--}$$
$$A^\Delta_{+-} = -A^\Delta_{-+} \qquad\qquad g_{+-} = g_{-+} \tag{2.11}$$

and we get

$$\sum_{\tau_i \lambda_i} | S(\tau_1, \tau_2, \lambda_1, \lambda_2) |^2 = \frac{1}{4} \frac{9}{4} \left(\sum_{\lambda_2 m_1} | g_{\lambda_2 m_1} |^2 \right) \cdot$$

$$\cdot \left(\sum_{n_1 n_2} | A^{\Delta}_{n_1 n_2} |^2 \right) \left(\sum_{m_2 \lambda_1} | g_{m_2 \lambda_1} |^2 \right) \qquad (2.12)$$

where

$$\sum_{\lambda_2 m_1} | g_{\lambda_2 m_1} |^2 = \sum_{m_2 \lambda_1} | g_{m_2 \lambda_1} |^2 = - \frac{2 k^2}{(2m)^2} \qquad (2.13)$$

(note that for $\underset{\sim}{f}_i = 0$ *we have* $k_1^2 = k_2^2 = k^2$*)*

We will express the invariant quantity $\sum_{n_1 n_2} | A^{\Delta}_{n_1 n_2} |^2$ *in the* πN *center of mass:*

$$\sum_{n_1 n_2} | A^{\Delta}_{n_1 n_2} |^2 = 2(1 + 3 \cos^2 \Theta_{\pi N}) \left(\frac{8\pi \sqrt{s_{\pi N}}}{2m \; q_{\pi N}} m_{\Delta} \Gamma \right)^2 \qquad (2.14)$$

where $\Theta_{\pi N}$ *and* $q_{\pi N}$ *are the scattering angle and pion momentum in this system.*

To take into account other isospin possibilities the amplitude must be multiplied by a factor 2.

The differential cross section in the pd center of mass is given by

$$\frac{d\sigma}{d\Omega} = \frac{(2m)^2}{64\pi^2 s} \cdot \frac{1}{6} \sum_{\text{spins}} | A |^2 \qquad (2.15)$$

so that finally we have

$$\frac{d\sigma}{d\Omega} = \frac{3m^2}{\pi^4 M_{10}^2 M_{20}^2} \left[\frac{G^2}{4\pi} F^2(k^2) \, k^2 \right]^2 \frac{s_{\pi N}}{s} (1 + 3 \cos^2 \Theta_{\pi N}) \frac{m_{\Delta}^2 \Gamma^2}{q_{\pi N}^2} .$$

$$\cdot \left| \int d^3 f_1 \, d^3 f_2 \, \Psi(\underset{\sim}{f_1}) \, \Psi(\underset{\sim}{f_2}) \, \frac{1}{M_\Delta^2 - Q^2} \, \frac{1}{K_1^2 - \mu^2} \right. \cdot$$

$$\cdot \left. \frac{1}{K_2^2 - \mu^2} \right|^2 \qquad (2.16)$$

In the next section we will estimate different approximations to the integral over the wave functions and poles.

In order to compare the double triangle and the one nucleon exchange contribution we have also calculated the cross section corresponding to the graph of figure 1a. Its Feynman amplitude is

$$M = \bar{u}\,(p_2) \quad C \, \frac{\Gamma_1 \cdot \varepsilon_1}{\sqrt{2}} \quad \frac{i(-\not{n}+m)}{n^2 - m^2} \; C \, \frac{\Gamma_2 \cdot \varepsilon_2}{\sqrt{2}} \quad u(p_1) \qquad (2.17)$$

Doing the same approximations as in 2.3-5-6 we get

$$M = -i \; 2^4 \, \pi^3 \; (n^2 - m^2) \; \Psi^2(\underset{\sim}{f}) \; \sum_\lambda \langle 1, \tau_1 \; | 1/2, 1/2 ; \lambda_2, \lambda \rangle \; \cdot$$

$$\cdot \; \langle 1, \tau_2 \; | \frac{1}{2}, \frac{1}{2} \; ; \lambda, \lambda_1 \rangle \qquad (2.18)$$

where $\underset{\sim}{f}$ is the momentum of the neutron in the lab sistem. After spin summation and using (2.15) we obtain

$$\frac{d\sigma}{d\Omega}^{ONE} = 3 \, \pi^4 \, \frac{(2m)^2}{s} \, (n^2 - m^2)^2 \, \Psi^4 \, (\underset{\sim}{f}) \qquad (2.19)$$

which is equivalent to the expression of Kerman and Kislinger [15,27]

3 - APPROXIMATIONS AND RESULTS

Now we will estimate the integral

$$I = \int d^3 f_1 \; d^3 f_2 \; \Psi(\underset{\sim}{f}_1) \quad \Psi(\underset{\sim}{f}_2) \; \frac{1}{M_\Delta^2 - Q^2} \; \frac{1}{K_1^2 - \mu^2} \; \frac{1}{K_2^2 - \mu^2} \tag{3.1}$$

for different approximations and compare the cross-sections, given by equation (2.16), to the experimental data. We will do the calculations for exponential-type wave functions and adopt the Mc Gee [28] parametrization, that for S-wave is:

$$\Phi(\underset{\sim}{r}) = \frac{1}{\sqrt{4\pi}} \; \frac{u(r)}{r} \tag{3.2}$$

$$u(r) = C \sum_{j=1}^{5} a_j \, e^{-\alpha_j r} \quad ; \quad \int_0^\infty u^2(r) \, dr = 1$$

so that in momentum space

$$\Psi(\underset{\sim}{f}) = \sqrt{\frac{1}{2\pi^2}} \; \tilde{u}(|\underset{\sim}{f}|) = \sqrt{\frac{1}{2\pi^2}} \; C \sum_{j=1}^{5} \frac{a_j}{\alpha_j^2 + |\underset{\sim}{f}|^2}$$

$$\tilde{u}(\,|\underset{\sim}{f}|\,) = \int_0^\infty u(r) \, j_0(|\underset{\sim}{f}| r) r \, dr$$

the kinematics and technical details of integration are respectively given in Appendices A and B

3a - WAVE FUNCTION INTEGRATION

The wave functions in equation (3.1) act as a weight function for the rest of the integrand and its very rapid fall-off favorizes small $|\underset{\sim}{f}|$ values. Then the simplest approximation is to take out of the integral the pions and delta poles for the value $\underset{\sim}{f}_i = 0$, as it was done in section 2 for the spin part of the amplitude. This approximation is equivalent to that used by Yao [19] and Barry [22] and gives a very simple formula for the cross-section:

$$\frac{d\sigma^{pd}}{d\Omega} = \frac{12}{m^2}\left[\frac{G^2\, F^2(k^2)k^2}{(k^2-\mu^2)}\right]^2 \frac{s_{\pi N}}{s}\, |\Phi(0)|^4 \,\frac{d\sigma^{\pi N}}{d\Omega} \qquad (3.3)$$

where $\frac{d\,\sigma^{\pi N}}{d\,\Omega}$ is the π-N differential cross section in the π-N c.m. system and $\Phi(0)$ is the value of the wave function for $\underset{\sim}{r} = 0$. For the Hulthen wave function

$$\Phi(\underset{\sim}{r}) = C_1 \left(\frac{e^{-\alpha r}}{r} - \frac{e^{-\beta r}}{r}\right) \qquad (3.4)$$

$$C_1 = \frac{1}{\beta-\alpha}\left[\frac{\alpha\beta(\alpha+\beta)}{2\pi}\right]^{1/2} \quad ; \quad \begin{array}{l}\alpha = .232 \ fm^{-1}\\ \beta = 1.202 \ fm^{-1}\end{array}$$

the constant $\Phi(0)$ is related to the Barry ξ parameter by

$$\xi = \frac{2\,\Phi(0)}{\alpha\, C_1} \simeq 8.36$$

which is the same value obtained in ref. [22] when a dispersive approach is used to calculate the OPE coupling strengh.

The cross section obtained with formula (3.3) is larger than the experimental one by a factor 40 in the 600 MeV proton kinetic energy region.

In fact this approximation is very crude because it neglects all $|\underset{\sim}{f}|$ dependence other than that contained in the wave function. Furthermore the constant $\phi(0)$ depends strongly on the assymptotic behaviour of $\Psi(\underset{\sim}{f})$, and consequently is a very badly known quantity.

3-b - PIONS POLE INTEGRATION

In this approximation we take out of the integral (3.1) the πN amplitude for $\underset{\sim}{f}_i = 0$ (here the delta pole) obtaining two integrals of the form

$$J(k_i) = \int \frac{d^3 f_i \; \Psi(\underset{\sim}{f}_i)}{K_i^2 - \mu^2} \tag{3.5}$$

where $K_i = k_i + f_i$, $k_i = (k_{i_0}, \underset{\sim}{k}_i)$

For the exponential type wave function of eq. (3.2) one gets [29]

$$J(k_i) = (2\pi)^{3/2} \frac{C}{\sqrt{4\pi}} \frac{1}{|\underset{\sim}{k}_i|} \left(\sum_{j=1}^{5} - a_j \; \mathrm{arctg} \frac{|\underset{\sim}{k}_i|}{\eta + \alpha_j} \right) \tag{3.6}$$

where $\eta^2 = \mu^2 - k_{i_0}^2$

The expression (3.6) is valid (it is not the case here) even when the pole is in the physical region. In such case $\eta^2 \leq 0$ and the function $J(k)$ becomes complex , its real and imaginary parts corresponding respectively to the principal part and delta part contribution of the particle propagator. Of course in this case one must take

$$\mathrm{arctg}\; x = \frac{1}{2i} \ln \frac{1 + ix}{1 - ix}$$

This approximation is more reliable than (3-a) because the presence of the pion pole increases the fall-off the integrand and consequently $J(k)$ is less sensible than $\Phi(0)$ to the large $|\underset{\sim}{f}|$ behaviour of $\Psi(\underset{\sim}{f})$. The results obtained for the backward cross sections are shown (curve (b)) in figure 5. In the 600 MeV region they are too big by a factor 2. In fact this approximation is expected to give better results at higher energy, i.e., in the region where the πN amplitude does not vary quickly with the incident energy.

3-c - DELTA POLE INTEGRATION

As we are in the delta excitation region and the pion is far from the mass-shell, integral (3.1) can be approximated by

$$I = \frac{1}{(k^2 - \mu^2)^2} \int d^3 f_1 \quad d^3 f_2 \; \Psi(\underset{\sim}{f}_1) \; \Psi(\underset{\sim}{f}_2) \frac{1}{M_\Delta^2 - Q^2(f_1, f_2)} \qquad (3.7)$$

where $Q = q + (f_1 + f_2)$; $q = (q_0, \underset{\sim}{q})$

Now it is convenient to introduce new variables

$$q_1 = -(f_1 + f_2)$$
$$q_2 = \frac{1}{2}(f_1 - f_2) \qquad (3.8)$$

so that we can write

$$I = -\frac{1}{(k^2 - \mu^2)^2} \int d^3 q_1 \frac{1}{M_\Delta^2 - Q^2(q_1)} \; G(\underset{\sim}{q}_1) \qquad (3.9)$$

where $G(\underset{\sim}{q}_1)$ is the deuteron form factor

$$G(\underset{\sim}{q}_1) = \int d^3 \underset{\sim}{q}_2 \; \Psi(\underset{\sim}{q}_2 - \frac{\underset{\sim}{q}_1}{2}) \; \Psi(\underset{\sim}{q}_2 + \frac{\underset{\sim}{q}_1}{2}) \qquad (3.10)$$

For the wave function (3.2) the integral (3.9) gives [30]

$$I = (2\pi)^3 \frac{C^2}{4\pi} \frac{1}{(k^2 - \mu^2)^2} \frac{1}{|\underset{\sim}{q}|} \left\{ \frac{|\underset{\sim}{q}|}{2} \left[\sum_{j=1}^{5} a_j^2 \ln (|\underset{\sim}{q}|^2 (2\alpha_j - i\eta)^2) \right.\right.$$
$$\left. + 2 \sum_{k>j} a_j a_k \ln (|\underset{\sim}{q}|^2 + (\alpha_j + \alpha_k - i\eta)^2) \right] +$$
$$+ \sum_j a_j^2 (2\alpha_j - i\eta) \operatorname{arctg} \frac{|\underset{\sim}{q}|}{2\alpha_j - i\eta} \qquad (3.11)$$

$$+ 2 \sum_{k>j} a_j a_k (\alpha_j + \alpha_k - i\eta) \text{ arctg } \frac{|\underset{\sim}{q}|}{(\alpha_j + \alpha_k - i\eta)} \Bigg\}$$

where $\eta^2 = q_0^2 - M_\Delta^2$

The backward cross-section corresponding to the delta-pole integration is shown in figure 5 (curve c) . The absolute normalization in the enhancement region is quite good. One can see how the pole integration is important when we are in the resonance region.

3-d - DELTA AND PION POLES INTEGRATION

The delta and pion poles in equation (3.1) can be integrated if one neglects the delta pole variation in one of the two internal loop variables. In this approximation we take

$$Q = q + f_1$$

and then equation (3.1) writes

$$I = \int \frac{d^3 f_1 \ \Psi(\underset{\sim}{f_1})}{(M_\Delta^2 - Q^2(f_1))(K_1^2 - \mu^2)} \int \frac{d^3 f_2 \ \Psi(\underset{\sim}{f_2})}{(K_2^2 - \mu^2)} \qquad (3.13)$$

The result of the last integral en (3.13) is just given by eq.(3.6). In appendix B, we have calculated the first integral whose final result is

$$L(q,k_1) = - \frac{\sqrt{2}\,\pi C}{r} \sum_j \frac{a_j}{A(T_+ - T_-)} \left\{ ln \frac{t_+ - T_+}{t_+ - T_-} + ln \frac{t_- - T_-}{t_- - T_+} \right\} \qquad (3.14)$$

The variables are defined in appendix B.

The backward cross-section corresponding to this approximation is represented in figure 5(curve d). The absolute normalization and the peak position are quite well reproduced. This result shows the importance of performing the loop integrals as complete as possible when one is interested in reproducing quantitatively the experimental data.

4 - RESULTS AND CONCLUSIONS

The backward ($\Theta_{c.m.} = 180^0$) elastic pd cross section as a function of the incident proton kinetic energy in the lab. is shown in figure 5. The experimental references are listed in the figure. The errors on data points include the incertainties of extrapolating the existing data to $\Theta_{c.m.} = 180^0$, except those from ref. [11] which are taken at $\Theta_{c.m.} = 170^0$. The data of ref. [8,13] are from neutron-deuteron elastic scattering. The continuous curves (b), (c) and (d) are the contribution of the double pion exchange graph (fig.4) when approximations (3-b), (3-c) and (3-d) of section 3 are used to calculate the integrals in equation (2.16). Calculations were performed with the Mc Gee[28] wave function. As one can see these approximations give similar results and reproduce, with no free parameter, the good shape and the good normalization for the cross section. To see how the normalization depends on the choice of the wave function we have also used the third Moravcsik[31] wave function. The results change less than 2% in the peak region. We also show in Fig. 5, for comparison, the one-nucleon-exchange contribution (dotted line) calculated from equation (2.19) where we took into account the D-wave component in the deuteron wave function. As we see the O.N.E. fails completely to explain the cross section behaviour near T=600 MeV. The dashed curve in figure 5 is the incoherent sum of the O.N.E. contribution and that of intermediate Δ excitation (curve (c)). Although this sum fits quite well the data[32], we must have in mind that we neglected the interference term and that duality claims for the possibility of double counting. These points become relevant mainly in the regions where the two contributions are of the same order.

We present in fig. 6 the differential cross section as a function of cos $\Theta_{c.m.}$ at fixed energy. All the curves correspond to the approximation (3-d). The continuous lines are obtained when cos Θ and cos $\Theta_{\pi N}$ are related by equation (A9). This choice gives a too large slope for the cross section what was somehow expected since we neglected the integrations over the Δ angular dependence and since there are incertainties about off-mass-shell extrapolations (see eq. A8 and A10). The dashed line is obtained when we keep cos $\Theta_{\pi N}=-1$ over the entire angular range.

In our calculations we neglected the deuteron D-wave coupling. We think however that, as it was shown in ref. [32] for the graph 3b, its contribution would not change apreciably the results.

To conclude we think that our results are a quantitative confirmation that intermediate $\Delta(1232)$ excitation is responsible for the enhancement observed in the energy dependence of the backward cross section for elastic pd scattering near T=600 MeV. Furthermore the methods used to calculate loop integrals can be applied in similar triangle-type graphs.

APPENDIX A: Kinematics

Here we shall follow the notation shown in figure 4. As an exemple, $p(p_1, \lambda_1)$ means that the particle is a proton with four-momentum p_1 and spin projection λ_1.

The energy-momentum conservation writes

$$p_1 + d_1 = p_2 + d_2 \ . \qquad (A1)$$

The internal nucleon momenta are defined by

$$N_i = \frac{d_i}{2} + f_i \qquad\qquad i = 1,2$$

$$M_i = \frac{d_i}{2} - f_i$$

where f_i 's are the Fermi momenta inside the deuterons.

The virtual pion momenta are

$$K_i = k_i + f_i \qquad\qquad i=1,2 \qquad (A2)$$

where

$$k_1 = p_2 - \frac{d_1}{2} \ ; \quad k_2 = p_1 - \frac{d_2}{2} \qquad (A3)$$

If we call T the proton kinetic energy in the lab. system ($\underset{\sim}{d}_1 = 0$), the square of the cm. energy is given by

$$s = (p_1 + d_1)^2 = 9m^2 + 4m\,T \qquad (A4)$$

where we took the deuteron mass equal to 2m. The other Mandelstam invariants are

$$t = (d_1 - d_2)^2 = -2\,\underset{\sim}{p}^2\,(1 - \cos\Theta)$$
$$u = (p_1 - d_2)^2 = 10\,m^2 - s - t \qquad (A5)$$

where $\underset{\sim}{p}$ *and* Θ *are the momentum and scattering angle in the pd c.m. system:*

$$\underset{\sim}{p} = P\,(s,\, m^2,\, 4m^2)$$
$$P\,(a,b,c) = \left[\frac{(a - b - c)^2 - 4bc}{4a}\right]^{1/2} \qquad (A6)$$

Of course in the final cross-sections formulae only f-independent quantities appear, such as the Mandelstam variables corresponding to intermediate πN *scatering for* $f_i = 0$*:*

$$s_{\pi N} = q^2 = \frac{1}{4}\,(p_1 + p_2)^2 = m^2 - t/4$$

$$u_{\pi N} = u \qquad (A7)$$

$$t_{\pi N} = 2(m^2 + k^2) - s_{\pi N} - u_{\pi N}$$

where $k^2 = k^2_1 = k^2_2 = \frac{1}{2}\,(u - m^2)$

The scattering angle in the πN *c.m. is given by*

$$\cos\,\Theta_{\pi N} = 1 + \frac{t_{\pi N}}{2\,q'^2_{\pi N}} \qquad (A8)$$

where $q'_N = P(s_{\pi N}, m^2, k^2)$ *is the off-shell pion momentum. Thus when the relative momenta* f_i *of nucleons inside the deuterons are neglected equations (A5) and (A7) give*

$$(\cos\Theta_{\pi N} - 1) = \frac{p^2}{4q'^2_{\pi N}}\,(\cos\Theta - 1) \qquad (A9)$$

For the delta width Γ *and for the pion exchange form factor* $F(k^2)$ *we take the Wolf*[33] *parametrization*

$$\Gamma(s_{\pi N}) = \frac{\Gamma_\Delta\ m_\Delta\ q_{\pi N}\ u_1(R_\Delta\, q_{\pi N})}{\sqrt{s_{\pi N}}\ \ q_\Delta\ u_1(R_\Delta\, q_\Delta)} \qquad (A10)$$

$$F(k^2) = \frac{u_1(R_\Delta\, q'_{\pi N})}{u_1(R_\Delta\, q_{\pi N})} \; \frac{(1 + R^2_N\, q^2_N)}{(1 + R^2_N\, q'^2_N)}$$

where $q_{\pi N} = P(\, s_{\pi N},\, m^2,\, \mu^2)$, $q_\Delta = P(m^2_\Delta,\, m^2,\, \mu^2)$, $q_N = P(m^2, m^2, \mu^2)$, $q'_N = P(m^2,\, m^2,\, k^2)$, $\Gamma_\Delta = 0.114$ GeV, $m_\Delta = 1.232$ GeV, $R_\Delta = 1.76\ \mathrm{GeV}^{-1}$, $R_N = 2.86\ \mathrm{GeV}^{-1}$ *and*

$$u_1(x) = \frac{1}{2x^2} \left[\frac{2x^2 + 1}{4x^2} \ln\,(4x^2 + 1) - 1 \right] \tag{A 11}$$

In section 3 the different results for the integral (3.1) are expressed in terms of the variables $k_{io}, |\underset{\sim}{k}_i|$, q_o *and* $|\underset{\sim}{q}|$. *In the lab. system we have*

$$k_{1_o} = \frac{m^2 - u}{4m} \quad ; \quad k_{2_o} = k_{1_o} - \frac{t}{8m}$$

$$\underset{\sim}{k}_i^2 = k^2_{i_o} - k^2$$

$$q_o = \frac{s - u}{8m} \quad ; \quad \underset{\sim}{q}^2 = q_o^2 - q^2 \tag{A 12}$$

$$(\underset{\sim}{q} - \underset{\sim}{k}_1)^2 = (q_o - k_{1_o})^2 - m^2$$

$$M_{2_o} = M_{1_o} - t/8m \quad ; \quad M_{1_o} = m$$

APPENDIX B:

We want to calculate the integral

$$L(q,k_1) = \int \frac{d^3 f_1 \; \Psi(\underset{\sim}{f}_1)}{(M_\Delta^2 - Q^2)(K_1^2 - \mu^2)} \tag{B1}$$

where $Q = q + f_1$ and $K_1 = k_1 + f_1$

The approximation (2.4) and the very rapid fall-off of the wave function suggest to take, for simplicity $f_{1_0} = 0$. Then we can put the integral (B1) in the form

$$L = - \int d^3 f_1 \Psi(\underset{\sim}{f}_1) \frac{1}{a^2 + (\underset{\sim}{q} + \underset{\sim}{f}_1)^2} \frac{1}{b^2 + (\underset{\sim}{k}_1 + \underset{\sim}{f}_1)^2} \tag{B2}$$

with $a^2 = M_\Delta^2 - q_0^2$; $b^2 = \mu^2 - k_{1_0}^2$

In fact a better approximation would be to take $f_{1_0} \simeq \frac{\underset{\sim}{f}_1^2}{2m}$. Even in this case, as it is shown in ref. [29] we can put the integral in the form (B2). However, as the numerical result does not change appreciably, we will keep $f_{1_0} = 0$.

Now, using the method of Feynman parameters,

$$\frac{1}{xy} = \int_0^1 \frac{d\beta}{(\beta x + (1-\beta)y)^2} \tag{B3}$$

and taking

$$x = a^2 + (\underset{\sim}{q} + \underset{\sim}{f}_1)^2 \quad ; \quad y = b^2 + (\underset{\sim}{k}_1 + \underset{\sim}{f}_1)^2$$

we have

$$L = - \int_0^1 d\beta \int \frac{d^3 f_1 \; \Psi(\underset{\sim}{f}_1)}{\left[D^2 + (\underset{\sim}{f}_1 + \underset{\sim}{v})^2\right]^2} \tag{B4}$$

where

$$\underset{\sim}{v} = \beta \underset{\sim}{q} + (1 - \beta) \underset{\sim 1}{k}$$

$$\underset{\sim}{r} = \underset{\sim}{q} - \underset{\sim 1}{k} \quad ; \; r = |\underset{\sim}{r}|$$

$$D^2 = r^2 (- \beta^2 + h\beta + g)$$

$$h = 1 + (a^2 - b^2) / r^2$$

$$g = b^2/r^2$$

Expressing $\Psi(\underset{\sim 1}{f})$ in the coordinate space and taking exponential type (3.2) wave function the integration over $\underset{\sim 1}{f}$ can be performed and we get

$$L = - \frac{\sqrt{2}\ \pi C}{2} \sum_j a_j \int_0^1 \frac{d\beta}{D \left[(\alpha_j + D)^2 + \underset{\sim}{v}^2 \right]} \qquad (B5)$$

using the Euler substitution

$$t - i\beta = (-\beta^2 + h\,\beta + g)^{1/2} = D/r$$

we obtain

$$L = - \frac{\sqrt{2}\ \pi C}{r} \sum_j \frac{a_j}{A} \int_{t_-}^{t_+} \frac{dt}{(t - t_+)(t - T_-)} \qquad (B6)$$

and then, finally,

$$L(q,k_1) = - \frac{\sqrt{2}\ \pi C}{r} \sum_j \frac{a_j}{A(T_+ - T_-)} \left\{ ln \frac{t_+ - T_+}{t_+ - T_-} + ln \frac{t_- - T_-}{t_- - T_+} \right\}$$

(B7)

where
$$t_+ = i + a/r$$
$$t_- = b/r$$
$$T_\pm = (-B \pm \sqrt{B^2 - 4AP})/2A$$
$$A = 2ir\alpha_j + S$$
$$B = 2iR + 2hr\alpha_j$$
$$P = hR + g(2ir\alpha_j - S)$$
$$R = \alpha_j^2 + b^2 + \underset{\sim}{k}^2$$
$$S = a^2 - b^2 + \underset{\sim}{q}^2 - \underset{\sim}{k}^2$$

The logarithm cuts must be chosen carefully in order to avoid crossing the integration path.

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FIGURE CAPTIONS:

fig.1 - *The one-baryon-exchange diagram for pd → dp scattering.*

fig.2 - *Impulse series diagrams: a) single scattering; b) double scattering.*

fig.3 - *The one-pion-exchange diagrams for the reactions:*
a) pp → πd ; b) pd → dp

fig.4 - *The double-pion-exchange graph for the pd → dp scattering, obtained by taking in diagram 3b the one-pion-exchange mechanism for the pp → dπ sub-reaction.*

fig.5 - *The backward c.m. cross section for elastic pd scattering as a function of the incident proton kinetic energy in the lab.. The three continuous curves represent the contribution of the double-pion-exchange graph with the following approximations, defined in section 3:*
Curve (b): pions pole integration;
Curve (c): delta pole integration;
Curve (d): delta and pion pole integration.
The dotted line represents the one-nucleon-exchange contribution and the dashed one is its incoherent sum with curve (c).

fig.6 - *Differential c.m. cross sections as a function of* $\cos\Theta_{c.m.}$. *The curves are obtained with the approximation (3-d); continuous line:* $\cos\Theta$ *and* $\cos\Theta_{\pi N}$ *linked by eq. (A9); dashed line:* $\cos\Theta_{\pi N} = -1$ *over the entire angular range. Similar behaviour occurs at other energies.*

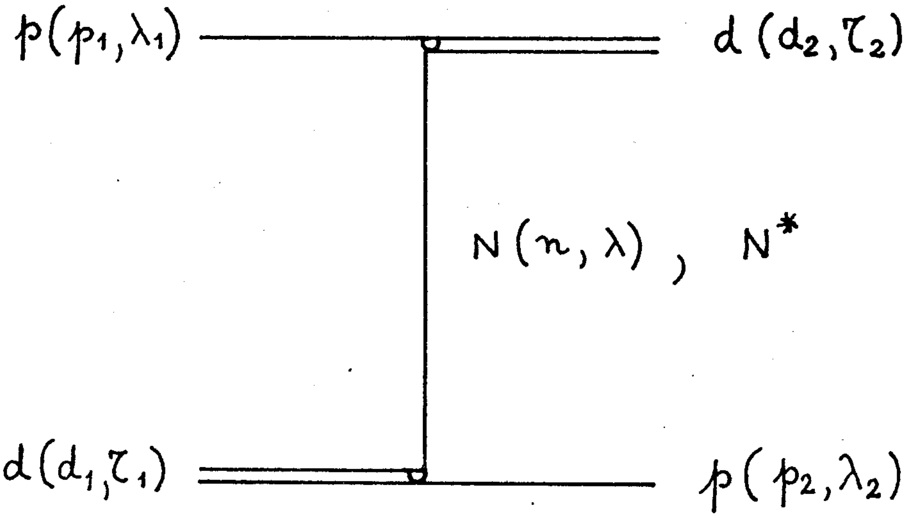

FIG. 1

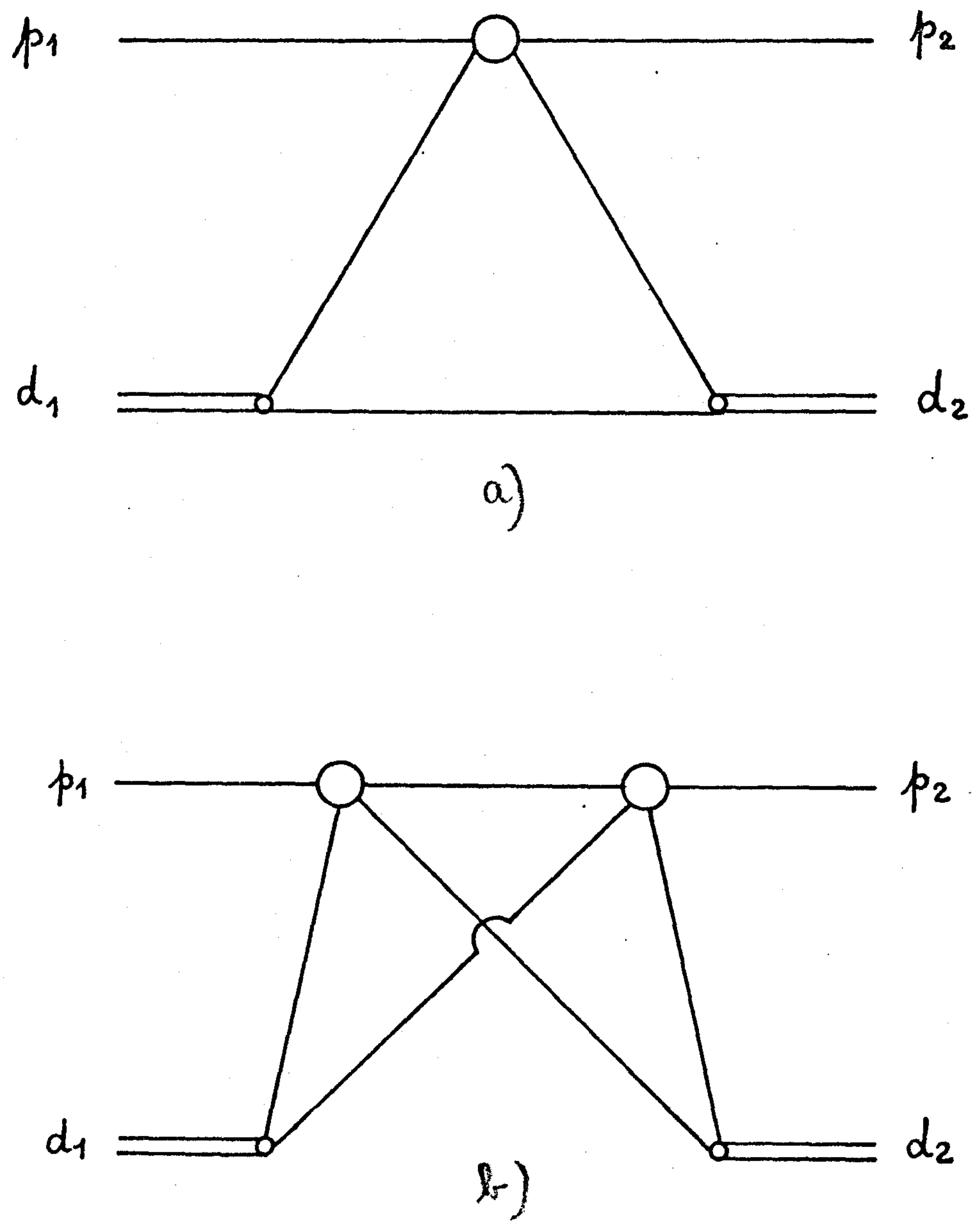

FIG. 2

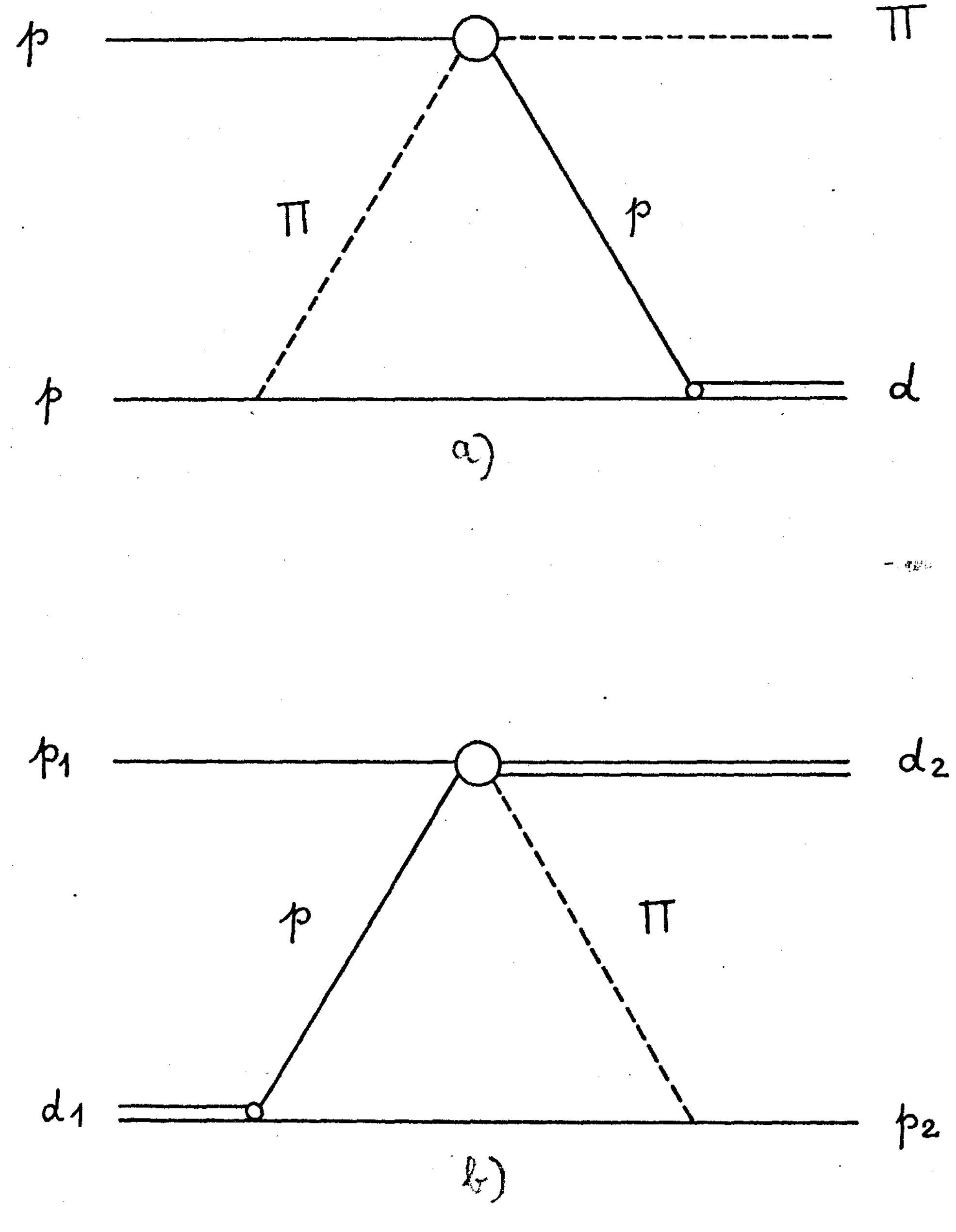

FIG. 3

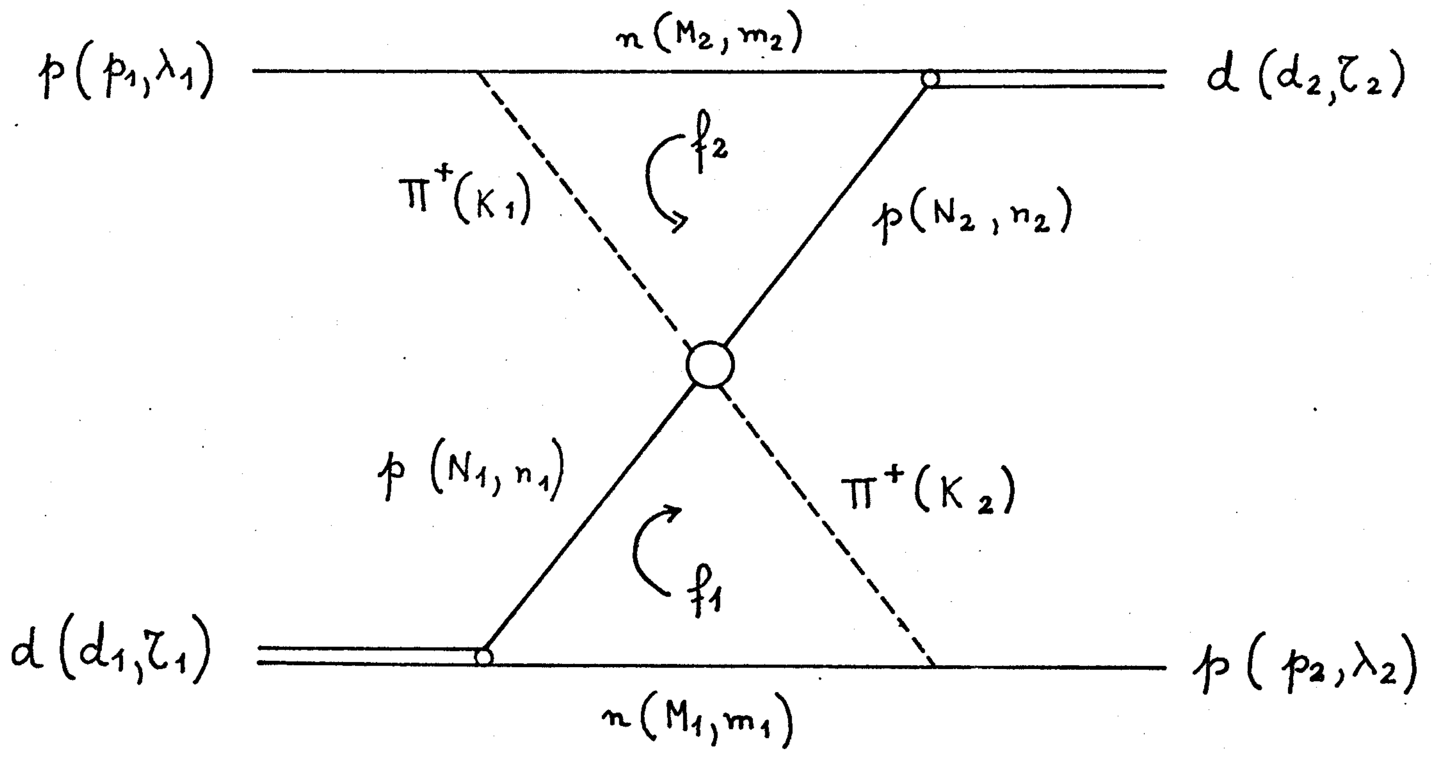
n (M2, m2)
p (p1, λ1)
d (d2, ζ2)
f2
π+ (K1)
p (N2, n2)
p (N1, n1)
π+ (K2)
f1
d (d1, ζ1)
p (p2, λ2)
n (M1, m1)

FIG. 4

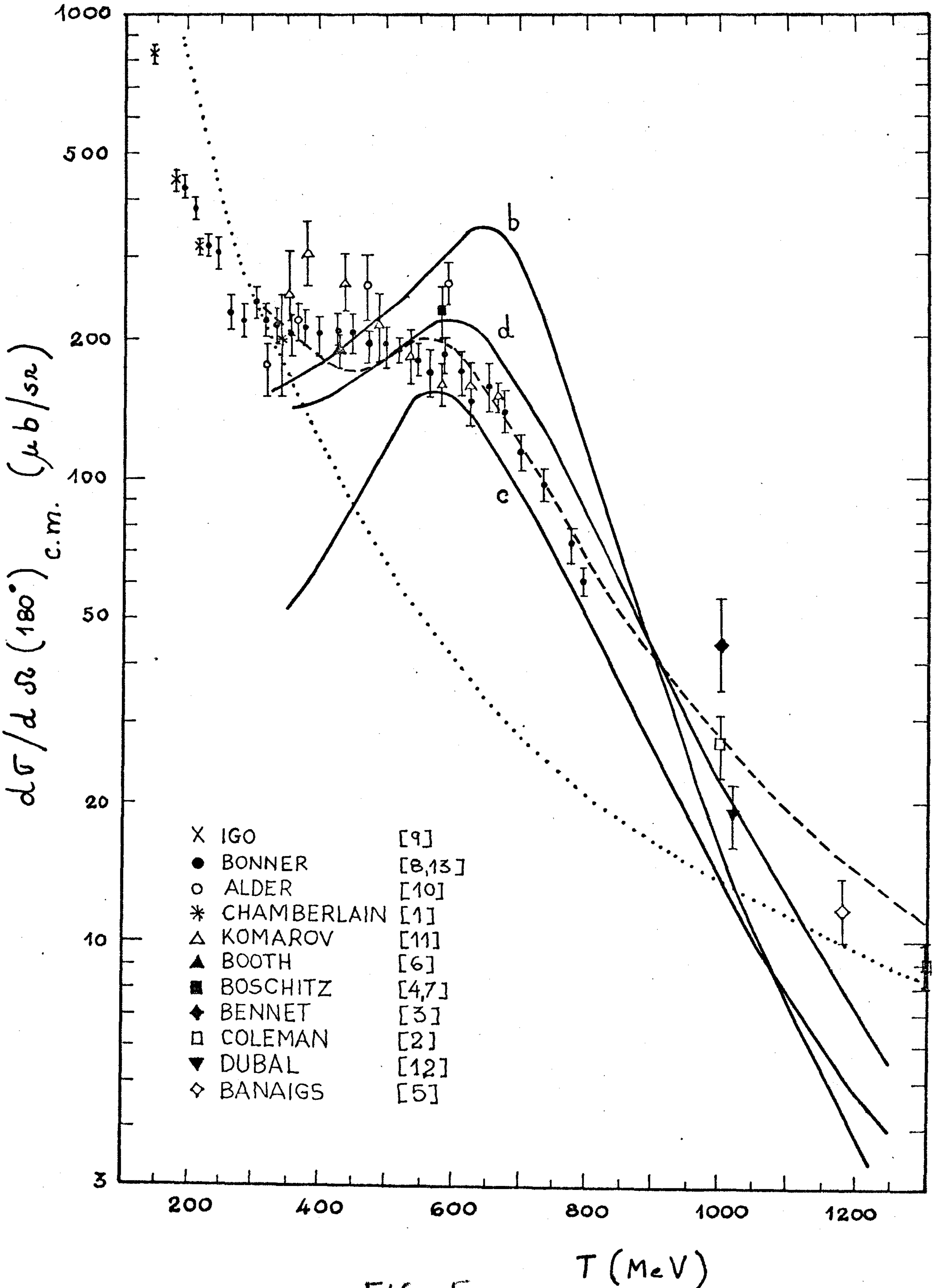
1000
500
200
100
50
20
10
3
200
400
600
800
1000
1200
dσ/dΩ (180°)c.m. (μb/sr)
T (MeV)
b
d
c
X IGO [9]
● BONNER [8,13]
○ ALDER [10]
✳ CHAMBERLAIN [1]
△ KOMAROV [11]
▲ BOOTH [6]
■ BOSCHITZ [4,7]
◆ BENNET [3]
□ COLEMAN [2]
▼ DUBAL [12]
◇ BANAIGS [5]

FIG. 5

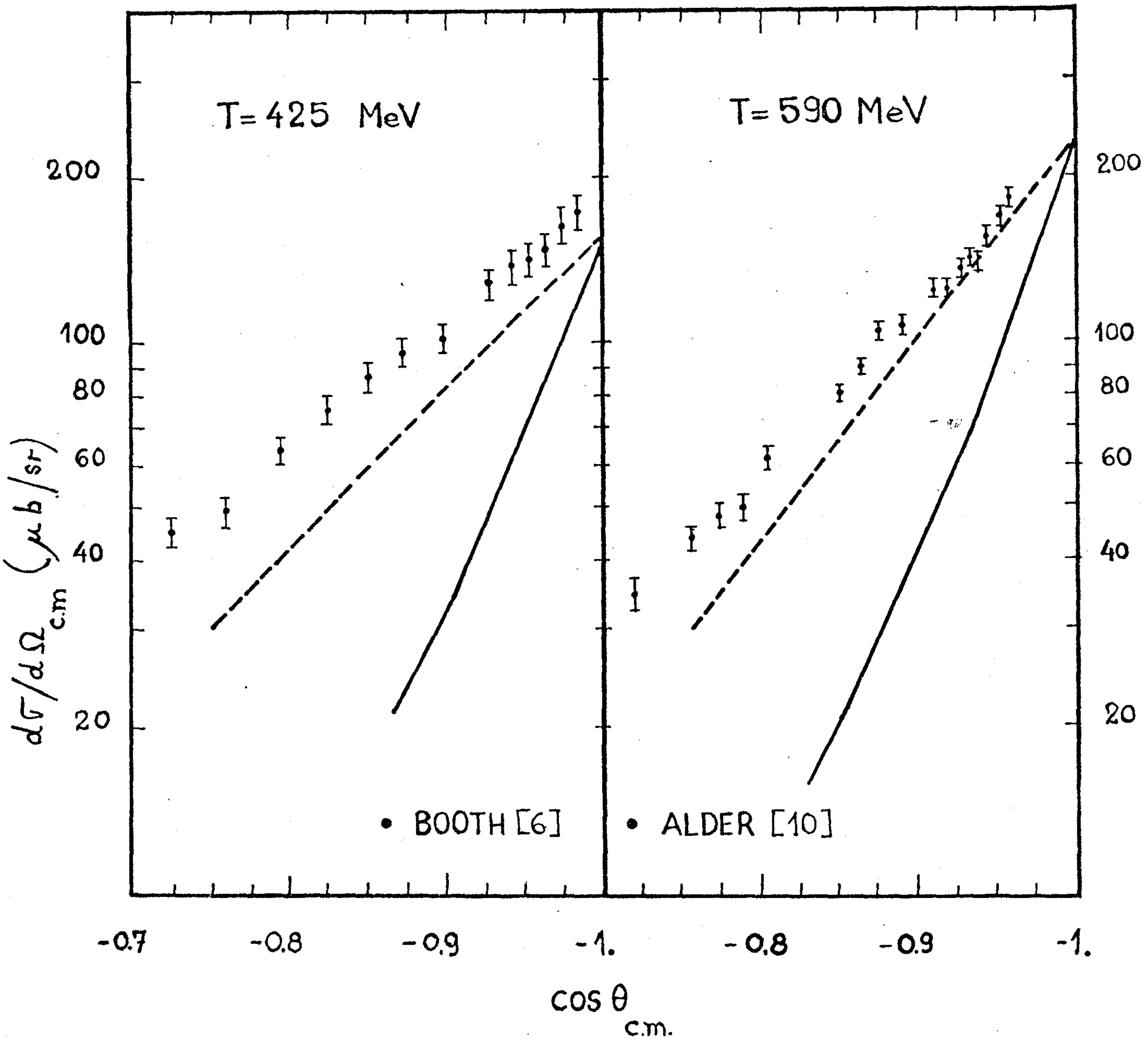
T= 425 MeV
T= 590 MeV
200
100
80
60
40
20
200
100
80
60
40
20
dσ/dΩ c.m (μb/sr)
• BOOTH [6]
• ALDER [10]
-0.7
-0.8
-0.9
-1.
-0.8
-0.9
-1.
cos θ c.m.

FIG. 6