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NON-LINEAR VISCOUS COSMOLOGY

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ABSTRACT: We treat the galactic fluid, in the early epochs of the Universe, as a non-linear Stokesian fluid. Some general comments on fluids with viscosity are made. Then we examine an exact solution and give a qualitative analysis for Bianchi type-I cosmological model generated by a fluid with quadratic viscosity dependence on the deformation tensor.

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I - INTRODUCTION

In the ambitious program to create a coherent global description of the Universe, cosmologists have been conducted to the analysis of some idealized configurations of the Cosmos. Starting with the assumption that Einstein's General Relativity is a good model for a theory of gravity, the essential task is to present a coupled system constituted by a given distribution of the energy contents in the Universe and the corresponding geometrical structure of the space-time. Due to its high degree of complexity one can achieve a good answer to this problem only through succesive idealized schemas.

The most acceptable cosmological models, i.e., Friedman's homogeneous and isotropic expanding Universe, assumes a continuous fluid description for the galactic matter plus radiation. Actually, the great majority of cosmical models deals with a perfect fluid behaviour. This has the great advantage of simplicity and far besides this, it seems to be in good agreement with current experimental cosmical observation.

Recently, however, the idea that Cosmology can go beyond the investigation of our present equilibrium era(1) has led some authors to try to incorporate dissipative terms on the energy-momentum tensor of the galactic fluid. Although such study is still at its begining it seems conceivable that viscosity effects will play an important role in Cosmology.

In 1968 Misner(2) suggested that neutrino viscosity could be an efficient mechanism by means of which arbitrary initial anisotropy dies away rappidly as the Universe expands.

As a phenomenological description of such process the

Cauchy linear relation between anisotropic pressure $\pi^{\mu}{}_{\nu}$ and shear deformation $\sigma^{\mu}{}_{\nu}$ was postulated. The second viscosity coefficient has been used by Klimek[3] and later by Murphy[4] in order to create a homogeneous and isotropic cosmological model without singularity. The effect of first and second viscosity coefficients on the structure of the cosmical singularity have been qualitatively investigated by Belinski and Khalatnikov[5]. Grischuk has considered the possibility of describing particle creation mechanism as a viscous effect in a non-stationary Universe, like Friedmann's models. The purpose of the present work is to initiate a program of systematic analysis of viscous cosmological model of a more general type than those that have been examined so far, by making an appeal to non-linear models of viscous fluid. This non-linearity may be intimately related to the feed-back mechanism of creation of particles in an expanding Universe and the modification on the geometry induced by the newly created matter. How this relation appears is a very interesting matter for future investigations.

II - STOKES FLUIDITY

The energy-momentum tensor of a viscous fluid, without heat conduction, is given by

$$T_{\mu\nu} = \rho V_{\mu} V_{\nu} - p\, h_{\mu\nu} + \pi_{\mu\nu} \tag{1}$$

where $h_{\mu\nu} \equiv g_{\mu\nu} - V_{\mu} V_{\nu}$ is the projector on the 3-dimensional rest-frame of the observer co-moving with the fluid velocity V_{μ};

$\pi_{\mu\nu}$ is the anisotropic pressure responsable for viscosity effects. The tensor $\pi_{\mu\nu}$ is symmetric, trace-free and orthogonal to V^{μ}, that is $\pi_{\mu\nu} = \pi_{\nu\mu}$, $\pi_{\mu\nu} g^{\mu\nu} = 0$, $\pi_{\mu\nu} V^{\nu} = 0$. In the analysis of the evanescence of some eventual primordial anisotropy some authors made the hypothesis by means of which the anisotropic pressure is linearly related to the dilatation tensor $\theta^{\alpha}{}_{\beta}$, actually to its trace-free part, the shear $\sigma^{\alpha}{}_{\beta} \equiv \theta^{\alpha}{}_{\beta} - \frac{\theta}{3} h^{\alpha}{}_{\beta}$. This hypothesis is a special case of a more general fluidity principle which has been set up by Stokes, among others. This principle is based on more fundamental assumptions which makes possible to relate the fluid dynamic quantities (as anisotropic pressure, heat conduction) to the kinematical ones (as dilatation, vorticity). Such relation, which is indeed the support of any phenomenological description of the fluid behavior represents the Principle of Generalized Viscosity. As a first typical example of such Principle we can take the Stokes fluidity definition(6), which states that the stress tensor of a fluid is a continuous function of the dilatation tensor $\theta^{\mu}{}_{\nu}$. By taking the time-like velocity vector $V^{\mu} = \delta^{\mu}{}_{o}$ and considering latin indices to vary in the domain $\{1,2,3\}$ we can write

(2) $$\pi^{i}{}_{j} = \phi_{o}\, \delta^{i}{}_{j} + \phi_{1}\, \theta^{i}{}_{j} + \phi_{2}\, \theta^{i}{}_{k}\, \theta^{k}{}_{j}$$

in which ϕ_o, ϕ_1 and ϕ_2 are polynomials in the principal invariants of the matrix $\theta^{i}{}_{j}$, that is, the scalars I, II and III which are given by

(3a) $$I \equiv \theta^{i}{}_{i} = \theta$$

(3b) $$II = \frac{1}{2} (\theta^2 - \theta^i{}_j \theta^j{}_i)$$

(3c) $$III = \varepsilon_{ijk} \, \theta^i{}_1 \theta^j{}_2 \theta^k{}_3$$

As a second example we could consider a non-Stokesian fluid in which the stress tensor is a function of the vortex matrix $\Omega^i{}_j \equiv \omega^i \omega_j - \frac{\omega^2}{3} h^i{}_j$ constructed with the vorticity vector ω_i. This vector is related to the vorticity tensor $\omega_{\mu\nu} = \frac{1}{2} h^{\lambda}_{[\mu} h^{\varepsilon}_{\nu]} V_{\lambda|\varepsilon}$ by the expression

$$\omega^i = - \frac{1}{2} \frac{1}{\sqrt{-g}} \varepsilon^{ijk} \omega_{jk} \quad .$$

(A simple bar means derivative, a double bar means co-variant derivative; the symbol [ ] means anti-symmetrization).

In this case we can write, for instance

(4) $$\pi^i{}_j = \alpha \, \Omega^i{}_j \quad .$$

Let us point out here that, as it has been remarked previously by A. Sommerfeld and others , the dependence of the stress on the vorticity is possible only in a quadratic regime, at least.

In any case, the presence of viscosity effects can change drastically the properties of the gravitational field induced by such stress. Among these, special interest deserves those which are related to entropy non-conservation induced by a non-null characteristic function $\Phi \equiv \pi^i{}_j \theta^j{}_i$. From conservation of energy-momentum projected in the rest-frame of the observer V^μ we obtain

(5) $$\dot{\rho} + (\rho+p)\theta - \Phi = 0$$

in which a dot means derivative in the V^{μ}-direction; we obtain also the acceleration equation

(6) $$(\rho+p)\,\dot{V}^{\alpha} - p_{|\mu}\, h^{\mu}{}_{\alpha} + \pi^{\mu\nu}{}_{\|\nu}\, h_{\mu}{}^{\alpha} = 0 \quad .$$

The characteristic function is a measure of the time--variation of the entropy - as one can obtain from (5) - and sets some restrictions on the possible values of the polynomials ϕ_o, ϕ_1 and ϕ_2 by the conditions imposed on it by the second law of Thermodynamics. Thus, Φ must either vanishes (entropy conservation) or be positive (increase of entropy) - see Table I.

The above phenomenological equations are set up in the believe that it can gives us a better understanding of how anisotropy can dies away as the Universe expands. The influence of the anisotropic pressure on the evolution of the shear can be investigated by the equations of motion of the kinematical quantities and the knowledge of the electric part of Weyl tensor $C_{\alpha\beta\mu\nu}$ which is given by $E_{\mu\nu} = - C_{\mu\alpha\nu\beta}V^{\alpha}V^{\beta}$.

If we call $\mu \equiv \sigma^{i}{}_{j}\sigma^{j}{}_{i}$, in the case there is neither acceleration nor rotation, the equation of evolution of shear gives

(7) $$\dot{\mu} + \frac{4}{3}\,\theta\mu = - 6\,\delta\,III + E^{i}{}_{j}\sigma^{j}{}_{i}$$

in which we have set $\pi^{i}{}_{j} = (1-\delta)(\sigma^{i}{}_{k}\sigma^{k}{}_{j} - \frac{1}{3}\,\sigma_{\ell m}\sigma^{\ell m}h^{i}{}_{j})$. Thus vanishing of μ, as time goes on, depends crucially on the determinant of the dilatation matrix and on the Electric tensor $E^{i}{}_{j}$.

π^i_j	$\Phi = \pi^i_j \theta^j_i$	constraints
$\alpha\theta^i_j + \beta\theta\delta^i_j$ (α,β are constants)	$(\alpha+\beta)\theta^2 - 2\alpha\ II$	$\alpha+3\ \beta = 0$
$a\theta^2\delta^i_j + b\theta\theta^i_j + c\theta^i_k\theta^k_j$ (a,b,c are constants)	$(a+b+c)\theta^3 - (2b+3c)\theta II + 3c\ III$	$(3a+b+c)\theta^2 - 2cII = 0$
$f\delta^i_j + g\theta^i_j + h\theta^i_k\theta^k_j$ (f,g,h are polynomials in I, II and III)	$f\theta + g(\theta^2 - 2II) + h(\theta^3 + 3III - 3\theta II)$	$3f + g\theta + h(\theta^2 - 2II) = 0$
$\lambda\omega^i_j + \eta\omega^2\delta^i_j$ (λ,η constants)	$\lambda\theta^{ij}\omega_i\omega_j + \eta\theta\omega^2$	$\lambda + 3\eta = 0$

[Table I - One can extract the conditions imposed by the second law of Thermodynamics by simple examination of this table. For instance, one obtains $\frac{\beta}{\alpha} \geq 2\,\frac{II}{\theta^2} - 1$; for a non-expanding fluid we have $\frac{g}{h} \leq \frac{3}{2}\,\frac{III}{II}$; if the second invariant vanishes, we must have $\frac{g}{h} \geq -\theta - \frac{g}{2}\,\frac{III}{\theta^2}$; and so on] .

These quantities should be analysed on each specific model, once very little can be said in generic terms. Thus it is not evident that the above mechanism could be efficient enough to give an explanation of the high value of isotropy of our Universe. However, one should make a more detailed analysis for each specific initial geometry and for each Stokesian fluid.

III - EINSTEIN'S EQUATIONS FOR BIANCHI TYPE-I AND THE QUADRATIC STOKESIAN REGIME

In order to gain some insight on the influence of the non-linear Stokesian regime on the behavior of the gravitational field, we will examine now a special geometry which represents a homogeneous but anisotropic expanding Universe, of type-I in Bianchi's classification scheme.

The infinitesimal element of length is given by

$$(8) \qquad ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)dy^2 - c^2(t)dz^2$$

Choosing a co-moving observer with the fluid velocity $V^\mu = \delta^\mu{}_o$, the energy-momentum tensor has the following non-null components:

$$(9) \qquad T^o{}_o = \rho$$

$$T^i{}_j = -p\delta^i{}_j + \alpha\,\theta^2\delta^i{}_j + \beta.\theta\;\theta^i{}_j + \gamma\theta^i{}_k\theta^k{}_j$$

Before proceding in this analysis, if seem worthwhile to make the following remark. In the discussion of viscosity, cosmologists have limited their analysis to the linearized case of Stokes fluid. Although it could be a difficult task to elaborate models by means of which one could evaluate the value of the generalized second-order coefficients of viscosity, there is no *a priori* reason to reject its presence. Further, in the region near the singularity the non-linear regime could dominate and probably could give a better approximation of the effect of newly created particles by gravitational field.

The entrance of the Universe in a full non-linear era, in Stokes expansion, has the effect of changing radically the early features of the Cosmos. For instance, it has been sugested by many cosmologists that in the early epochs of the Universe matter should be gravitationally unimportant. Thus, going back in time one should enter a region (which is called vacuum stage) where gravitation is sustained by itself. If this is true, then the properties of the singularity in such models are independent of matter behavior. This has been proved by Lifshitz *et al.* in an equilibrium era in which matter is treated as a perfect fluid. However, it is straightforward to prove that such vacuum stage could not exist in a quadratic regime for the Stokes fluid, for instance. The reason for this is simple: the matter terms are no more negligible - in Einstein's equations - by comparison with pure gravity terms, e.g. the Ricci tensor.

Now, the dilatation tensor is diagonal and its non-null terms are $\theta^1_1 = \frac{\dot{a}}{a}$, $\theta^2_2 = \frac{\dot{b}}{b}$, $\theta^3_3 = \frac{\dot{c}}{c}$. Einstein's equations are:

(10-a) $$\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{c}}{c} + \frac{\dot{b}}{b}\frac{\dot{c}}{c} = \rho$$

(10-b) $$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}}{b}\frac{\dot{c}}{c} = - p + \alpha\theta^2 + \beta\theta\frac{\dot{a}}{a} + \gamma(\frac{\dot{a}}{a})^2$$

(10-c) $$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\dot{a}}{a}\frac{\dot{c}}{c} = - p + \alpha\theta^2 + \beta\theta\frac{\dot{b}}{b} + \gamma(\frac{\dot{b}}{b})^2$$

(10-d) $$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} = - p + \alpha\theta^2 + \beta\theta\frac{\dot{c}}{c} + \gamma(\frac{\dot{c}}{c})^2$$

Further, we have the constraint relation:

(11) $$(3\alpha+\beta)\theta^2 + \gamma\left[(\frac{\dot{a}}{a})^2 + (\frac{\dot{b}}{b})^2 + (\frac{\dot{c}}{c})^2\right] = 0 \quad .$$

From the above set of equations, we can infer that in the case of a power law dependence of the anisotropic functions a,b and c on time, there is no posibility of a vacuum stage.

We will make now two very different discussion of these equations. Firstly, we give a special exact solution in order to have some feeling on its properties, and secondly we will turn to a qualitative analysis of equations (10) by using the standard techniques of qualitative investigation of ordinary non-linear differential equations.

Let us set

$$a(t) = e^{\mu t}\, t^{A}$$
$$b(t) = e^{\nu t}\, t^{B} \qquad (12)$$
$$c(t) = e^{\eta t}\, t^{C}$$

in which μ, ν, η and A, B, C are constants.

Using the ansatz (12) in equations (10) we obtain

(13-a)
$$\mu\nu + \mu\eta + \nu\eta + \left[A(\nu+\eta) + B(\mu+\eta) + C(\mu+\nu)\right] t^{-1} + (AB + AC + BC)\, t^{-2} = \rho$$

(13-b)
$$\nu^2 + \eta^2 + \nu\eta + \left[2\nu B + 2\eta C + \eta B + \nu C\right] t^{-1} + \left[B(B-1) + C(C-1) + BC\right] t^{-2} = -p + \alpha\Phi^2 + \beta\Phi\mu + \gamma\mu^2 + \left[2\alpha\Phi\Omega + \beta\mu\Omega + \beta\Phi A + 2\mu\gamma A\right] t^{-1} + \left[\alpha\Omega^2 + \beta\Omega A + \gamma A^2\right] t^{-2}$$

(13-c)
$$\nu^2 + \mu^2 + \nu\mu + \left[2\nu B + 2\mu A + \mu B + \nu A\right] t^{-1} +$$

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$$
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a(t) &= e^{\mu t}\, t^{A} \\
b(t) &= e^{\nu t}\, t^{B} \\
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\end{aligned}
\tag{12}
$$

in which μ, ν, η and A, B, C are constants.

Using the ansatz (12) in equations (10) we obtain

(13-a) $$\mu\nu + \mu\eta + \nu\eta + \left[A(\nu+\eta) + B(\mu+\eta) + C(\mu+\nu)\right] t^{-1} + (AB + AC + BC)\, t^{-2} = \rho$$

(13-b) $$\nu^2 + \eta^2 + \nu\eta + \left[2\nu B + 2\eta C + \eta B + \nu C\right] t^{-1} + \left[B(B-1) + C(C-1) + BC\right] t^{-2} = -p + \alpha\Phi^2 + \beta\Phi\mu + \gamma\mu^2 + \left[2\alpha\Phi\Omega + \beta\mu\Omega + \beta\Phi A + 2\mu\gamma A\right] t^{-1} + \left[\alpha\Omega^2 + \beta\Omega A + \gamma A^2\right] t^{-2}$$

(13-c) $$\nu^2 + \mu^2 + \nu\mu + \left[2\nu B + 2\mu A + \mu B + \nu A\right] t^{-1} +$$

$$+ \left[A(A-1) + B(B-1) + AB\right] t^{-2} = -p + \alpha\Phi^2 +$$

$$+ \beta\Phi\eta + \gamma\eta^2 + \left[2\alpha\Phi\Omega + \beta\eta\Omega + \beta\Phi C + 2\eta\gamma C\right] t^{-1} +$$

$$+ \left[\alpha\Omega^2 + \beta\Omega C + \gamma C^2\right] t^{-2}$$

(13-d) $$\eta^2 + \mu^2 + \eta\mu + \left[2\mu A + 2\eta C + \eta A + \mu C\right] t^{-1} +$$

$$+ \left[A(A-1) + C(C-1) + AC\right] t^{-2} = - p + \alpha\Phi^2 + \beta\Phi\nu +$$

$$+ \gamma\nu^2 + \left[2\alpha\Phi\Omega + \beta\Omega\nu + \beta\Phi B + 2\nu\gamma B\right] t^{-1} +$$

$$+ \left[\alpha\Omega^2 + \beta\Omega B + \gamma B^2\right] t^{-2}$$

in which we have set $\theta = \Phi + \Omega t^{-1}$; that is $\Phi = \mu+\nu+\eta$ and $\Omega = A+B+C$.

Let us take the very special situation in which the coefficients of viscosity are such that

(14-a) $$3\alpha + \beta = 0$$

(14-b) $$\gamma = 0$$

Then we look for a solution which has $\Phi = 0$ and $A = B = C$, in order to obtain the simplest generalization of Friedmann's Cosmos with Euclidean section . We set:

(15-a) $$\rho = \rho_o + \rho_1 t^{-1} + \rho_2 t^{-2}$$

(15-b) $$p = p_o + p_1 t^{-1} + p_2 t^{-2}$$

where ρ_o, ρ_1, ρ_2, p_o, p_1, p_2 are constants.

Then a straightforward calculation gives, for the compatibility of equations (13) with our ansatz:

$$
\begin{aligned}
p &= \rho \\
\alpha &= 1/3 \\
\beta &= -1
\end{aligned}
\qquad (16)
$$

$A = 1/3$; $\rho_o = -\mu^2 - \nu^2 - \mu\nu$; $\rho_1 = 0$; $\rho_2 = 1/3$.

Let us make some comments on this solution. First of all we remark that this is a very special solution and it may not, certainly, be considered as a typical one.

The density ρ is not strictly positive for all time t. Indeed, we have $\rho = \rho_o + 1/3\ t^{-2}$ in which ρ_o is given above. It is positive definite only for those times in the range $0 < t < \frac{1}{3}\ \frac{1}{|\rho_o|} \equiv t_c$. The domain of positivity, measured by the value of t_c, depends on the anisotropy. Indeed, smaller the anisotropy (that is, ρ_o) bigger t_c . There is thus, apparently, a difficulty here, due to the non-positivity of energy for all times. However the above solution is to be considered in a circunstance in which the viscosity is non-linear in the dilatation tensor, and thus we are facing the hypothesis by which this could occur in some restricted epoch of the history of the Universe, but not for all times. Thus, for other periods the galactic fluid behaves differently and thus we should match our solution with other geometry with different matter behavior.

The unique non-vanishing components of the dilatation tensor are:

$$\theta^1_1 = \mu + \frac{1}{3}\ t^{-1}\ ;\ \theta^2_2 = \nu + \frac{1}{3}\ t^{-1}\ ;\ \theta^3_3 = \eta + \frac{1}{3}\ t^{-1}\ .$$

Finally, let us remark that the characteristic function Φ equals $2\rho_o\, t^{-1}$ and thus is a negative increasing function of time. Thus, the evolution of such Universe is in the direction of decreasing time, in accord with the second law of Thermodynamics. We face here the same situation as encountered in[5], of break of invariance with respect to time inversion, due to the effects of viscosity.

Let us turn now to a different approach[7] of the investigation of the set of equations (10).

In order to simplify our preliminar analysis of equations (10) we will limit the anisotropy to a plane by assuming b = c, for instance. Then we define two new variables by the relation

$$U = \frac{\dot{a}}{a} \tag{17-a}$$

$$V = \frac{\dot{b}}{b} \tag{17-b}$$

In the new variables, the old system reduces to

$$2V\,(U + V) = \rho \tag{18-a}$$

$$\dot{U} = -U^2 + \frac{1}{2}V^2 - UV - \frac{p}{2} + 2L_I(U,V) \tag{18-b}$$

$$\dot{V} = -\frac{3}{2}V^2 - \frac{1}{2}p - L_I(U,V) \tag{18-c}$$

in which

$$L_I \equiv \alpha(U + 2V)^2 + \beta(U+2V)V + \gamma V^2$$

and we will assume there is an equation of state $p = \lambda\rho$, λ is a constant. We will simplify further our discussion here by assuming conditions (14) for the coefficient of viscosity.

The existence of a plane of anisotropy has the effect to reduce our system to equations (18) which defines what is called an autonomous planar system of differential equations, that is, we have

(19-a) $$\dot{U} = R(U,V)$$

(19-b) $$\dot{V} = \tau\ (U,V)$$

with

(20-a) $$R(U,V) \equiv (2\alpha-1)\ U^2 + \left[\frac{1-\lambda}{2} - 4\alpha\right]V^2 + (2\alpha-1-\lambda)UV$$

(20-b) $$\tau(U,V) \equiv (2\alpha - \frac{3}{2} - \frac{\lambda}{2})V^2 - \alpha U^2 - (\lambda+\alpha)\ UV\ .$$

The origin, in the (U,V) plane is an isolated singularity for the system (19). Both R and τ are homogeneous functions, of degree two, on U and V. Thus, in order to analyse the properties of the above system[19] it is conveniente to change coordinates going into polar variables r, ϕ defined by $U = r\cos\phi$, $V = r\sin\phi$. Thus, (19) goes into

(21-a) $$\dot{r} = r^2 R\ [\phi]$$

(21-b) $$\dot{\phi} = r\ T\ [\phi]$$

where $R[\phi]$ and $T[\phi]$ are given by

(22-a) $$R[\phi] = \sin^3\phi(2\alpha - \frac{3}{2} - \frac{\lambda}{2}) + (2\alpha - 1)\cos^3\phi\ +$$

$$+ (\alpha-\lambda-1)\ \sin\phi\ \cos^2\phi - \left[\frac{3}{2}\lambda + 5\alpha - \frac{1}{2}\right]\sin^2\phi\cos\phi$$

(22-b) $$T[\phi] = -\alpha\cos^3\phi - \left[\frac{1-\lambda}{2} - 4\alpha\right]\sin^3\phi + \left(\frac{\lambda-1}{2}\right)\cos\phi\sin^2\phi -$$

$$- (3\alpha + \lambda - 1)\cos^2\phi\sin\phi$$

In the investigation of the behavior of the trajectories of the system (21) in the (r,ϕ) plane the particular solution wich pass through the (singular) origin are determined by the real roots of the equation $T[\phi] = 0$. We examine two special cases characterized by a dependence between the coefficient of viscosity α and λ , which are:

case (i) : $\lambda = 1 - 6\alpha$; case (ii) : $\lambda = 1 - 8\alpha$.

Firstly, let us consider case i). A simple inspection shows that there is a unique characteristic line which makes an angle of 45^o with the U-axis. This line, which equation $U = V$, is nothing but Friedmann isotropic solution. The behaviour of the trajectories (of the solutions) are given in figure 1 in which we have made a conformal mapping in order to represent the infinite as the boundary of a circle. The curves go approximatively parallel to Friedmann's solution whith a small (not catastrophic) atraction near the origin.

There are two singular points at infinity which are the contact points of Friedmann solution with the boundary: points P_1 and $\tilde{P}_1$ of figure 1. These two points are two-tangent nodes for the trajectories of our system. Thus, all solution starts at $\tilde{P}_1$ at past infinite and ends at P_1 at future infinite. The solution pass through a region in which the total energy is negative that is $U(2U+V) < 0$. However the solution does not stay at this region for long periods of time. Remark that there is no possibility of interchanging the axis of expansion/contraction. Both axis begin contracting, pass through a region of minimum and them both starts to expand. This general behavior is stable for those perturbations that do not destroy the constraint relation between λ and α .

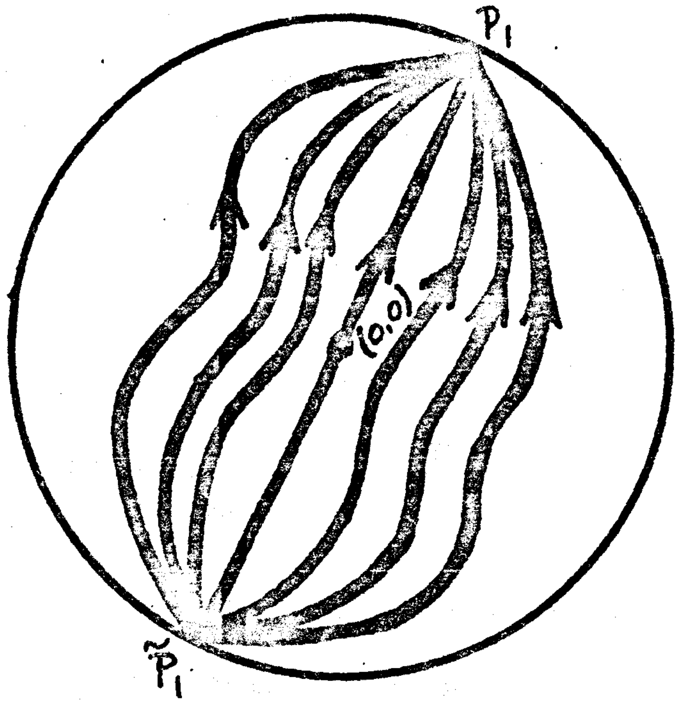

Fig. 1 - Conformal representation of the characteristics of the plane autonomous system (19) in the special case $\lambda = 1-6\alpha$. The arrows point in the direction of increasing t.

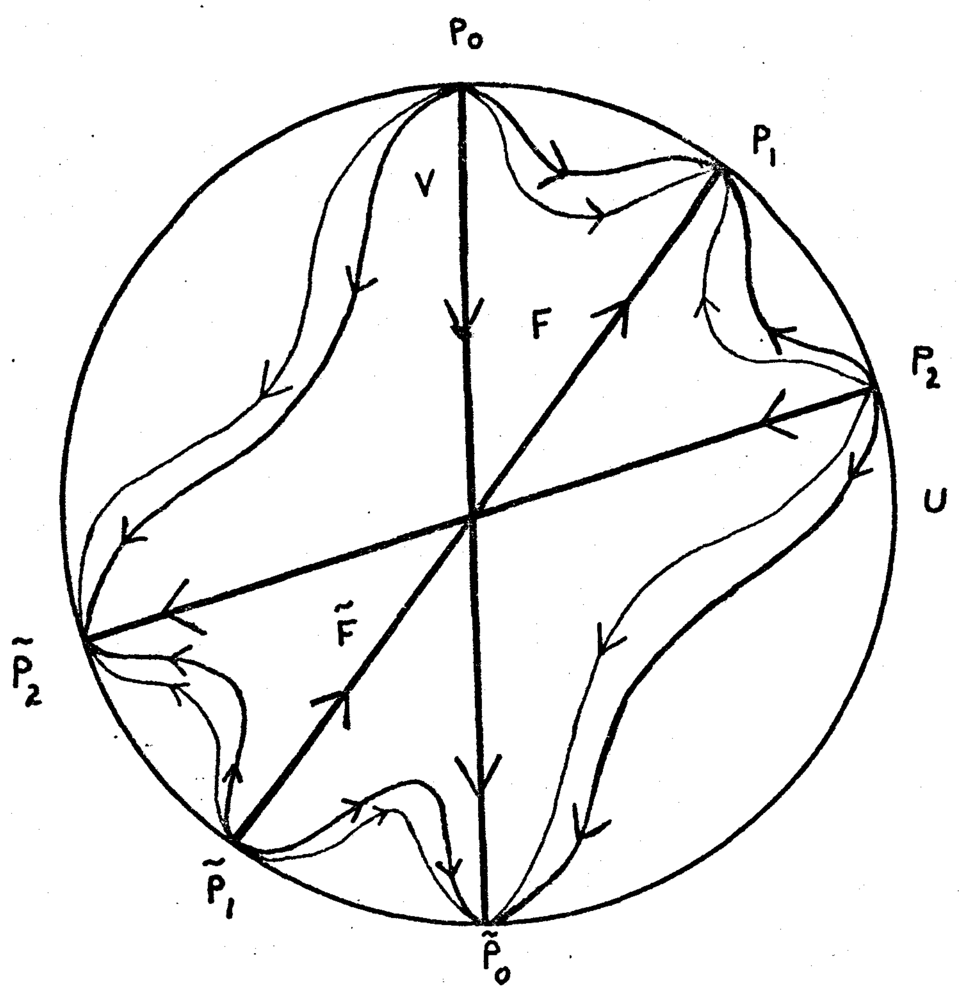

Fig. 2 - Conformal representation of the characteristics of the plane autonomous system (19) in the special case $\lambda = 1-8\alpha$, $\frac{7}{24} < \alpha < \frac{1}{8}$. The arrows point in the direction of increasing t. $F\tilde{F}$ is Friedmann's solution.

Let us turn now to discuss case ii) $\lambda = 1-8\alpha$

The roots of $T[\phi]$ are given by the solution of $\cos\theta_o = 0$; $\cos\theta_1 = \sin\theta_1$; $\cos\theta_2 = 4\sin\theta_2$. Contrary to the previous situation, here the behavior of the trajectories depend not only on the relation between λ and α but on the value of α itself. From the systematics of qualitative analysis for homogeneous system we know that the behavior of the characteristics depends on the sign of $R[\phi]$ at the neighborhood of the invariant rays

$\phi_o = \theta_o$, $\phi_1 = \theta_1$, $\phi_2 = \theta_2$. This sign, as one sees from (22-a) depends on the sign of the difference α - 7/24. Now, from the natural limit on λ , we obtain $0 < \alpha < 1/8$. Let us discuss the situation in which $7/24 < \alpha < 1/8$. In figure 2 we represent the conformal mapping of infinity into a circle. The contact points of the axis $\phi_o = \theta_o$, $\phi_1 = \theta_1$ and $\phi_2 = \theta_2$ with the circle are the singular points at infinity, which we represent by P_o, P_1, P_2 and its symmetrically related points $\tilde{P}_o$, $\tilde{P}_1$, $\tilde{P}_2$. All these points are two-tangent nodes. The trajectory $P_1\tilde{P}_1$ represents Friedmann solution. As trajectories in the planar system cannot be crossed, figure 2 tells us that only in the $P_o\tilde{P}_2$ and $\tilde{P}_oP_2$ regions the phenomenon of alternation role of the axis - expansion turned into contraction and/or vice-versa - can occur. Thus, in our case the expanding (contracting) plane y-z can turn into a contracting (expanding) era, although the x-axis cannot change the sign of its expansion (or contraction). Particularly interesting is the behavior of the model under perturbations of the Friedmann solution. In the region inside the arc $\tilde{P}_o\tilde{P}_1\tilde{P}_2$, perturbations of Friedmann solution are unstable; in the region $P_oP_1P_2$ perturbations of Friedmann solution are stable.

Such time assymmetry of the behavior of the above model Universe is a consequence of the viscosity effects, as has been pointed out previously by some authors(5).

We could like to call attention of the reader to the high degree of instability in (Friedmann) $\tilde{F}$-solution. A small departure from it may be responsible for the model to annihilate at $\tilde{P}_2$ or $\tilde{P}_o$. If $\tilde{F}$ solution happens to occur, then its trajectory until the point 0 must not be perturbed in order to the solution enter the F-region. Thus, it seems very unlikely that the previous era of our Cosmos, before the actual period of expansion, should share both properties that the geometry to be of Friedmann type and the matter to behave like a quadratic Stokesian fluid.

IV - CONCLUSION

In this paper we have presented the basic idea of how to treat the viscosity of the galactic matter as a Stokesian non-linear fluid. This could be a good description of the behavior of the energy of the cosmic fluid in that highly compressed early epoch in which large-scale anisotropy could be so important as to induce non-linear response.

In the preliminar analysis we have presented here the basic features of solutions of Einstein's equations under such non-linear Stokesian regime reveals a lot of new results. Among these, one can quote the absence of vacuum stage and consequently the need of a study of the behavior of matter coupled to geometry near the singularity; the very sensitive dependence of the stability of Friedmann solution on the values of the quadratic coefficients of viscosity; and, finally the non-symmetric behavior of the Cosmos under time-inversion.

Finally, let us remark that, as it has been pointed out by some authors, like Grishchuk[8], viscosity effects may be related to the effects on the geometry due to created particles by a non-stationary Cosmos. Thus, our present model could be related to the created particle mechanism. We will come back to this point elsewhere.

REFERENCES

(1) One could say that cosmologists must look for more general models which could give a justification for the present highly isotropic stage which we live and possibly clear up our ideas about the behavior of the Cosmos near the global singularity present in Friedmann models.

(2) C.W. Misner - Astrophys. J. vol. 151, 431 (1968).

(3) Z. Klimek - Acta Cosmologica, 2, 49 (1973).

(4) G. Murphy - Phys. Rev. D Vol. 8, 4231(1973).

(5) Belinski - Khalatinkov - Sov. Phys. JETP Vol. 42 nº 2, 205 (1976).

(6) J.L. Ericksen - Appl. Mech. Rev. 20, 1029 (1967).

(7) G. Sansone - R. Conti - Non-Linear Differential Equations (Pergamon Press, 1964).

(8) L.P. Grishchuck - Sov. Phys. JETP, 40, 409 (1975).