MASS REVERSAL AND THE UNIVERSAL INTERACTION *§

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In this paper a new invariance principle is introduced for the relativistic quantum theory of fields. It is assumed that the results of such theories should be invariant under the change in sign of the mass term in the Dirac equation. This invariance holds indeed both for Quantum Electrodynamics and Meson Theory. If several Dirac particles interact with several Boson fields this principle introduces selection rules against certain interactions. The consequences for the Universal Fermi Interaction are that only such mixtures as scalar, pseudo-scalar and tensor (S,P,T) interactions <u>or</u> mixtures of fector and axial vector interactions (V,A) are possible. If also the interactions of π mesons with nucleons and with μ mesons are known we may be led to the exclusion of one of these possibilities.

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Finally, the consequence of further imposing a symmetry principle of the kind proposed by Pursey is examined. It is found that even considering other possible symmetry principles the only possible Fermi interactions are the following mutually excluding ones:

$(S + P - T)$, $(A - V)$, $(S + P + \frac{T}{3})$; $(S + P + T)$, $(A + V)$, $(S + P - \frac{T}{3})$;

(S,P), A, V and T.

I. Invariance of Quantum Electrodynamics under mass reversal.

It is known that all physical predictions in quantum electrodynamics are independent of the sign of the mass m in the Dirac equation:

$$i\hbar\frac{\partial\psi}{\partial t} = V\psi + c\vec{\alpha}\cdot(\vec{p} - e\vec{A})\psi \pm mc^2\beta\psi \qquad (1)$$

Most authors use the positive sign and then the two upper components of ψ (in the usual representation of the Dirac matrices) are the "large" ones. Others use the negative sign and the "large" components are then the lower ones.

This is the same as to say that Quantum Electrodynamics is invariant under the transformation

$$\begin{aligned} m &\rightarrow -m \\ \psi &\rightarrow \gamma_5\psi \\ A^\mu &\rightarrow A^\mu \end{aligned} \qquad (2)$$

We shall call this transformation "Mass Reversal", after Peaslee[1] who used it in a somewhat different manner than in the present paper.

The above result can be stated in a more general way as follows: if we have several spinor fields ψ_r (in what follows ψ_r may be considered as one particle wave function or as field operators) with masses m_r, in interaction only with the electromagnetic potentials, the equations are invariant under "mass reversal" for <u>any</u> of the particles (r_0), say, under the transformation:

$$m_{r_0} \rightarrow -m_{r_0} \quad ; \quad m_r \rightarrow m_r , \; r \neq r_0$$

$$\psi_{r_0} \rightarrow \eta_{r_0} \gamma_5 \psi_{r_0} \quad ; \quad \psi_r \rightarrow \eta_r \psi_r , \; r \neq r_0 \tag{3}$$

η_r in these transformations are arbitrary phase factors:

$$|\eta_r|^2 = 1 .$$

In general the phases $\eta_r (r \neq r_0)$ are not essential.

In what follows ψ_r will refer to the positive particles; the corresponding anti-particle field

$$\psi'_r = C\bar{\psi}_r ,$$

(say the negative particles), will transform then as

$$\psi'_r \rightarrow \eta^* \psi'_r \; ; \; \psi'_{r_0} \rightarrow -\eta^*_{r_0} \gamma_5 \psi'_{r_0} \; ; \; r \neq r_0$$

II. <u>Invariance of the meson theory</u>

The Dirac equation for nucleons in interaction with the $\pi^{\pm}, \pi_0$ meson field ϕ_i are assumed to be (pseudoscalar theory , pseudoscalar interaction):

$$i\frac{\partial\psi}{\partial t} = V\psi + c\vec{\alpha}\cdot(\vec{p} - e\vec{A})\psi + Mc^2\beta\psi + i\beta\gamma_5\vec{\phi}\cdot\vec{\tau}\psi \tag{4}$$

where $\vec{\tau}$ is the isotopic spin operator for the nucleons whose wave operators are

$$\psi = \begin{pmatrix} \psi_P \\ \psi_N \end{pmatrix}$$

This equation is invariant under both the "mass inversal" transformations:

$$M \to -M \quad ; \quad \psi \to \gamma_5\psi \; ; \quad \phi_i \to -\phi_i \tag{5}$$

and

$$M \to -M ; \psi \to \gamma_5\tau_3\psi ; \phi_1 \to \phi_1 , \phi_2 \to \phi_2 , \phi_3 \to -\phi_3 \tag{6}$$

Here $\psi \to \gamma_5 \tau_3 \psi$ is the same as: $\psi_P \to \gamma_5 \psi_P$, $\psi_N \to \gamma_5 \psi_N$. Then we see that if the π meson interacts (with pseudoscalar coupling) also with another spinor χ (say the $\mu^{\pm}$, ν fields), then, in opposition to the case of the electromagnetic fields, the equation for

$$\chi = \begin{pmatrix} \psi_\mu \\ \psi_\nu \end{pmatrix}$$

cannot be kept in general unchanged just by allowing phase factors changes for ψ_μ and ψ_ν . We can overcome this difficulty either by assuming that this interaction (which leads to the $\pi \to \mu$ decay) is not an elementary interaction[2] but a secondary one. Another possibility is to assume that only charged π mesons interact with the χ field (consistently with the experimentally observed cases). In this case the phase factor change for the μ and ν fields should be the same, if we use transformation (6) for the mass reversal of the nucleon field, or they should have opposite sign if we use (5).

$$\psi_\mu \to \eta \psi_\mu \quad ; \quad \psi_\nu \longrightarrow -\eta \psi_\nu \qquad (5a)$$

$$\psi_\mu \to \eta \psi_\mu \quad ; \quad \psi_\nu \longrightarrow \eta \psi_\nu \qquad (6a)$$

Now the referred interaction of (μ, ν) with the $\pi^\pm$ field will not be invariant under mass reversal of the μ field unless a similar transformation is also performed for the ν field; this implies the assumption that μ and ν form <u>one</u> field in the sense that (P,N) form the nucleon field. In this paper we shall assume, for the sake of simplicity that the neutral particle produced in the $\pi \to \mu$ decay is not a neutrino but a particle μ_o with small mass:

$$\pi \to \mu + \mu_o' \quad .$$

So the field $\chi = \begin{pmatrix} \psi_\mu \\ \psi_{\mu_o} \end{pmatrix}$ will transform under mass reversal in a way similar to the nucleon field ψ, as given by (5), (5a).

The extension of the above results to other forms of interaction is straightforward. In the case of several fermions interacting with several boson fields, restrictions of the type above referred to and relation rules for elementary interactions (similar to the exclusion of interactions of μ and ν with π_o field) will appear.

III. <u>Invariance of the Universal Fermi Interaction.</u>

The well known fact that the interactions which lead to β-decay,

μ-decay and μ-capture have practically the same strength leads to the assumption that the Fermi interaction is Universal for particles of spin 1/2 and thus is symmetrical in all particles. If all particles are considered different the complete symmetry[3] leads to the Wigner-Critchfield interaction (S - A - P) which is unsatisfactory in many respects. In particular this interaction does not obey the mass reversibility principle. If we assume that all particles appear in pairs, such as (P, N), (μ, μ_0), (e, ν) then the symmetry between these (pair) fields does not introduce any restriction. We shall adopt this point of view. So we assume the interaction schemes:

$$P \longrightarrow N + e^+ + \nu' \qquad (\beta \text{ decay}) \qquad (7a)$$

$$\mu^+ \longrightarrow \mu_0 + e^+ + \nu' \qquad (\mu \text{ decay}) \qquad (7b)$$

$$P + \mu^- \longrightarrow N + \mu_0' \qquad (\mu\text{-capture}) \qquad (7c)$$

$$\pi^+ \longrightarrow \mu^+ + \mu_0' \qquad (\pi - \text{decay}) \qquad (7d)$$

The mass reversal transformation should be made simultaneously for both particles in a pair, as in (5), (6).

If, now, we assume that the theory should be invariant under mass reversal for each particle (pair) we are led to the following result:

The only possible forms of Fermi interaction which are invariant under mass reversal are <u>either</u> a linear combination of <u>vector and pseudoscalar</u> interactions, or a linear combination of <u>Scalar, Tensor and Pseudoscalar</u> interactions.

Both cases are consistent with transformations of the types:

a) $M_P \longrightarrow -M_P$; $M_N \longrightarrow -M_N$ $\quad \Psi_{P_N} \gamma_5 \Psi_{P_N}$,

the other pairs of particles which interact with (P N) transforming with equal phases for (V,A) interactions, and with phases of <u>opposite</u> signs for (S,P.T) interactions (such as in (5),(6)).

b) $M_{P_N} \longrightarrow - M_{P_N}$; $\psi_{P_N} \longrightarrow \pm \gamma_5 \Psi_{P_N}$,

the other pairs of particles transforming with equal phases for (S,P,T) and with opposite phases for (V, A) interactions.

The extension to the case of non-Universal interaction is immediate. In this case we find that <u>all</u> Fermi interactions which correspond to the processes (7a,b,c) should be of the same type (either (V,A) or (S,P,T)) the only arbitrariness being in the coeficients of the linear combinations.

We see that the consequence of the invariance under mass reversal is to exclude interactions which lead to Fierz interferences [4], which appear when interactions of the group (A,V) are mixed with those of (S,P,T). This could be seen from the beginning by observing that these interference terms, say in β-decay, are linear in the mass of the electron; so they change sign under mass reversal for the (e,ν) field.

Now if we consider simultaneously the π-nucleon interaction and those which lead to processes (7) we may be led to a choice between (S,P,T) and (V,A) theories. Indeed, if we assume pseudoscalar coupling of π-mesons with both (P,N) and (μ, μ_0) then the transformations used for (P,N) imply the relations between the phases of (μ, μ_0) given by (5a) and (6a) which are correct, for the invariance of the Fermi interaction, only for the (S,P,T) theory). So a definite knowledge of the π meson interactions may lead to a choice in favour of (S,P.T) or (A,V) Fermi interaction.

IV. Unique determinations of the Universal Interaction.

Now that we are left with only (S,P,T) and (A,V) mixtures, we may attempt to a dd a further principle to obtain the coeficients of the linear combination. A natural assumption is to impose symmetry among the several particles. The complete symmetry which led to the Wigner-Critchfield theory is known to be unsatisfactory. Less strong symmetry properties have already been proposed by several authors. Thus Pursey [5] has proposed to assume symmetry between the anticommuting operators for the neutral particles (and then also for the charged ones). So he is led (in the usual ordering of β -decay interaction $\bar{\psi}_P \psi_N \bar{\psi}_e \psi_\nu$), to a linear combination of (S + P - T), (A - V) and (S - A - P). Now if we impose invariance under mass reversal we are restricted to either (S + P - T) or (A - V): A final selection between them could result as referred in sec. III.

This is, however, correct only if we assume anticommutation not only between the wave functions for two particles of the same pair (P,N) (μ, μ_0) (e, ν) as it is necessary for the Hamiltonian formalism but also between particles in different pairs. If we suppress the last restriction, say, if we assume that the wave operators for two particles in different pairs commute (say N and ν) then the interaction which is symmetrical between neutral particles is now a linear combination of [1] $(S + P + \frac{T}{3})$ and (2 S - 2P + A + V). Thus the principle of mass reversal leads us to the unique possibility, $(S + P + \frac{T}{3})$.

Pursey's symmetry principle can be expressed in a formal way by stating that the Universal Fermi interaction Hamiltonian is given by

$$H=\sum_{r,s}\sum_{k} G^{k} O^{k}_{\alpha\beta} O^{k}_{\gamma\delta}\left(\bar{\Psi}^{r}_{\alpha}\bar{\Psi}'^{s}_{\gamma}+\bar{\Psi}'^{s}_{\alpha}\bar{\Psi}^{r}_{\gamma}\right)\left(\varphi^{r}_{\beta}\varphi'^{s}_{\delta}+\varphi'^{s}_{\beta}\varphi^{r}_{\delta}\right)+C.C \quad (8)$$

where Ψ_r, φ_r are anihilation operators for the positive and neutral particles of the r^{th} kind. Ψ' and φ' are the charge conjugate operators. O_K represent the 5 kinds of covariant operators. According to whether Ψ_r and Ψ'_s anticommute or commute, we obtain Pursey's result or the other one referred to above.

Another possibile symmetry principle is that of "symmetry between particles and between antiparticles" (or between emitted particles and between absorbed particles). In this case we write:

$$H=\sum_{r,s}\sum_{k} G_{k} O^{k}_{\alpha\beta} O^{k}_{\gamma\delta}\left(\bar{\Psi}^{r}_{\alpha}\bar{\varphi}^{s}_{\gamma}+\bar{\varphi}^{s}_{\alpha}\bar{\Psi}^{r}_{\gamma}\right)\left(\varphi^{r}_{\beta}\Psi^{s}_{\delta}+\Psi^{s}_{\beta}\varphi^{r}_{\delta}\right)+C.C \quad (9)$$

Here we find that if Ψ_r anticommutes with φ_s then H is a linear combination of $(S + P - \frac{T}{3})$ and $(2S - 2P + A - V)$:

Thus, if we impose again the mass reversal invariance we are restricted to the 3 unique possibilities: $(S + P + T)$, $(A + V)$, in the intercommuting case, and $(S + P - \frac{T}{3})$ in the commuting.

Finally, we could impose symmetry between particles in the same pair:

$$H=\sum_{r,s}\sum_{k} G^{k} O^{k}_{\alpha\beta} O^{k}_{\gamma\delta}\left(\bar{\Psi}^{r}_{\alpha}\bar{\varphi}'^{s}_{\beta}\pm\bar{\varphi}'^{r}_{\alpha}\bar{\Psi}^{r}_{\beta}\right)\left(\varphi^{s}_{\gamma}\Psi'^{s}_{\delta}\pm\Psi'^{s}_{\gamma}\varphi^{s}_{\delta}\right)+C.C \quad (10)$$

In this case, as ψ_r and φ'_r anticommute necessarily, we find that H is of the type (S, P, A) if the plus sign is taken in (10), and (V,T) if we use the minus sign [6]. So the mass reversibility principle leads in this case to the mutually excluding possibilities: (S,P), A, V and T. In this case β^+ and β^- emission would be completely symmetrical [6] except for the Coulomb distortion. These interactions are, however, known to be unsatisfactory for β -decay.

1 Peaslee, D. C., Phys. Rev. 91, 1447, 1953:

2 A similar situation occurs for the interaction of nucleons with the electromagnetic field. Here, if we introduce in the Hamiltonian a Pauli term a $\gamma_{\mu\nu} F^{\mu\nu}$ together with the usual term i.e. $e\gamma^{\mu} A_{\mu}$ (for protons) we find that the theory cannot be made invariant under mass reversal. If we start, however, from a theory without the Pauli term but involving a meson field (in a way invariant under mass reversal) it is known that the Pauli term may result as a secondary effect.

3 Yang, C.N., and Tiomno, J. Phys. Rev. 79, 495, 1950.

4 Fierz, M., Zeits. Phys. 104, 553, 1947.

5 Pursey, D., Physica 18, 1017, 1952.

6 de Groot, S.R. and Tolhock, H.A., Physica 16, 456, 1950.