CBPF-NF-019/05

Superparticles with constrained generalized supersymmetries*

Zhanna Kuznetsova[a†] and *Francesco Toppan*[b‡]

[a]Dep. de Física, Universidade Estadual de Londrina,
Caixa Postal 6001, Londrina (PR), Brazil
[b]CBPF, Rua Dr. Xavier Sigaud 150,
cep 22290-180 Rio de Janeiro (RJ), Brazil

Abstract

This talk is based on the paper appeared in JHEP 0505 (2005) 060 [1], where a classification of the constrained complex generalized supersymmetries is presented. The generalized superparticle models (i.e., whose target superspaces are generalized supersymmetries) are formulated in arbitrary space-times. The consistency conditions for the constrained generalized complex superparticles are derived.

1 Introduction.

The elusive nature of the M-theory forces us to understand the role of the bosonic tensorial central charges appearing in the M-algebra and going beyond the Haag-Łopuszański-Sohnius scheme [2]. This is particularly true if we want to understand the dynamics of the non-minkowskian twelve-dimensional F-theory [3], based on the F-algebra presentation of the M-algebra, see e.g. [4], admitting only higher-rank bosonic tensors and no translations at all.

From this point of view, in order to understand this generalized dynamical setting, it is quite convenient to investigate at first the simplest classes of models that can be based on generalized supersymmetries. The generalized superparticles models fit nicely into this framework. It is worth recalling that the first theory of this kind was introduced by Rudychev-Sezgin [5] as a generalization of the Brink-Schwarz superparticle [6], in terms of a generalized supersymmetric target with extra, tensorial, bosonic coordinates. The [5] model was based on real spinors. Later, Bandos-Lukierski [7] analyzed a corresponding model for complex spinors. They surprisingly proved, see also [8], that the dynamical content of the four-dimensional superparticle model with six extra rank-two bosonic coordinates, describes a tower of higher helicity massless particles, making the physical implications of these theories, orginally regarded as toy-models, particularly deep.

In this talk we discuss several aspects of this class of models. We point out that they can be derived in a unified framework, dimensionally reduced models being obtained

*Talk delivered by Francesco Toppan as invited lecturer at the SQS'05 International Workshop, "Supersymmetries and Quantum Symmetries", July 27-31, 2005 held at JINR, Dubna, Russia.

†*e-mail: zhanna@cbpf.br*
‡*e-mail: toppan@cbpf.br*

from the associated oxidized (read, maximal) form of the generalized supersymmetries. Inequivalent models are specified in terms of the different admissible choices for the spinorial metric. Complex generalized supesymmetries can, finally, be consistently constrained. In various cases, these admissible algebraic constraints lead to admissible constraints on the Equations Of Motion of their associated complex generalized superparticles.

2 Constrained complex generalized supersymmetries.

A complex generalized supersymmetry algebra is expressed in terms of complex spinors Q_a and their complex conjugate ${Q^*}_{\dot{a}}$. The most general (with a saturated r.h.s.) algebra is in this case given by

$$\{Q_a, Q_b\} = \mathcal{P}_{ab} \qquad , \qquad \{{Q^*}_{\dot{a}}, {Q^*}_{\dot{b}}\} = {\mathcal{P}^*}_{\dot{a}\dot{b}}, \tag{2.1}$$

together with

$$\{Q_a, {Q^*}_{\dot{b}}\} \;=\; \mathcal{R}_{a\dot{b}}, \tag{2.2}$$

where the matrix $\mathcal{P}_{ab}$ (${\mathcal{P}^*}_{\dot{a}\dot{b}}$ is its conjugate and does not contain new degrees of freedom) is symmetric, while $\mathcal{R}_{a\dot{b}}$ is hermitian.

The maximal number of allowed components in the r.h.s. is given, for complex fundamental spinors with n complex components, by $n(n+1)$ (real) bosonic components entering the symmetric $n \times n$ complex matrix $\mathcal{P}_{ab}$ plus n^2 (real) bosonic components entering the hermitian $n \times n$ complex matrix $\mathcal{R}_{a\dot{b}}$.

The saturated r.h.s. are given by the most general combination of rank-k antisymmetric tensors which are either symmetric in the $a \leftrightarrow b$ exchange (they are constructed with the help of the charge conjugation matrix C) or hermitian (these tensors are constructed with the matrix A used to define barred spinors).

The following division-algebra compatible constraints can be imposed on both $\mathcal{P}$ and $\mathcal{R}$. We obtain the table, whose entries specify the total number of bosonic components (in the real counting), while the columns represent the restrictions on $\mathcal{R}$ and the rows the restrictions on $\mathcal{P}$ (an imaginary condition on $\mathcal{P}$ is equivalent to the reality condition and therefore is not reported)

$\mathcal{P}\backslash\mathcal{R}$	1) *Full*	2) *Real*	3) *Imag.*	4) *Abs.*
a) *Full*	$2n^2+n$	$\frac{3}{2}(n^2+n)$	$\frac{1}{2}(3n^2+n)$	n^2+n
b) *Real*	$\frac{1}{2}(3n^2+n)$	n^2+n	n^2	$\frac{1}{2}(n^2+n)$
c) *Abs.*	n^2	$\frac{1}{2}(n^2+n)$	$\frac{1}{2}(n^2-n)$	0

(2.3)

Some comments are in order. The above list of constraints is not necessarily implemented for any given supersymmetric dynamical system. One should check, e.g., that the above restrictions are indeed compatible with the equations of motion. On a purely algebraic basis, however, they are admissible restrictions which require a careful investigation.

One can notice that certain numbers appear twice as entries in the above table. This is related with the fact that the same constrained superalgebra can admit a different, but equivalent, presentation. We refer to these equivalent presentations as "dual formulations" of the constrained supersymmetries. It is worth stressing that in application to dynamical systems, which need more data than just superalgebraic data, one should explicitly verify whether the above related constraints indeed lead to equivalent theories.

The inequivalent constrained generalized supersymmetries can be listed as follows

I	$(a1)$	$2n^2+n,$	$k=3,$	$l=1$
II	$(a2)$	$\frac{3}{2}(n^2+n),$	$k=3,$	$l=0$
III	$(a3\,\&\,b1)$	$\frac{1}{2}(3n^2+n),$	$k=2,$	$l=1$
IV	$(a4\,\&\,b2)$	$n^2+n,$	$k=2,$	$l=0$
V	$(b3\,\&\,c1)$	$n^2,$	$k=1,$	$l=1$
VI	$(b4\,\&\,c2)$	$\frac{1}{2}(n^2+n),$	$k=1,$	$l=0$
VII	$(c3)$	$\frac{1}{2}(n^2-n),$	$k=0,$	$l=1$

(2.4)

The integral numbers k,l have the following meaning. For the given constrained supersymmetry the bosonic r.h.s. can be presented in the following form

$$Z \;=\; kX+lY, \quad k=0,1,2,3, \quad l=0,1, \tag{2.5}$$

where X and Y denote the bosonic sectors associated with the VI and respectively VII constrained supersymmetry.

In association with the maximal Clifford algebras in D-dimensional spacetimes (with no dependence on their signature), the X and Y bosonic sectors are given by the following set of rank-k antisymmetric tensors

	X	Y
$D=3$	M_1	M_0
$D=5$	M_2	M_0+M_1
$D=7$	M_0+M_3	M_1+M_2
$D=9$	$M_0+M_1+M_4$	M_2+M_3
$D=11$	$M_1+M_2+M_5$	$M_0+M_3+M_4$
$D=13$	$M_2+M_3+M_6$	$M_0+M_1+M_4+M_5$

(2.6)

Formula (2.5) specifies the admissible class of division-algebra related, constrained bosonic sectors.

3 Superparticles with tensorial central charges.

The most general action S involving real spinors is constructed in terms of the real superspace coordinates X^{ab}, Θ^a conjugated to the superalgebra generators $\mathcal{Z}_{ab}$ and Q_a [5] (X^{ab} is symmetric in the $a\leftrightarrow b$ exchange). We have

$$S \;=\; \frac{1}{2}\int d\tau tr\left[\mathcal{Z}\cdot\Pi - e(\mathcal{Z})^2\right], \tag{3.7}$$

where

$$\Pi^{ab} = dX^{ab} - \Theta^{(a}d\Theta^{b)}, \tag{3.8}$$

while e^{ab} denotes the Lagrange multipliers whose (anti)symmetry property is the same as the one of the charge conjugation matrix C^{ab}, i.e.

$$e^T = \varepsilon e \qquad for \qquad C^T = \varepsilon C. \tag{3.9}$$

By construction

$$(\mathcal{Z})^2{}_{ab} = \mathcal{Z}_{ac}C^{cd}\mathcal{Z}_{db}, \tag{3.10}$$

namely the charge conjugation matrix is used as a metric to raise and lower spinorial indices.

The massless constraint

$$(\mathcal{Z})^2_{ab} = 0 \tag{3.11}$$

is obtained from the variation δe^{ab} of the Lagrange multipliers.

A symmetric charge conjugation matrix ($\varepsilon = 1$) allows us [5] to construct a massive model by simply performing a shift $\mathcal{Z} \to \mathcal{Z} + mC$ in the action (3.7).

In order to introduce the action for the superparticle with complex spinors we mimick, as much as possible, the real formulation. The bosonic matrix $\mathcal{Z}_{ab}$ is now replaced by the pair of matrices $\mathcal{P}_{ab}$ and $\mathcal{R}_{a\dot{b}}$ (respectively symmetric and hermitian) entering (2.1) and (2.2). They can be accommodated in a symmetric matrix $\mathbf{P}$ ($\mathbf{P}^T = \mathbf{P}$) as follows

$$\mathbf{P} = \begin{pmatrix} \mathcal{P} & \mathcal{R} \\ \mathcal{R}^* & \mathcal{P}^* \end{pmatrix}. \tag{3.12}$$

The supercoordinates conjugated to $\mathcal{P}_{ab}$, $\mathcal{R}_{a\dot{b}}$, Q_a and ${Q^*}_{\dot{a}}$ are given by X^{ab}, $Y^{a\dot{b}}$, Θ^a and $\Theta^{*\dot{a}}$.

It is convenient to use the notation

$$\mathbf{\Pi} = \begin{pmatrix} dX - \Theta d\Theta & dY - \Theta d\Theta^* \\ dY^* - \Theta^* d\Theta & dX^* - \Theta^* d\Theta^* \end{pmatrix}. \tag{3.13}$$

We will also need the matrix

$$\mathbf{P}^2 = \mathbf{P}\mathcal{C}\mathbf{P}, \tag{3.14}$$

whose indices are raised by the metric $\mathcal{C}$. There are three possible choices for $\mathcal{C}$, given by

i)

$$\mathcal{C} = \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix}, \tag{3.15}$$

in this case $\mathcal{C}$ is (anti)symmetric in accordance with the sign of ϵ;

ii)

$$\mathcal{C} = \left(\begin{array}{cc} 0 & A \\ \xi A^* & 0 \end{array} \right), \tag{3.16}$$

where ξ is an arbitrary sign ($\xi = \pm 1$); in this case the (anti)symmetry property of $\mathcal{C}$ is specified by the sign of $\delta\xi$;

iii)

$$\mathcal{C} = \left(\begin{array}{cc} C & A \\ \epsilon\delta A^* & C^* \end{array} \right), \tag{3.17}$$

the (anti)symmetry property of $\mathcal{C}$ is specified by the sign of ϵ. It should be noticed that in this last case an (anti)symmetric matrix $\mathbf{P}^2$ ($\mathbf{P}^2 = \mathbf{P}\mathcal{C}\mathbf{P}$) is only possible, for both non-vanishing $\mathcal{P}$, $\mathcal{R}$ entering $\mathbf{P}$, if the condition

$$\epsilon = \delta \tag{3.18}$$

is matched.

The (anti)-symmetry property of $\mathbf{P}^2$ coincides with the (anti)-symmetry property of $\mathcal{C}$.

The Lagrange multipliers enter a matrix

$$\mathbf{E} = \left(\begin{array}{cc} e & f \\ g & h \end{array} \right). \tag{3.19}$$

In general, for any $\mathbf{U}$ (for our purposes $\mathbf{U} \equiv \mathbf{P}^2$) s.t.

$$\mathbf{U} = \left(\begin{array}{cc} U & V \\ \lambda\mu V^* & U^* \end{array} \right) \tag{3.20}$$

with $U^T = \lambda U$, $V^\dagger = \mu V$ (therefore $\mathbf{U}^T = \lambda \mathbf{U}$), the reality of the term $tr(\mathbf{EU})$ requires

$$\begin{aligned} g &= \lambda\mu f^*, \\ h &= e^*. \end{aligned} \tag{3.21}$$

A reality (imaginary) condition imposed on either $\mathbf{U}$ or $\mathbf{V}$ implies a reality (imaginary) condition for the lagrange multipliers e and f respectively.

We are now in the position to write the action S for the superparticle with bosonic tensorial central charges and complex spinors as

$$S = \frac{1}{2}\int d\tau tr \left[\mathbf{P\Pi} - \mathbf{E}(\mathbf{P})^2\right]. \tag{3.22}$$

As in the real case, a massive model can be introduced in correspondence of a symmetric $\mathcal{C}$ through the shift $\mathbf{P} \rightarrow \mathbf{P} + m\mathcal{C}$ in the action (3.22).

4 Constrained complex superparticles with tensorial central charges.

In the previous Section we formulate the complex generalized superparticle model. It is clear at this point that we can investigate whether its equations of motion are compatible with the constraints on complex generalized supersymmetries discussed in Section **2**. This investigation should be performed for each one of the three available choices for the spinorial metric $\mathcal{C}$. As a necessary condition for the consistency of the theory, the number of lagrange multipliers constraints should not exceed the number of bosonic degrees of freedom entering $\mathcal{P}$ and $\mathcal{R}$.

The complete list of results, which we cannot report here for lack of space, has been furnished in [1]. Here we limit ourselves to mention that the constraints II and III of (2.4) are never compatible with the equations of motion of the (constrained) generalized complex superparticles. The remaining constraints, on the other hand, can be imposed for suitable values of the ϵ, δ, ξ signs entering the construction of the model, as discussed in Section **3**. For "generic" values of the space-time we obtain the following table which reports the set of consistent constraints for the allowed choices of the metric $\mathcal{C}$

	i	ii	iii
I	yes	yes	yes
$IV\,(a4)$	yes	yes	no
$IV\,(b2)$	yes	yes	$yes^*\,(\epsilon=1)$
$V\,(b3)$	yes	yes	$yes^*\,(\epsilon=1)$
$V\,(c1)$	yes	yes	no
$VI\,(b4)$	$yes^*\,(\epsilon=-1)$	yes	no
$VI\,(c2)$	$yes^*\,(\epsilon=-1)$	yes	no
VII	$yes^*\,(\epsilon=-1)$	yes	no

(4.23)

The "$*$" denotes which choices are consistent only for a specific value of ϵ.

The above result is the starting point for investigating the consequences of the constrained generalized supersymmetries in a dynamical setting. The importance of (one class of) constrained generalized supersymmetries was noticed in [9]. It was proven that they are required in order to perform the functional quantization of any model constructed with the minkowskian M-algebra.

References

[1] Z. Kuznetsova and F. Toppan, JHEP 0505 (2005) 060.

[2] R. Haag, J. Łopuszański and M. Sohnius, Nucl.Phys. **B 88** (1975) 257.

[3] C. Vafa, Nucl. Phys. **B 469** (1996) 403.

[4] F. Toppan, JHEP 09 (2004) 016.

[5] E. Sezgin and I. Rudychev, "Superparticles, p-form coordinates and the BPS condition", hep-th/9711128.

[6] L. Brink and J. Schwarz, Phys. Lett. **B 100** (1981) 310.

[7] I. Bandos and J. Lukierski, Mod. Phys. Lett. **A 14** (1999) 1257.

[8] I. Bandos, J. Lukierski and D. Sorokin, Phys. Rev. **D 61** (2000) 045002.

[9] J. Lukierski and F. Toppan, Phys. Lett. **B 584** (2004) 315.